Analysis Methods for RF, Microwave, and Millimeter-Wave Planar Transmission Line Structures

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To my wife, Ngọc-Diệp,
and my children, Christine (Nhã-Uyên) and Andrew (An)
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RF integrated circuit (RFICs) and microwave integrated circuits (MICs), both hybrid and monolithic, have advanced rapidly in the last two decades. This progress has been achieved not only because of the advance of solid-state devices, but also due to the progression of planar transmission lines. Many milestones have been achieved: one of them being the development of various analysis methods for RF microwave and millimeter-wave passive structures, in general, and planar transmission lines, in particular. These methods have played an important role in providing accurate transmission line parameters for designing RFICs and MICs, as well as in investigating and developing new planar transmission lines.

The primary objective of this book is to present the Green’s function, conformal-mapping, variational, spectral-domain, and mode-matching methods, which are some of the most useful and commonly used techniques for analyzing planar transmission lines. Information for these methods in the literature is at a level that is not very suitable for the majority of first-year graduate students and practicing RF and microwave engineers. The material in this book is self-contained and presented in a way that allows readers with only fundamental knowledge in electromagnetic theory to easily understand and implement the techniques. The book also includes problems at the end of each chapter, allowing readers to reinforce their knowledge and to practice their understanding. Some of these problems are relatively long and difficult, and thus are more suitable for class projects. The book can therefore serve not only as a textbook for first-year graduate students, but also as a reference book for practicing RF and microwave engineers. Another purpose of the book is to use these methods as means to present the principles of applying electromagnetic theory to the analysis of microwave boundary-value problems. This knowledge is essential for microwave students and engineers, as it allows them to modify and improve these methods, as well as to develop new techniques.

This book is based on the material of a graduate course on field theory for microwave passive structures offered at Texas A&M University. It is completely self-contained and requires readers to have only the fundamentals
of electromagnetic theory, which is normally fulfilled by the first undergraduate course in electromagnetics.

I sincerely appreciate the patience of Professor Kai Chang, Editor of the Wiley Series in Microwave and Optical Engineering, and Mr. George Telecki, Executive Editor of Wiley-Interscience, during the writing of the manuscript for this book. I am also grateful to my former students who took the course and provided me with a purpose for writing this book. Finally, I wish to express my heartfelt thanks and deepest appreciation to my wife, Ngoc-Diep, for her constant encouragement and support, and my children, Christine and Andrew, for their understanding during the writing of this book.

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CHAPTER ONE

Introduction

MICrowave integrated circuits (MICs) were introduced in the 1950s. Since then, they have played perhaps the most important role in advancing the radiofrequency (RF) and microwave technologies. The most noticeable and important milestone was possibly the emergence of monolithic microwave integrated circuits (MMICs). This progress of MICs would not have been possible without the advances of solid-state devices and planar transmission lines. Planar transmission lines refer to transmission lines that consist of conducting strips printed on surfaces of the transmission lines’ substrates. These structures are the backbone of MICs, and represent an important and interesting research topic for many microwave engineers. Along with the advances of MICs and planar transmission lines, numerous analysis methods for microwave and millimeter-wave passive structures, in general, and planar transmission lines, in particular, have been developed in response to the need for accurate analysis and design of MICs. These analysis methods have in turn helped further investigation and development of new planar transmission lines.

This book presents the Green’s function, conformal-mapping, variational, spectral-domain, and mode-matching methods. They are useful and commonly used techniques for analyzing microwave and millimeter-wave planar transmission lines, in particular, and passive structures, in general. Information for these methods in the literature is at a level that is not very suitable for the majority of first-year graduate students and practicing microwave engineers. This book attempts to present the materials in such a way as to allow students and engineers with basic knowledge in electromagnetic theory to understand and implement the techniques. The book also includes problems for each chapter so readers can reinforce and practice their knowledge.

1.1 PLANAR TRANSMISSION LINES AND MICROWAVE INTEGRATED CIRCUITS

Planar transmission lines are essential components of MICs. They have been used to realize many circuit functions, such as baluns, filters, hybrids, and couplers, as well as simply to carry signals. Figure 1.1 shows some commonly used planar
Figure 1.1 Common planar transmission lines.
Figure 1.1 (Continued)
## TABLE 1.1 Properties of Planar Transmission Lines Shown in Fig. 1.1

<table>
<thead>
<tr>
<th>Transmission Line</th>
<th>Operating Frequency (GHz)</th>
<th>Characteristic Impedance Range (Ohm)</th>
<th>Dimension</th>
<th>Loss</th>
<th>Power Handling</th>
<th>Solid-State Device Mounting</th>
<th>Low-Cost Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microstrip line</td>
<td>≤110 GHz</td>
<td>10–100</td>
<td>Small</td>
<td>High</td>
<td>Low</td>
<td>Fair</td>
<td>Good</td>
</tr>
<tr>
<td>Strip line</td>
<td>≤60 GHz</td>
<td>20–150</td>
<td>Moderate</td>
<td>Low</td>
<td>Low</td>
<td>Moderate</td>
<td>Good</td>
</tr>
<tr>
<td>Suspended</td>
<td>≤220 GHz</td>
<td>20–150</td>
<td>Moderate</td>
<td>Low</td>
<td>Low</td>
<td>Moderate</td>
<td>Fair</td>
</tr>
<tr>
<td>Fin line</td>
<td>≤220 GHz</td>
<td>20–400</td>
<td>Moderate</td>
<td>Moderate</td>
<td>Low</td>
<td>Easy</td>
<td>Fair</td>
</tr>
<tr>
<td>Slot line</td>
<td>≤110 GHz</td>
<td>60–200</td>
<td>Small</td>
<td>High</td>
<td>Low</td>
<td>Easy</td>
<td>Good</td>
</tr>
<tr>
<td>Inverted microstrip line</td>
<td>≤220 GHz</td>
<td>25–130</td>
<td>Small</td>
<td>Moderate</td>
<td>Low</td>
<td>Moderate</td>
<td>Fair</td>
</tr>
<tr>
<td>Coplanar waveguide</td>
<td>≤110 GHz</td>
<td>40–150</td>
<td>Small</td>
<td>High</td>
<td>Low</td>
<td>Very easy</td>
<td>Good</td>
</tr>
<tr>
<td>Coplanar strips</td>
<td>≤110 GHz</td>
<td>30–250</td>
<td>Small</td>
<td>High</td>
<td>Low</td>
<td>Easy</td>
<td>Good</td>
</tr>
</tbody>
</table>
transmission lines and Table 1.1 summarizes their properties. Each transmission line has its own unique advantages and disadvantages and, depending on circuit types, either an individual transmission line or a combination of them is needed to achieve desired circuit functions as well as optimum performances. The most viable planar transmission lines are perhaps the conventional microstrip line and coplanar waveguide (CPW), from which many other planar transmission lines have evolved. Multilayer planar transmission lines, such as that shown in Fig. 1.2, are especially attractive for MICs due to their flexibility and ability to realize complicated circuits, ultimately allowing very compact, high-density circuit integration. They also allow thin dielectric layers to be deposited on conductor-backed semiconducting substrates for achieving ultracompact MICs. Furthermore, multilayer transmission lines have significantly less cross talk and distortion via appropriate selection of dielectric layers.

There are two classes of MICs: hybrid and monolithic circuits. Hybrid MIC refers to a planar circuit in which only parts of the circuit are formed on surfaces of the circuit’s substrates by some deposition schemes. A typical hybrid MIC has all the transmission lines deposited on the dielectric surfaces, except solid-state devices such as transistors and other passive components like capacitors. These solid-state devices and passive elements are discrete components and connected to the transmission lines by bonding, soldering, or conducting epoxy. The substrates of a hybrid MIC are generally low-loss insulators, used solely for supporting the circuit components and delivering the signals. Advantages of hybrid MICs include small size, light weight, easy fabrication, low cost, and high-volume production. In practice, hybrid MICs are normally referred to simply as MICs. Figures 1.3–1.7 show photographs of some hybrid MICs employing planar transmission lines.

Figure 1.2 A multilayer planar transmission line.
Figure 1.3  S-band (2–4 GHz) MIC push–pull field effect transistor (FET) amplifier using CPW and slot line.

Figure 1.4  W-band (75–110 GHz) MIC diode balanced mixer using fin line, CPW, and suspended strip line.
The monolithic MIC (MMIC) is a special class of MICs, in which all the circuit elements, including passive components and solid-state devices, are formed into the bulk or onto the surface of a semi-insulating semiconductor substrate by some deposition technique. In contrast to hybrid MICs, the substrates are used in MMICs not only as a signal-propagating medium and a supporting structure for passive components, but also as a material onto which semiconducting layers with good properties for realizing microwave solid-state devices are grown or deposited. Compared to hybrid MICs, the advantages of MMICs are lower-cost circuits through batch processing, improved reliability and reproducibility through minimization of wire bonds and discrete components, smaller size and weight, more circuit design flexibility, and multifunction performance on a single chip. MMICs are very important for microwave technology. Most microwave and millimeter-wave applications are expected eventually to employ all MMICs. Figure 1.8 shows a photograph of a Ka-band (26.5–40 GHz) push–push MMIC oscillator.

1.2 ANALYSIS METHODS FOR PLANAR TRANSMISSION LINES

In using planar transmission lines in MICs, analysis methods are needed in order to determine the transmission lines’ characteristics such as characteristic
impedance, effective dielectric constant, and loss. The design of MICs depends partly on accurate analysis of planar transmission lines.

The microwave technology is changing rapidly and, in connecting with it, useful analysis methods for microwave and millimeter-wave planar transmission lines, either completely brand new or modifications of existing techniques, appear constantly. In fact, microwave engineers are now faced with many different techniques and a vast amount of information, making the techniques difficult to understand and hence to implement in the short time normally encountered in an industrial setting. Each method has its own unique advantages and disadvantages for particular problems and needs. However, they are all based on Maxwell’s
equations, in general, and wave equations and boundary conditions, in particular. These are the fundamentals of these methods and, while techniques can change steadily, the fundamentals always remain the same. They, in fact, provide a foundation for the derivation, modification, and implementation of all current and future analysis methods. In this book, we describe particularly the details of the Green’s function, conformal-mapping, variational, spectral-domain, and mode-matching methods. These methods not only represent some of the most useful and commonly used techniques for analyzing planar transmission lines, but also serve as means to present the fundamentals of applying electromagnetic theory to the analysis of microwave boundary-value problems. This knowledge would then allow readers to modify and improve these methods, or to develop new techniques.

1.3 ORGANIZATION OF THE BOOK

The book is organized into eight chapters and is self-contained. Chapter 2 gives the fundamentals of electromagnetic theory, which are needed for the formulation of the methods addressed in this book. Chapter 3 covers Green’s functions used in various methods. Chapter 4 discusses the fundamentals of planar transmission lines and provides useful equations for commonly used transmission lines.
Figure 1.8  Ka-band MMIC push–push FET oscillator using microstrip line.