SCATTERING OF ELECTROMAGNETIC WAVES
WILEY SERIES IN REMOTE SENSING

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ADVANCED TOPICS
To my family, Hannah, Clarisse, and Kaleb for their love.

— L. Tsang

To my family.

— J. A. Kong
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Scattering of Electromagnetic Waves

Volume I: Theories and Applications (Tsang, Kong, and Ding)
Volume II: Numerical Simulations (Tsang, Kong, Ding, and Ao)
Volume III: Advanced Topics (Tsang and Kong)
PREFACE

Electromagnetic wave scattering is an active, interdisciplinary area of research with myriad practical applications in fields ranging from atomic physics to medical imaging to geoscience and remote sensing. In particular, the subject of wave scattering by random discrete scatterers and rough surfaces presents great theoretical challenges due to the large degrees of freedom in these systems and the need to include multiple scattering effects accurately. In the past three decades, considerable theoretical progress has been made in elucidating and understanding the scattering processes involved in such problems. Diagrammatic techniques and effective medium theories remain essential for analytical studies; however, rapid advances in computer technology have opened new doors for researchers with the full power of Monte Carlo simulations in the numerical analysis of random media scattering. Numerical simulations allow us to solve the Maxwell equations exactly without the limitations of analytical approximations, whose regimes of validity are often difficult to assess. Thus it is our aim to present in these three volumes a balanced picture of both theoretical and numerical methods that are commonly used for tackling electromagnetic wave scattering problems. While our book places an emphasis on remote sensing applications, the materials covered here should be useful for students and researchers from a variety of backgrounds as in, for example, composite materials, photonic devices, optical thin films, lasers, optical tomography, and X-ray lithography. Introductory chapters and sections are also added so that the materials can be readily understood by graduate students. We hope that our book would help stimulate new ideas and innovative approaches to electromagnetic wave scattering in the years to come.

The increasingly important role of numerical simulations in solving electromagnetic wave scattering problems has motivated us to host a companion web site that contains computer codes on topics relevant to the book. These computer codes are written in the MATLAB programming language and are available for download from our web site at www.emwave.com. They are provided to serve two main purposes. The first is to supply our readers a hands-on laboratory for performing numerical experiments, through which the concepts in the book can be more dynamically relayed. The second is to give new researchers a set of basic tools with which they could quickly build on projects of their own. The fluid nature of the web site would also allow us to regularly update the contents and keep pace with new research developments.

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The present volume deals with multiple scattering and analytic wave theories. It also contains in-depth discussion of topics introduced in Volumes I and II. In Volume I, the small perturbation method (SPM) and Kirchhoff approach (KA) were introduced for wave scattering by one-dimensional random rough surfaces. In Chapters 1 and 2, these methods are extended to electromagnetic scattering by two-dimensional dielectric random rough surfaces. In recent years, there has been a great deal of theoretical progress aimed at bridging the parametric gap between the applicability of SPM and KA. The phase perturbation method bears much similarity to SPM with the important difference that the perturbation series is made in the exponent. It successfully reduces to SPM and KA in their respective regimes. On the other hand, some rough surfaces, e.g. ocean surfaces, are better characterized by small rapidly varying roughness superimposed on a smoothly undulating surface. The two-scale, or composite surface, model is used to compute emissivities from rough ocean surfaces.

Radiative transfer (RT) equation is usually derived heuristically by considering the scattering and attenuation of specific intensity in an elemental volume. However, at a more fundamental level, the fields satisfy the wave equation. Thus it is more rigorous to start with analytic wave theories and try to derive the RT equation from there. Before doing so, we consider in Chapter 3 a simple volume scattering model where scattering by a thick layer is replaced by a cascading of thin layers consisting of point scatterers. It is shown that the Foldy's approximation and the RT equation in difference form can be derived by this simple model. The cascading layer approach illustrates in a more intuitive way the role of multiple scattering in a thick layer and helps us understand when RT type equation is expected to be valid.

In Chapter 4, analytic wave theories are developed using diagrammatic expansion techniques. We derive Dyson's equation for the mean field and the Bethe-Salpeter equation for the field covariance. Practical computations usually require approximations be made to these equations. We show that the ladder approximation for the Bethe-Salpeter equation and nonlinear approximation for Dyson's equation are consistent with energy conservation. We also introduce the strong permittivity fluctuation theory where by properly extracting the singularity of the dyadic Green's function, the bilocally-approximated Dyson equation is made applicable for random medium with large permittivity fluctuations.
In Chapter 5, multiple scattering equations for random discrete scatterers are derived using the operator formalism. We discuss approximations of the multiple scattering equations through conditional averaging. This results in effective medium theories such as Foldy's approximation and the quasicrystalline approximation (QCA). The method of coherent potential (CP) can be used in conjunction with these approximations to improve the results. QCA can also be conveniently formulated using the T-matrix multiple scattering approach introduced in Volume II. This is discussed in Chapter 6. We develop a QCA based dense medium radiative transfer (DMRT) theory and apply it to remote sensing problems.

In Chapter 7, the DMRT equation is derived from Dyson's equation with QCA-CP and the Bethe-Salpeter equation with the correlated ladder approximation. These approximations are shown to be consistent with energy conservation. Active and passive remote sensing applications are provided for illustrations. Finally, in Chapter 8, we address the interesting phenomenon of backscattering enhancement through multiple scattering theories for both isotropic and anisotropic scatterers.

Acknowledgments

We would like to acknowledge the collaboration with our colleagues and graduate students. In particular, we wish to thank Professor Chi Chan of City University of Hong Kong, Professor Joel T. Johnson of Ohio State University, Dr. Robert T. Shin of MIT Lincoln Laboratory, and Dr. Dale Winebrenner of University of Washington. The graduate students who completed their Ph.D. theses from the University of Washington on random media scattering include Boheng Wen (1989), Kung-Hau Ding (1989), Shu-Hsiang Lou (1991), Charles E. Mandt (1992), Richard D. West (1994), Zhengxiao Chen (1994), Lisa M. Zurk (1995), Kyung Pak (1996), Guifu Zhang (1998), and Qin Li (2000). Much of their dissertation works are included in this book. Financial supports from the Air Force Office of Scientific Research, Army Research Office, National Aeronautics and Space Administration, National Science Foundation, Office of Naval Research, and Schlumberger-Doll Research Center for research materials included in this book are gratefully acknowledged. We also want to acknowledge the current UW graduate students who have helped to develop the numerical codes used throughout this book. These include to Chi-Te Chen, Houfei Chen, Jianjun Guo, Chung-Chi Huang, and Lin Zhou. Special thanks are due to Kung-Hau Ding for careful
proofreading of the manuscript and Bae-Ian Wu for production assistance. We would also like to thank Chi On Ao for his help in editing and typsetting the manuscript.

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February 2001
SCATTERING OF ELECTROMAGNETIC WAVES
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Chapter 1
TWO-DIMENSIONAL RANDOM ROUGH SURFACE SCATTERING BASED ON SMALL PERTURBATION METHOD

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In Volume I, we have studied the small perturbation method (SPM) for one-dimensional surface. The small perturbation method is valid for small rms height and small slope. Over the years, SPM has found extensive applications. In this chapter, we apply SPM to three-dimensional problem with two-dimensional random rough surfaces. The analyses are performed up to second order. Second order calculations are important to ensure energy conservation and also to calculate cross-polarization. We also give detailed derivation of the second order solution of dielectric surfaces. The formulation of SPM is based on extinction theorem.

The most common analytical treatments of scattering by random rough surfaces are the Kirchhoff approach (KA) and the small perturbation method (SPM). Improvements upon these two methods include the phase perturbation method [Winebrenner and Ishimaru, 1985a,b; Broschat et al. 1988], second order Kirchhoff method [Ishimaru and Chen, 1990, 1991]. Integral equation method (IEM) [Fung, 1994; Chen and Fung, 1995; Fung and Pan, 1987; Chen et al. 2000], Wiener method [Ito, 1985; Ogura and Takahashi, 1995], small slope approximation (SSA) [Broschat, 1993; Voronovich, 1994a,b; Thorsos and Broschat, 1995]. For the case of absorption and emissivity calculations, it was shown that the results of SSA and SPM are identical for the half-space case [Irisov, 1994, 1997]. Two-scale model is used to superimpose large scale roughness and small scale roughness [Plant, 1986; Yueh, 1997; Voronovich, 1996; Durden and Vesecky, 1985, 1990]. Other notable methods include the Feynman diagrammatic method [DeSanto, 1974; DeSanto and Shisha, 1974; DeSanto and Wombell, 1991], full wave methods [Collin, 1994; Bahar and Fitzwater, 1989] as well as the mean field theory [Sentenac and Greffet, 1998; Greffet and Nieto-Vesperinas, 1998].

In this chapter, we use the small perturbation method for electromagnetic wave scattering by two-dimensional random rough surfaces. In the next chapter, the Kirchhoff approach, the phase perturbation method, and the two-scale model are discussed.

1 Electromagnetic Wave Scattering by a Perfect Electric Conductor

Consider a plane electromagnetic wave

\[ \mathbf{E}_i = \hat{e}_i \exp(ik_{ix} x + ik_{iy} y - ik_{iz} z) \]  

(1.1.1)

incident upon a rough surface of perfect electric conductor. In (1.1.1), \( k_{ix} = k \sin \theta_i \cos \phi_i \), \( k_{iy} = k \sin \theta_i \sin \phi_i \), and \( k_{iz} = k \cos \theta_i \). We also have \( k_{ip} = k \sin \theta_i \). The rough surface is characterized by a random height function
Electromagnetic Wave Scattering by a Perfect Electric Conductor

Figure 1.1.1: Electromagnetic scattering by a 2-D perfectly conducting rough surface. $z = f(x, y)$, where $f(x, y)$ is a random function with zero mean ($f(x, y) = 0$). Let $f_{\text{min}}$ and $f_{\text{max}}$ be the minimum and maximum values, respectively, of the surface profile $f(x,y)$.

From Huygens' principle, the extinction theorem, and setting the tangential electric field equal to zero, we have

$$E_{\text{tan}}(x, y) = 0$$

where $r_{\text{tan}}$ denotes vector $x\hat{i} + y\hat{j}$ in $x$-$y$ plane.

We can define surface field unknowns. For $z' = f(x', y')$, the tangential magnetic field is defined as

$$H_{\text{tan}}(x, y) = 0$$

We make use of the integral representation of dyadic Green's function as given in equation (2.1.20) of Volume I. For $z < f_{\text{min}}$, we use the lower half of (1.1.2) to get

Equation (1.1.4) is the extended boundary condition and can be used to solve for the surface field $a(r_{\text{tan}})$. Since $n_x H$ is tangential to the surface, we also have the condition

Using the definition of the normal vector $n(x', y')$, we have
After the surface field is evaluated, the scattered field is, for \( z > f_{\text{max}} \), using the plane wave representation of Green's function in equation (2.1.20) of Volume I.

\[
E_s(\vec{r}) = -\frac{1}{8\pi^2} \int d\vec{k}_\perp e^{i\vec{k}_\perp \cdot \vec{r}_\perp} e^{ik_z z} \int d\vec{r}' e^{-i\vec{k}_\perp \cdot \vec{r}'_\perp} e^{-ik_z f(\vec{r}'_z)} \\
\times \left[ \hat{e}(k_z)\hat{e}(k_z) + \hat{h}(k_z)\hat{h}(k_z) \right] \cdot \vec{a}(\vec{r}'_\perp)
\]  

Equation (1.1.7) shows that the scattered wave consists of a spectrum of plane waves. The incident wave, on the other hand, as represented by (1.1.1), consists of a plane wave in a single direction. Equations (1.1.4) through (1.1.7) are exact. We shall solve (1.1.4) and (1.1.7) by the perturbation method to the second order. A higher-order solution can be calculated in a similar manner with more complicated algebra.

To solve for the surface fields, the perturbation method makes use of series expansions. Let

\[
\vec{a}(\vec{r}'_\perp) = \sum_{m=0}^{\infty} \vec{a}^{(m)}(\vec{r}'_\perp) = \vec{a}^{(0)}(\vec{r}'_\perp) + \vec{a}^{(1)}(\vec{r}'_\perp) + \vec{a}^{(2)}(\vec{r}'_\perp) + \cdots
\]  

where \( \vec{a}^{(m)} \) is the \( m \)th-order solution of \( \vec{a} \). We also have

\[
e^{\pm ik_z f(\vec{r}'_\perp)} = \sum_{m=0}^{\infty} [\pm ik_z f(\vec{r}'_\perp)]^m
\]  

\[
\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \ll 1
\]  

Equation (1.1.11) is a small slope approximation because the assumption of the smallness of the slope is used to balance the order of solutions. That is,

As a result of the small slope approximation, we note that the \( \hat{z} \) component of the surface field \( \vec{a} \) is one order lower than the \( \hat{x} \)- and \( \hat{y} \)-components. We define the Fourier transform of the surface field \( \vec{a} \) by

\[
\overline{A}(\vec{q}_\perp) = \frac{1}{(2\pi)^2} \int d\vec{r}'_\perp \overline{a}(\vec{r}'_\perp) e^{-i\vec{q}_\perp \cdot \vec{r}'_\perp}
\]  

Assuming small height, we obtain

\[
|k_z f| \ll 1
\]
We note that both the extinction theorem of (1.1.4) and the scattered field of (1.1.7) are of similar form. On expansion to the second order,

\[
\int d\vec{r}_\perp \, \vec{a}(\vec{r}_\perp) \, e^{-i\vec{k}_\perp \cdot \vec{r}_\perp \pm i k_z f(\vec{r}_\perp)}
= \int d\vec{r}_\perp \, \vec{a}(\vec{r}_\perp) \, e^{-i\vec{k}_\perp \cdot \vec{r}_\perp} \left[ 1 \pm ik_z f(\vec{r}_\perp) - \frac{k_z^2 f^2(\vec{r}_\perp)}{2} \right]
= 4\pi^2 \left\{ A(\vec{k}_\perp) \pm i k_z \int d\vec{k}_\perp A(\vec{k}_\perp) F(\vec{k}_\perp - \vec{k}_\perp') \right.
\left. - \frac{k_z^2}{2} \int d\vec{k}_\perp A(\vec{k}_\perp) F^{(2)}(\vec{k}_\perp - \vec{k}_\perp') \right\} \tag{1.1.15}
\]

where \( F(\vec{k}_\perp) \) is the Fourier transform of \( f(\vec{r}_\perp) \) and \( F^{(2)}(\vec{k}_\perp) \) is the Fourier transform of \( f^2(\vec{r}_\perp) \), that is,

\[
F^{(2)}(\vec{k}_\perp) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} d\vec{r}_\perp \, e^{-i\vec{k}_\perp \cdot \vec{r}_\perp} f^2(\vec{r}_\perp) \tag{1.1.16}
\]

In (1.1.15), the + sign in the exponent corresponds to the extinction theorem and the − sign corresponds to the scattered field.

If we use (1.1.15), then (1.1.4) and (1.1.7) assume the following forms. For \( z < f_{\text{min}} \) we have

\[
\vec{E}_i(\vec{r}) = \hat{e}_i \exp(i k_{ix} x + i k_{iy} y - i k_{iz} z)
= \frac{1}{2} \int \, d\vec{k}_\perp \, e^{i \vec{k}_\perp \cdot \vec{r}_\perp - i k_{iz} z} \frac{k_z}{k_z^2} \left[ \hat{e}(-k_z) \hat{e}(-k_z) + \hat{h}(-k_z) \hat{h}(-k_z) \right]
\cdot \left[ \overline{A}(\vec{k}_\perp) + i k_z \int \, d\vec{k}_\perp \, \overline{A}(\vec{k}_\perp) F(\vec{k}_\perp - \vec{k}_\perp') \right.
\left. - \frac{k_z^2}{2} \int \, d\vec{k}_\perp \, \overline{A}(\vec{k}_\perp) F^{(2)}(\vec{k}_\perp - \vec{k}_\perp') \right] \tag{1.1.17}
\]

and for \( z > f_{\text{max}} \) we have

\[
\vec{E}_s(\vec{r}) = -\frac{1}{2} \int \, d\vec{k}_\perp \, e^{i \vec{k}_\perp \cdot \vec{r}_\perp + i k_{iz} z} \frac{k_z}{k_z^2} \left[ \hat{e}(k_z) \hat{e}(k_z) + \hat{h}(k_z) \hat{h}(k_z) \right]
\cdot \left[ \overline{A}(\vec{k}_\perp) - i k_z \int \, d\vec{k}_\perp \, \overline{A}(\vec{k}_\perp) F(\vec{k}_\perp - \vec{k}_\perp') \right.
\left. - \frac{k_z^2}{2} \int \, d\vec{k}_\perp \, \overline{A}(\vec{k}_\perp) F^{(2)}(\vec{k}_\perp - \vec{k}_\perp') \right] \tag{1.1.18}
\]

We use (1.1.17) to solve for the unknown \( \overline{A}(\vec{k}_\perp) \) in a perturbative manner. Then we use (1.1.10) to calculate the scattered fields. Taking the Fourier
transform of (1.1.17) gives the simple relation
\[ \hat{e}_i \delta(k_{\perp} - k_{i,\perp}) = \frac{1}{2} \frac{k}{k_z} [\hat{e}(-k_z)\hat{e}(-k_z) + \hat{h}(-k_z)\hat{h}(-k_z)] \]
\[ \cdot \left[ A(k_{\perp}) + i k_z \int d\bar{k}_{\perp}' A(k_{\perp}')F(k_{\perp} - k_{i,\perp}') - \frac{k_z^2}{2} \int d\bar{k}_{\perp}' A(k_{\perp}')F(2)(k_{\perp} - k_{i,\perp}') \right] \] (1.1.19)

Taking (1.1.11) in spectral domain gives
\[ A^{(m)}(k_{\perp}) = i \int d\bar{k}_{\perp}' F(k_{\perp} - k_{i,\perp}')(k_{\perp} - k_{i,\perp}') \cdot A^{(m-1)}(k_{\perp}') \] (1.1.20)

Equations (1.1.19) and (1.1.20) are the two equations which can be solved by balancing terms to the zeroth, first, and second orders. It is convenient to form an orthonormal vector system to calculate surface fields. Define
\[ k_{\perp} = k_x \hat{x} + k_y \hat{y} = k_\rho \cos \phi_k \hat{x} + k_\rho \sin \phi_k \hat{y} \] (1.1.21a)
\[ \hat{q}(k_{\perp}) = \hat{e}(-k_z) = \frac{\hat{x} k_y - \hat{y} k_x}{k_\rho} = \hat{x} \sin \phi_k - \hat{y} \cos \phi_k \] (1.1.21b)
\[ \hat{p}(k_{\perp}) = \hat{z} \times \hat{q}(k_{\perp}) = \frac{\hat{x} k_x + \hat{y} k_y}{k_\rho} = \cos \phi_k \hat{x} + \sin \phi_k \hat{y} = \hat{k}_{\perp} \] (1.1.21c)
so that
\[ \hat{h}(\pm k_z) = \pm \frac{k_z}{k_\rho} \hat{p}(k_{\perp}) + \frac{k_\rho}{k_\rho} \hat{z} = \pm \frac{k_z}{k_\rho} (\cos \phi_k \hat{x} + \sin \phi_k \hat{y}) + \frac{k_\rho}{k_\rho} \hat{z} \] (1.1.22)

Thus we use two orthonormal systems, \((\hat{h}, \hat{e}, \hat{k})\) and \((\hat{z}, \hat{q}, \hat{p})\). The relation is \(\hat{q} = \hat{e}\). Also \(\hat{p}\) is the unit vector denoting the projection of \(\hat{k}\) on the \(x-y\) plane. Hence \(\hat{p}\) is the direction vector of the projection of \(\hat{h}(-k_z)\) in \(x-y\) plane. We let
\[ k_x = k_\rho \cos \phi_k \] (1.1.22a)
\[ k_y = k_\rho \sin \phi_k \] (1.1.22b)
\[ k_x' = k_\rho' \cos \phi_k' \] (1.1.22c)
\[ k_y' = k_\rho' \sin \phi_k' \] (1.1.22d)
then
\[ \hat{e}(\pm k_z) \cdot \hat{q}(k_{\perp}') = \frac{k_y k_y' + k_x k_x'}{k_\rho k_\rho'} = \cos(\phi_k - \phi_k') \] (1.1.22e)
\[ \hat{e}(\pm k_z) \cdot \hat{p}(k_{\perp}') = \frac{k_y k_x' - k_x k_y'}{k_\rho k_\rho'} = \sin(\phi_k - \phi_k') \] (1.1.22f)
### §1.1 Zeroth- and First-Order Solutions

#### Zeroth-Order Solution

Balancing (1.1.19) and (1.1.20) to the zeroth order, we obtain

\[ A_z^{(0)}(\overline{k}_\perp) = 0 \]  

(1.1.23)

\[ \hat{e}_i \delta(\overline{k}_\perp - \overline{k}_{i\perp}) = \frac{1}{2k_z} \left[ \hat{e}(-k_z)\hat{e}(-k_z) + \hat{h}(-k_z)\hat{h}(-k_z) \right] \cdot \overline{A}_\perp^{(0)}(\overline{k}_\perp) \]  

(1.1.24)

Solution of (1.1.24) can easily be calculated in the \((\hat{q}, \hat{p}, \hat{z})\) system defined in (1.1.21) and (1.1.22). Let

\[ \hat{q}_i = \hat{q}(\overline{k}_{i\perp}) \]  

(1.1.25)

\[ \hat{p}_i = \hat{p}(\overline{k}_{i\perp}) \]  

(1.1.26)

be the \(\hat{q}\) and \(\hat{p}\) for the incident direction. Then the solution of (1.1.24) is

\[ \overline{A}_\perp^{(0)}(\overline{k}_\perp) = (\hat{q}_i a_q^{(0)} + \hat{p}_i a_p^{(0)}) \delta(\overline{k}_\perp - \overline{k}_{i\perp}) \]  

(1.1.27)

where

\[ a_q^{(0)} = 2\hat{e}(-k_{iz}) \cdot \hat{e}_i \frac{k_{iz}}{k} \]  

(1.1.28a)

\[ a_p^{(0)} = 2\hat{h}(-k_{iz}) \cdot \hat{e}_i \]  

(1.1.28b)

The Dirac delta function in (1.1.27) indicates that the zeroth-order surface field consists of only a single spectral component that corresponds to specular reflection. Substituting (1.1.27) in (1.1.18) gives the zeroth-order solution of the scattered field as

\[ \overline{E}_s^{(0)}(\overline{\tau}) = \left\{ -\hat{e}(k_{iz})\left(\hat{e}_i \cdot \hat{e}(-k_{iz})\right) + \hat{h}(k_{iz})\left(\hat{e}_i \cdot \hat{h}(-k_{iz})\right) \right\} e^{i\overline{k}_{i\perp} \cdot \overline{\tau}_\perp + ik_{iz}z} \]  

(1.1.29)

which is the response of a flat surface.
First-Order Solution

Balancing (1.1.19) to the first order gives

\[ 0 = \left\{ \hat{e}(-k_z)\hat{e}(-k_z) + \hat{h}(-k_z)\hat{h}(-k_z) \right\} \cdot \left[ \overline{A}^{(1)}(\vec{k}_\perp) + ik_z \int d\vec{k}'_\perp \overline{A}^{(0)}(\vec{k}'_\perp)F(\vec{k}_\perp - \vec{k}'_\perp) \right] \]  

(1.1.30)

Note that (1.1.30) has only two independent components since it states that the projection of the vector in the square bracket onto the two polarization directions are equal to zero. The third component can be obtained by balancing (1.1.20) to the first order that gives

\[ A^{(1)}_z(\vec{k}_\perp) = i \int d\vec{k}'_\perp F(\vec{k}_\perp - \vec{k}'_\perp)(\vec{k}_\perp - \vec{k}'_\perp) \cdot \overline{A}^{(0)}(\vec{k}'_\perp) \]  

(1.1.31)

From (1.1.27) and (1.1.31), we have

\[ A^{(1)}_z(\vec{k}_\perp) = iF(\vec{k}_\perp - \vec{k}_i\perp)(\vec{k}_\perp - \vec{k}_i\perp) \cdot \left( \hat{q}_i a_q^{(0)} + \hat{p}_i a_p^{(0)} \right) \]

\[ = iF(\vec{k}_\perp - \vec{k}_i\perp) \left\{ -k_\rho \sin(\phi_k - \phi_i)a_q^{(0)} \right. \]

\[ + \left. [k_\rho \cos(\phi_k - \phi_i) - k_{i\rho}] a_p^{(0)} \right\} \]  

(1.1.32)

To solve (1.1.30), let

\[ \overline{A}^{(1)}(\vec{k}_\perp) = A^{(1)}_q(\vec{k}_\perp)\hat{q}(\vec{k}_\perp) + A^{(1)}_p(\vec{k}_\perp)\hat{p}(\vec{k}_\perp) + A^{(1)}_z(\vec{k}_\perp)\hat{z} \]  

(1.1.33)

Note that in here we use \( \hat{q}(\vec{k}_\perp) \) and \( \hat{p}(\vec{k}_\perp) \) as basis vectors. Previously [Shin, 1984; Tsang et al. 1985], the representation of \( \overline{A}^{(1)}(\vec{k}_\perp) \) was made differently using \( \hat{q}_i \) and \( \hat{p}_i \) as basis vectors. The present set of basis vectors simplifies subsequent calculations. We find it more convenient to use \( \hat{q} \) and \( \hat{p} \) as defined by the scattered directions. Substituting (1.1.33) and (1.1.27) into (1.1.30) gives two equations in which the dot product of \( \hat{e}(-k_z) \) with the square-bracketed terms in (1.1.30) gives zero and the dot product of \( \hat{h}(-k_z) \) with the square-bracketed terms in (1.1.30) also gives zero.

From (1.1.33) and (1.1.30) the dot product of \( \hat{e}(-k_z) \) with the square-bracketed terms gives

\[ A^{(1)}_q(\vec{k}_\perp) = -ik_z \hat{e}(-k_z) \cdot \left[ a_q^{(0)}\hat{q}_i + a_p^{(0)}\hat{p}_i \right] F(\vec{k}_\perp - \vec{k}_i\perp) \]  

(1.1.34)

The dot product of \( \hat{h}(-k_z) \) with the square-bracketed terms in (1.1.30) gives

\[ \frac{k_z}{k} A^{(1)}_p(\vec{k}_\perp) = -\frac{k_\rho}{k} A^{(1)}_z(\vec{k}_\perp) - ik_z \hat{h}(-k_z) \cdot \left[ a_q^{(0)}\hat{q}_i + a_p^{(0)}\hat{p}_i \right] F(\vec{k}_\perp - \vec{k}_i\perp) \]  

(1.1.35)
Thus the two components $A_q^{(1)}$ and $A_p^{(1)}$ depend on the projection of the polarizations $\hat{e}(-k_z)$ and $\hat{h}(-k_z)$ on the incident polarizations as projected on the $x$-$y$ plane. Substituting (1.1.35) into (1.1.34) and using (1.1.22) gives the explicit expressions for $A_q^{(1)}(\overline{k}_\perp)$ and $A_p^{(1)}(\overline{k}_\perp)$:

\[ A_q^{(1)}(\overline{k}_\perp) = iF(\overline{k}_\perp - \overline{k}_i\perp)\tilde{A}_q^{(1)}(\overline{k}_\perp) \]  \hspace{1cm} (1.1.36a)

\[ A_p^{(1)}(\overline{k}_\perp) = iF(\overline{k}_\perp - \overline{k}_i\perp)\tilde{A}_p^{(1)}(\overline{k}_\perp) \]  \hspace{1cm} (1.1.36b)

where

\[ \tilde{A}_q^{(1)}(\overline{k}_\perp) = -k_z \left[ a_q^{(0)} \cos(\phi_k - \phi_i) + a_p^{(0)} \sin(\phi_k - \phi_i) \right] \] \hspace{1cm} (1.1.36c)

\[ \tilde{A}_p^{(1)}(\overline{k}_\perp) = \frac{k^2}{k_z} \left[ a_q^{(0)} \sin(\phi_k - \phi_i) - a_p^{(0)} \cos(\phi_k - \phi_i) \right] + \frac{k_p k_i p a_p^{(0)}}{k_z} \] \hspace{1cm} (1.1.36d)

For TE excitation, $a_q^{(0)} = 2k_{iz}/k$, $a_p^{(0)} = 0$:

\[ \tilde{A}_q^{(1)}(\overline{k}_\perp) = -\frac{2k_z k_{iz}}{k} \cos(\phi_k - \phi_i) \] \hspace{1cm} (1.1.36e)

\[ \tilde{A}_p^{(1)}(\overline{k}_\perp) = \frac{2k k_{iz}}{k_z} \sin(\phi_k - \phi_i) \] \hspace{1cm} (1.1.36f)

For TM excitation, $a_q^{(0)} = 0$, $a_p^{(0)} = 2$ we have:

\[ \tilde{A}_q^{(1)}(\overline{k}_\perp) = -2k_z \sin(\phi_k - \phi_i) \] \hspace{1cm} (1.1.36g)

\[ \tilde{A}_p^{(1)}(\overline{k}_\perp) = \frac{2}{k_z} \left\{ k_p k_i p - k^2 \cos(\phi_k - \phi_i) \right\} \] \hspace{1cm} (1.1.36h)

Substituting into (1.1.18) gives the first-order scattered field as:

\[ \overline{E}_s^{(1)}(\overline{r}) = -\frac{1}{2} \int d\overline{k}_\perp e^{i\overline{k}_\perp \cdot \overline{r}} \frac{k}{k_z} \left[ \hat{e}(k_z)\hat{e}(k_z) + \hat{h}(k_z)\hat{h}(k_z) \right] \]

\[
\cdot \left[ A_q^{(1)}(\overline{k}_\perp)\hat{q}(\overline{k}_\perp) + A_p^{(1)}(\overline{k}_\perp)\hat{p}(\overline{k}_\perp) + \hat{z}A_z^{(1)}(\overline{k}_\perp) \right. \\
- \left. ik_z F(\overline{k}_\perp - \overline{k}_i\perp) \left( \hat{q}_i a_q^{(0)} + \hat{p}_i a_p^{(0)} \right) \right]\]

\[ = -\frac{1}{2} \int d\overline{k}_\perp e^{i\overline{k}_\perp \cdot \overline{r} + ik_z \frac{k}{k_z}} \left\{ \hat{e}(k_z) \left[ A_q^{(1)}(\overline{k}_\perp) - ik_z F(\overline{k}_\perp - \overline{k}_i\perp)\hat{e}(k_z) \cdot \left( \hat{q}_i a_q^{(0)} + \hat{p}_i a_p^{(0)} \right) \right] \\
+ \hat{h}(k_z) \left[ -\frac{k_z}{k} A_p^{(1)}(\overline{k}_\perp) + \frac{k_p}{k} A_z^{(1)}(\overline{k}_\perp) \right. \\
- \left. ik_z F(\overline{k}_\perp - \overline{k}_i\perp)\hat{h}(k_z) \cdot \left( \hat{q}_i a_q^{(0)} + \hat{p}_i a_p^{(0)} \right) \right\} \right\} \] \hspace{1cm} (1.1.37a)
In view of (1.1.34), (1.1.35), and (1.1.22a)–(1.1.22d) and the fact that \( \hat{h}(k_z) \cdot \hat{\dot{q}}_i = -\hat{h}(-k_z) \cdot \hat{\dot{q}}_i \) and \( \hat{h}(k_z) \cdot \hat{\dot{p}}_i = -\hat{h}(-k_z) \cdot \hat{\dot{p}}_i \), the terms inside the two square brackets are the same and we get a factor of 2. Thus

\[
\bar{E}_s^{(1)}(\vec{r}) = -\int d\vec{k}_\perp e^{i\vec{k}_\perp \cdot \vec{r}_\perp + ik_z z} \left[ \hat{e}(k_z)\tilde{A}_q^{(1)}(\vec{k}_\perp) + \hat{h}(k_z) \left( -\frac{k_z}{k} \tilde{A}_p^{(1)}(\vec{k}_\perp) \right) \right] \\
\cdot iF(\vec{k}_\perp - \vec{k}_{i\perp})
\]

(1.1.37b)

The incident power per unit area is

\[
\bar{S}_i \cdot \hat{z} = -\frac{\cos \theta_i}{2\eta}
\]

(1.1.38)

The power per unit area associated with the first-order fields (which is also that of the incoherent wave) is,

\[
\langle \bar{S}_s \cdot \hat{z} \rangle = \frac{1}{2} \text{Re} \left( \bar{E}_s^{(1)}(\vec{r}) \times \bar{H}_s^{(1)*}(\vec{r}) \right) \cdot \hat{z}
\]

(1.1.39a)

Using (1.1.37b), we obtain

\[
\bar{H}_s^{(1)}(\vec{r}) = \frac{1}{\eta} \int d\vec{k}'_\perp e^{i\vec{k}'_\perp \cdot \vec{r}_\perp + ik'_z z} \left[ \hat{h}(k'_z)\tilde{A}_q^{(1)}(\vec{k}'_\perp) + \hat{e}(k'_z) \left( -\frac{k_z}{k} \tilde{A}_p^{(1)}(\vec{k}'_\perp) \right) \right] \\
\cdot iF(\vec{k}_\perp - \vec{k}_{i\perp})
\]

(1.1.39b)

Also

\[
\langle F(\vec{k}_\perp - \vec{k}_{i\perp})F^*(\vec{k}'_\perp - \vec{k}'_{i\perp}) \rangle = \delta(\vec{k}_\perp - \vec{k}_{i\perp})W(\vec{k}_\perp - \vec{k}_{i\perp})
\]

(1.1.40)

and we have

\[
\langle \bar{S}_s \cdot \hat{z} \rangle = \frac{1}{2\eta} \int d\vec{k}_\perp \int dk_z \left( \frac{k_z}{k} \right) W(\vec{k}_\perp - \vec{k}_{i\perp}) \left[ |\tilde{A}_q^{(1)}(\vec{k}_\perp)|^2 + \left| \frac{\tilde{A}_p^{(1)}(\vec{k}_\perp)}{k} \right|^2 \right]
\]

(1.1.41)

where \( P \) stands for propagating waves with \( k_\rho \leq k \). This is because the real part is taken to get power in (1.1.39), and only the propagating waves of the spectrum in (1.1.41) have nonzero real part and contribute to power.

Casting (1.1.41) in terms of directions in angular variables \((\theta_s, \phi_s)\), we have

\[
\begin{align*}
&k_x = k \sin \theta_s \cos \phi_s \\
&k_y = k \sin \theta_s \sin \phi_s \\
&k_z = k \cos \theta_s \\
dk_x \ dk_y &= k^2 \sin \theta_s \cos \theta_s d\theta_s d\phi_s
\end{align*}
\]

(1.1.42a)

(1.1.42b)

(1.1.42c)

(1.1.42d)

We can write (1.1.41) as

\[
\langle \bar{S}_s \cdot \hat{z} \rangle = |\bar{S}_i \cdot \hat{z}| \int_0^{\pi/2} \sin \theta_s \int_0^{2\pi} d\phi_s \frac{\gamma(k_z, \hat{k}_i)}{4\pi}
\]

(1.1.43)
where
\[ \gamma(\hat{k}_s, \hat{k}_i) = \frac{4\pi k^2}{\cos \theta_i} W(\overline{k}_\perp - \overline{k}_i\perp) \left[ |\tilde{A}_q^{(1)}(\overline{k}_\perp)|^2 + |\tilde{A}_p^{(1)}(\overline{k}_\perp)|^2 \overline{k}_z/k \right] \] (1.1.44)

To get \( \gamma_{hh} \) and \( \gamma_{vh} \), we let incident wave be TE with \( \hat{e}_i = \hat{e}(-k_{iz}) \) so that \( a_q^{(0)} = 2k_{iz}/k \), \( a_p^{(0)} = 0 \). Then \( \gamma_{hh} \) corresponds to the \( |\tilde{A}_q^{(1)}|^2 \) in (1.1.44) while \( \gamma_{vh} \) corresponds to the \( |\tilde{A}_p^{(1)}k_z/k|^2 \) in (1.1.44). Here \( h \) stands for horizontal polarization (TE) and \( v \) stands for vertical polarization (TM).

\[ \gamma_{hh} = 16\pi k^4 W(\overline{k}_\perp - \overline{k}_i\perp) \cos^2(\phi_s - \phi_i) \cos \theta_i \cos^2 \theta_s \] (1.1.45a)
\[ \gamma_{vh} = 16\pi k^4 W(\overline{k}_\perp - \overline{k}_i\perp) \sin^2(\phi_s - \phi_i) \cos \theta_i \] (1.1.45b)

To get \( \gamma_{vv} \) and \( \gamma_{hv} \), we let \( \hat{e}_i = \hat{h}(-k_{iz}) \) so that \( a_q^{(0)} = 0 \) and \( a_p^{(0)} = 2 \). Then \( \gamma_{hv} \) corresponds to the \( |\tilde{A}_q^{(1)}|^2 \) in (1.1.44) while \( \gamma_{vv} \) corresponds to the \( |\tilde{A}_p^{(1)}k_z/k|^2 \) in (1.1.44). Thus

\[ \gamma_{hv} = \frac{16\pi k^4}{\cos \theta_i} W(\overline{k}_\perp - \overline{k}_i\perp) \cos^2 \theta_s \sin^2(\phi_s - \phi_i) \] (1.1.45c)
\[ \gamma_{vv} = \frac{16\pi k^4}{\cos \theta_i} W(\overline{k}_\perp - \overline{k}_i\perp) [\sin \theta_s \sin \theta_i - \cos(\phi_s - \phi_i)]^2 \] (1.1.45d)

In the backscattering direction (\( \theta_s = \theta_i \) and \( \phi_s = \pi + \phi_i \)) one has \( \overline{k}_\perp = -\overline{k}_i\perp \) and

\[ \sigma_{\alpha\beta} = \gamma_{\alpha\beta} \cos \theta_i \] (1.1.46)

so that

\[ \sigma_{vv} = 16\pi k^4 W(-2\overline{k}_i\perp)(1 + \sin^2 \theta_i)^2 \] (1.1.47a)
\[ \sigma_{vh} = 0 \] (1.1.47b)
\[ \sigma_{hv} = 0 \] (1.1.47c)
\[ \sigma_{hh} = 16\pi k^4 W(-2\overline{k}_i\perp) \cos^4 \theta_i \] (1.1.47d)

It is noteworthy that in the backscattering direction, there is no depolarization for a linearly polarized incident wave. Also, \( \sigma_{vv} \) is larger than \( \sigma_{hh} \). If \( \theta_i \) is close to grazing, so that \( \theta_i \to 90^\circ \), then \( \sigma_{vv} \) is much larger than \( \sigma_{hh} \).

1.2 Second-Order Solutions

For the second-order solution, let

\[ \overline{A}^{(2)}(\overline{k}_\perp) = A_q^{(2)}(\overline{k}_\perp)\hat{q}(\overline{k}_\perp) + A_p^{(2)}(\overline{k}_\perp)\hat{p}(\overline{k}_\perp) + A_z^{(2)}(\overline{k}_\perp)\hat{z} \] (1.1.48)