Microstrip Filters for RF/Microwave Applications

JIA-SHENG HONG
M. J. LANCASTER

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Preface

Filters play important roles in many RF/microwave applications. Emerging applications such as wireless communications continue to challenge RF/microwave filters with ever more stringent requirements—higher performance, smaller size, lighter weight, and lower cost. The recent advances in novel materials and fabrication technologies, including high-temperature superconductors (HTS), low-temperature cofired ceramics (LTCC), monolithic microwave integrated circuits (MMIC), microelectromechanic system (MEMS), and micromachining technology, have stimulated the rapid development of new microstrip and other filters for RF/microwave applications. In the meantime, advances in computer-aided design (CAD) tools such as full-wave electromagnetic (EM) simulators have revolutionized filter design. Many novel microstrip filters with advanced filtering characteristics have been demonstrated. However, up until now there has not been a single book dedicated to this subject.

Microstrip Filters for RF/Microwave Applications offers a unique and comprehensive treatment of RF/microwave filters based on the microstrip structure, providing a link to applications of computer-aided design tools and advanced materials and technologies. Many novel and sophisticated filters using computer-aided design are discussed, from basic concepts to practical realizations. The book is self-contained—it is not only a valuable design resource but also a handy reference for students, researchers, and engineers in microwave engineering. It can also be used for RF/microwave education.

The outstanding features of this book include discussion of many novel microstrip filter configurations with advanced filtering characteristics, new design techniques, and methods for filter miniaturization. The book emphasizes computer analysis and synthesis and full-wave electromagnetic (EM) simulation through a large number of design examples. Applications of commercially available software are demonstrated. Commercial applications are included as are design theories and methodologies, which are not only for microstrip filters, but also directly applicable to other types of filters, such as waveguide and other transmission line filters. Therefore, this book is more than just a text on microstrip filters.

Much of work described herein has resulted from the authors' research. The authors wish to acknowledge the financial support of the UK EPSRC and the European Commission through the Advanced Communications Technologies and Services (ACTS) program. They would also like to acknowledge their national and international collaborators, including Professor Heinz Chaloupka at Wuppertal University (Germany), Robert Greed at Marconi Research Center (U.K.), Dr. Jean-Claude Mage at Thomson-CSF/CNRS (France), and Dieter Jedamzik, formerly with GEC-Marconi Materials Technology (U.K.).

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In addition, Jia-Sheng Hong would like to thank Professor John Allen at the University of Oxford (U.K.), Professor Werner Wiesbeck at Kalsruhe University (Germany), and Dr. Nicholas Edwards at British Telecom (U.K.) for their many years of support and friendship. Professor Joseph Helszajn at Heriot-Watt University (U.K.), who sent his own book on filters to Jia-Sheng Hong, is also acknowledged.

Finally, Jia-Sheng Hong would like to express his deep appreciation to his wife, Kai, and his son, Haide, for their tolerance and support, which allowed him to write the book at home over many evenings, weekends, and holidays. In particular, without the help of Kai, completing this book on time would not have been possible.

CHAPTER 1

Introduction

The term *microwaves* may be used to describe electromagnetic (EM) waves with frequencies ranging from 300 MHz to 300 GHz, which correspond to wavelengths (in free space) from 1 m to 1 mm. The EM waves with frequencies above 30 GHz and up to 300 GHz are also called *millimeter waves* because their wavelengths are in the millimeter range (1–10 mm). Above the millimeter wave spectrum is the infrared, which comprises electromagnetic waves with wavelengths between 1 μm (10^{-6} m) and 1 mm. Beyond the infrared spectrum is the visible optical spectrum, the ultraviolet spectrum, and x-rays. Below the microwave frequency spectrum is the radio frequency (RF) spectrum. The frequency boundary between RF and microwaves is somewhat arbitrary, depending on the particular technologies developed for the exploitation of that specific frequency range. Therefore, by extension, the RF/microwave applications can be referred to as communications, radar, navigation, radio astronomy, sensing, medical instrumentation, and others that explore the usage of frequency spectrums in the range of, say, 300 kHz up to 300 GHz (Figure 1.1). For convenience, some of these frequency spectrums are further divided into many frequency bands as indicated in Figure 1.1.

Filters play important roles in many RF/microwave applications. They are used to separate or combine different frequencies. The electromagnetic spectrum is limited and has to be shared; filters are used to select or confine the RF/microwave signals within assigned spectral limits. Emerging applications such as wireless communications continue to challenge RF/microwave filters with ever more stringent requirements—higher performance, smaller size, lighter weight, and lower cost. Depending on the requirements and specifications, RF/microwave filters may be designed as lumped element or distributed element circuits; they may be realized in various transmission line structures, such as waveguide, coaxial line, and microstrip.

The recent advance of novel materials and fabrication technologies, including monolithic microwave integrated circuit (MMIC), microelectromechanic system (MEMS), micromachining, high-temperature superconductor (HTS), and low-temperature cofired ceramics (LTCC), has stimulated the rapid development of new mi-

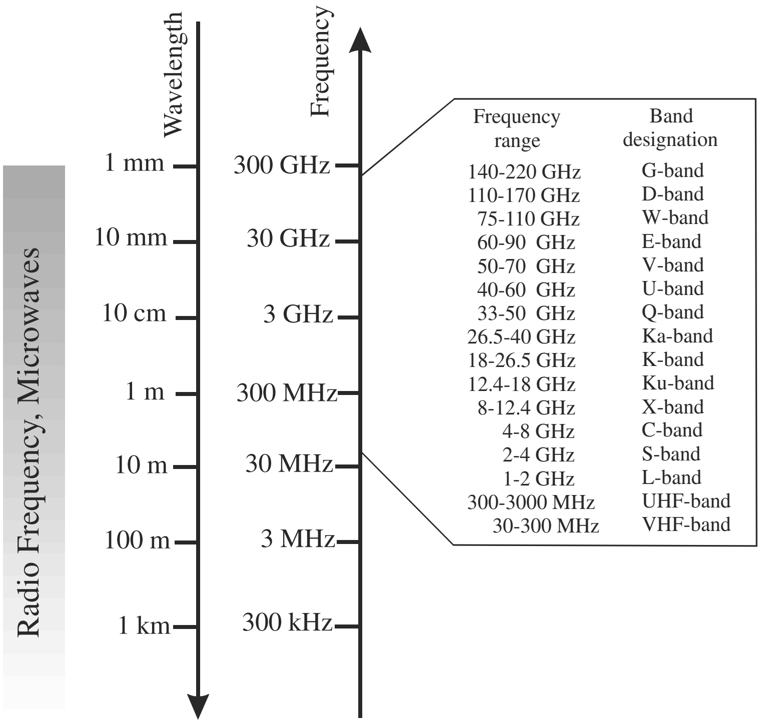


FIGURE 1.1 RF/microwave spectrums.

crostrip and other filters. In the meantime, advances in computer-aided design (CAD) tools such as full-wave electromagnetic (EM) simulators have revolutionized filter design. Many novel microstrip filters with advanced filtering characteristics have been demonstrated.

It is the main objective of this book to offer a unique and comprehensive treatment of RF/microwave filters based on the microstrip structure, providing a link to applications of computer-aided design tools, advanced materials, and technologies (see Figure 1.2). However, it is not the intention of this book to include everything that has been published on microstrip filters; such a work would be out of scale in terms of space and knowledge involved. Moreover, design theories and methods described in the book are not only for microstrip filters but directly applicable to other types of filters, such as waveguide filters.

Although the physical realization of filters at RF/microwave frequencies may vary, the circuit network topology is common to all. Therefore, the technique content of the book begins with Chapter 2, which describes various network concepts and equations; theses are useful for the analysis of filter networks. Chapter 3 then introduces basic concepts and theories for design of general RF/microwave filters (including microstrip filters). The topics cover filter transfer functions (such as Butterworth, Chebyshev, elliptic function, all pass, and Gaussian response), lowpass

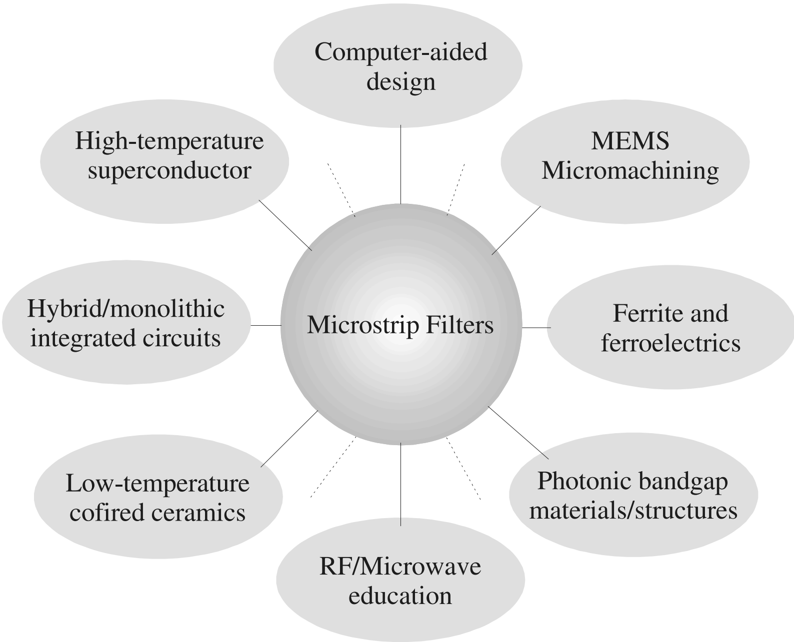


FIGURE 1.2 Microstrip filter linkage.

prototype filters and elements, frequency and element transformations, immittance (impedance/admittance) inverters, Richards' transformation, and Kuroda identities for distributed elements. Effects of dissipation and unloaded quality factors of filter elements on filter performance are also discussed.

Chapter 4 summarizes basic concepts and design equations for microstrip lines, coupled microstrip lines, and discontinuities, as well as lumped and distributed components, which are useful for design of filters. In Chapter 5, conventional microstrip lowpass and bandpass filters, such as stepped-impedance filters, open-stub filters, semilumped element filters, end- and parallel-coupled half-wavelength resonator filters, hairpin-line filters, interdigital and combline filters, pseudocombline filters and stub-line filters, are discussed with instructive design examples.

Chapter 6 discusses some typical microstrip highpass and bandstop filters. These include quasilumped element and optimum distributed highpass filters, narrow-band and wide-band bandstop filters, as well as filters for RF chokes. Design equations, tables and examples are presented for easy references.

The remaining chapters of the book deal with more advanced topics, starting with Chapter 7, which introduces some of advanced materials and technologies for RF/microwave filter applications. These include high-temperature superconductors (HTS), ferroelectrics, MEMS or micromachining, hybrid or monolithic microwave integrated circuits (MMIC), active filters, photonic bandgap (PBG) materials/structures, and low-temperature cofired ceramics (LTCC).

Chapter 8 presents a comprehensive treatment of subjects regarding coupled resonator circuits. These are of importance for design of RF/microwave filters, in particular the narrow-band bandpass filters, which play a significant role in many applications. There is a general technique for designing coupled resonator filters, which can be applied to any type of resonator despite its physical structure. For example, it can be applied to the design of waveguide filters, dielectric resonator filters, ceramic combline filters, microstrip filters, superconducting filters, and micromachined filters. This design method is based on coupling coefficients of intercoupled resonators and the external quality factors of the input and output resonators. Since this design technique is so useful and flexible, it would be desirable to have a deep understanding not only of its approach, but also its theory. For this purpose, the subjects cover the formulation of the general coupling matrix, which is of importance for representing a wide range of coupled-resonator filter topologies, the general theory of couplings for establishing the relationship between the coupling coefficient and the physical structure of coupled resonators. This leads to a very useful formulation for extracting coupling coefficients from EM simulations or measurements. Formulations for extracting the external quality factors from frequency responses of the externally loaded input/output resonators are derived next. Numerical examples are followed to demonstrate how to use these formulations to extract coupling coefficients and external quality factors of microwave coupling structures for filter designs.

Chapter 9 is concerned with computer-aided design (CAD). Generally speaking, any design that involves using computers may be called CAD. There have been extraordinary recent advances in CAD of RF/microwave circuits, particularly in full-wave electromagnetic (EM) simulations. They have been implemented both in commercial and specific in-house software and are being applied to microwave filter simulation, modeling, design, and validation. The developments in this area are certainly being stimulated by increasing computer power. Another driving force for the developments is the requirement of CAD for low-cost and high-volume production. In general, besides the investment for tooling, materials and labor mainly affect the cost of filter production. Labor costs include those for design, fabrication, testing, and tuning. Here the costs for the design and tuning can be reduced greatly by using CAD, which can provide more accurate design with less design iterations, leading to first-pass or tuneless filters. This chapter discusses computer simulation and/or computer optimization. It summarizes some basic concepts and methods regarding filter design by CAD. Typical examples of the applications, including filter synthesis by optimization, are described. Many more CAD examples, particularly those based on full-wave EM simulation, can be found through this book.

In Chapter 10, we discuss the designs of some advanced filters, including selective filters with a single pair of transmission zeros, cascaded quadruplet (CQ) filters, trisection and cascaded trisection (CT) filters, cross-coupled filters using transmission line inserted inverters, linear phase filters for group delay equalization, and extracted-pole filters. These types of filters, which are different from conventional Chebyshev filters, must meet the stringent requirements of RF/microwave systems, particularly wireless communications systems.

Chapter 11 is intended to describe novel concepts, methodologies, and designs for compact filters and filter miniaturization. The new types of filters discussed include ladder line filters, pseudointerdigital line filters, compact open-loop and hairpin resonator filters, slow-wave resonator filters, miniaturized dual-mode filters, multilayer filters, lumped-element filters, and filters using high-dielectric constant substrates.

The final chapter of the book (Chapter 12) presents a case study of high-temperature superconducting (HTS) microstrip filters for cellular base station applications. The study starts with a brief discussion of typical HTS subsystems and RF modules that include HTS microstrip filters for cellular base stations. This is followed by more detailed descriptions of the developments of duplexers and preselect bandpass filters, including design, fabrications, and measurement. The work presented in this chapter has been carried out mainly for a European research project called Superconducting Systems for Communications (SUCOMS), in which the authors have been involved. The objective of the project is to construct an HTS-based transceiver for mast-mounted DCS1800 base stations, but it can be interfaced with the Global System for Mobile Communication or GSM-1800 base station. It can also be modified for other mobile communication systems such as the Personal Communication System (PCS) and the future Universal Mobile Telecommunication System (UMTS).

CHAPTER 2

Network Analysis

Filter networks are essential building elements in many areas of RF/microwave engineering. Such networks are used to select/reject or separate/combine signals at different frequencies in a host of RF/microwave systems and equipment. Although the physical realization of filters at RF/microwave frequencies may vary, the circuit network topology is common to all.

At microwave frequencies, voltmeters and ammeters for the direct measurement of voltages and currents do not exist. For this reason, voltage and current, as a measure of the level of electrical excitation of a network, do not play a primary role at microwave frequencies. On the other hand, it is useful to be able to describe the operation of a microwave network such as a filter in terms of voltages, currents, and impedances in order to make optimum use of low-frequency network concepts.

It is the purpose of this chapter to describe various network concepts and provide equations that are useful for the analysis of filter networks.

2.1 NETWORK VARIABLES

Most RF/microwave filters and filter components can be represented by a two-port network, as shown in Figure 2.1, where V_1, V_2 and I_1, I_2 are the voltage and current variables at the ports 1 and 2, respectively, Z_{01} and Z_{02} are the terminal impedances, and E_s is the source or generator voltage. Note that the voltage and current variables are complex amplitudes when we consider sinusoidal quantities. For example, a sinusoidal voltage at port 1 is given by

$$v_1(t) = |V_1|\cos(\omega t + \phi) \tag{2.1}$$

We can then make the following transformations:

$$v_1(t) = |V_1|\cos(\omega t + \phi) = \mathrm{Re}(|V_1|e^{j(\omega t+\phi)}) = \mathrm{Re}(V_1 e^{j\omega t}) \tag{2.2}$$

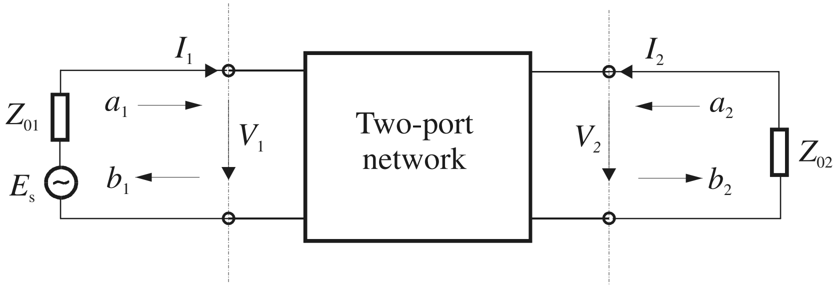


FIGURE 2.1 Two-port network showing network variables.

where Re denotes "the real part of" the expression that follows it. Therefore, one can identify the complex amplitude V_1 defined by

$$V_1 = |V_1|e^{j\phi} \tag{2.3}$$

Because it is difficult to measure the voltage and current at microwave frequencies, the wave variables a_1, b_1 and a_2, b_2 are introduced, with a indicating the incident waves and b the reflected waves. The relationships between the wave variables and the voltage and current variables are defined as

$$\begin{aligned} V_n &= \sqrt{Z_{0n}}(a_n + b_n) \\ I_n &= \frac{1}{\sqrt{Z_{0n}}}(a_n - b_n) \end{aligned} \qquad n = 1 \text{ and } 2 \tag{2.4a}$$

or

$$\begin{aligned} a_n &= \frac{1}{2}\left(\frac{V_n}{\sqrt{Z_{0n}}} + \sqrt{Z_{0n}}\, I_n\right) \\ b_n &= \frac{1}{2}\left(\frac{V_n}{\sqrt{Z_{0n}}} - \sqrt{Z_{0n}}\, I_n\right) \end{aligned} \qquad n = 1 \text{ and } 2 \tag{2.4b}$$

The above definitions guarantee that the power at port n is

$$P_n = \tfrac{1}{2}\mathrm{Re}(V_n \cdot I_n^*) = \tfrac{1}{2}(a_n a_n^* - b_n b_n^*) \tag{2.5}$$

where the asterisk denotes a conjugate quantity. It can be recognized that $a_n a_n^*/2$ is the incident wave power and $b_n b_n^*/2$ is the reflected wave power at port n.

2.2 SCATTERING PARAMETERS

The scattering or S parameters of a two-port network are defined in terms of the wave variables as

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$
$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \tag{2.6}$$

where $a_n = 0$ implies a perfect impedance match (no reflection from terminal impedance) at port n. These definitions may be written as

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{2.7}$$

where the matrix containing the S parameters is referred to as the scattering matrix or S matrix, which may simply be denoted by [S].

The parameters S_{11} and S_{22} are also called the reflection coefficients, whereas S_{12} and S_{21} the transmission coefficients. These are the parameters directly measurable at microwave frequencies. The S parameters are in general complex, and it is convenient to express them in terms of amplitudes and phases, i.e., $S_{mn} = |S_{mn}|e^{j\phi_{mn}}$ for $m, n = 1, 2$. Often their amplitudes are given in decibels (dB), which are defined as

$$20 \log|S_{mn}| \text{ dB} \qquad m, n = 1, 2 \tag{2.8}$$

where the logarithm operation is base 10. This will be assumed through this book unless otherwise stated. For filter characterization, we may define two parameters:

$$L_A = -20 \log|S_{mn}| \text{ dB} \qquad m, n = 1, 2 (m \neq n)$$
$$L_R = 20 \log|S_{nn}| \text{ dB} \qquad n = 1, 2 \tag{2.9}$$

where L_A denotes the insertion loss between ports n and m and L_R represents the return loss at port n. Instead of using the return loss, the voltage standing wave ratio $VSWR$ may be used. The definition of $VSWR$ is

$$VSWR = \frac{1 + |S_{nn}|}{1 - |S_{nn}|} \tag{2.10}$$

Whenever a signal is transmitted through a frequency-selective network such as a filter, some delay is introduced into the output signal in relation to the input signal. There are other two parameters that play role in characterizing filter performance related to this delay. The first one is the phase delay, defined by

$$\tau_p = \frac{\phi_{21}}{\omega} \text{ seconds} \tag{2.11}$$

where ϕ_{21} is in radians and ω is in radians per second. Port 1 is the input port and port 2 is the output port. The phase delay is actually the time delay for a steady sinusoidal signal and is not necessarily the true signal delay because a steady sinusoidal signal does not carry information; sometimes, it is also referred to as the carrier delay [5]. The more important parameter is the group delay, defined by

$$\tau_d = -\frac{d\phi_{21}}{d\omega} \text{ seconds} \tag{2.12}$$

This represents the true signal (baseband signal) delay, and is also referred to as the envelope delay.

In network analysis or synthesis, it may be desirable to express the reflection parameter S_{11} in terms of the terminal impedance Z_{01} and the so-called input impedance $Z_{in1} = V_1/I_1$, which is the impedance looking into port 1 of the network. Such an expression can be deduced by evaluating S_{11} in (2.6) in terms of the voltage and current variables using the relationships defined in (2.4b). This gives

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{V_1/\sqrt{Z_{01}} - \sqrt{Z_{01}}I_1}{V_1/\sqrt{Z_{01}} + \sqrt{Z_{01}}I_1} \tag{2.13}$$

Replacing V_1 by $Z_{in1}I_1$ results in the desired expression

$$S_{11} = \frac{Z_{in1} - Z_{01}}{Z_{in1} + Z_{01}} \tag{2.14}$$

Similarly, we can have

$$S_{22} = \frac{Z_{in2} - Z_{02}}{Z_{in2} + Z_{02}} \tag{2.15}$$

where $Z_{in2} = V_2/I_2$ is the input impedance looking into port 2 of the network. Equations (2.14) and (2.15) indicate the impedance matching of the network with respect to its terminal impedances.

The S parameters have several properties that are useful for network analysis. For a reciprocal network $S_{12} = S_{21}$. If the network is symmetrical, an additional property, $S_{11} = S_{22}$, holds. Hence, the symmetrical network is also reciprocal. For a lossless passive network the transmitting power and the reflected power must equal to the total incident power. The mathematical statements of this power conservation condition are

$$\begin{aligned} S_{21}S_{21}^* + S_{11}S_{11}^* = 1 \text{ or } |S_{21}|^2 + |S_{11}|^2 = 1 \\ S_{12}S_{12}^* + S_{22}S_{22}^* = 1 \text{ or } |S_{12}|^2 + |S_{22}|^2 = 1 \end{aligned} \tag{2.16}$$

2.3 SHORT-CIRCUIT ADMITTANCE PARAMETERS

The short-circuit admittance or Y parameters of a two-port network are defined as

$$
\begin{aligned}
Y_{11} &= \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad & Y_{12} &= \left.\frac{I_1}{V_2}\right|_{V_1=0} \\
Y_{21} &= \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad & Y_{22} &= \left.\frac{I_2}{V_2}\right|_{V_1=0}
\end{aligned}
\tag{2.17}
$$

in which $V_n = 0$ implies a perfect short-circuit at port n. The definitions of the Y parameters may also be written as

$$
\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{2.18}
$$

where the matrix containing the Y parameters is called the short-circuit admittance or simply Y matrix, and may be denoted by $[Y]$. For reciprocal networks $Y_{12} = Y_{21}$. In addition to this, if networks are symmetrical, $Y_{11} = Y_{22}$. For a lossless network, the Y parameters are all purely imaginary.

2.4 OPEN-CIRCUIT IMPEDANCE PARAMETERS

The open-circuit impedance or Z parameters of a two-port network are defined as

$$
\begin{aligned}
Z_{11} &= \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad & Z_{12} &= \left.\frac{V_1}{I_2}\right|_{I_1=0} \\
Z_{21} &= \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad & Z_{22} &= \left.\frac{V_2}{I_2}\right|_{I_1=0}
\end{aligned}
\tag{2.19}
$$

where $I_n = 0$ implies a perfect open-circuit at port n. These definitions can be written as

$$
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{2.20}
$$

The matrix, which contains the Z parameters, is known as the open-circuit impedance or Z matrix and is denoted by $[Z]$. For reciprocal networks, $Z_{12} = Z_{21}$. If networks are symmetrical, $Z_{12} = Z_{21}$ and $Z_{11} = Z_{22}$. For a lossless network, the Z parameters are all purely imaginary.

Inspecting (2.18) and (2.20), we immediately obtain an important relation

$$
[Z] = [Y]^{-1} \tag{2.21}
$$

2.5 *ABCD* PARAMETERS

The *ABCD* parameters of a two-port network are give by

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \tag{2.22}$$

These parameters are actually defined in a set of linear equations in matrix notation

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \tag{2.23}$$

where the matrix comprised of the *ABCD* parameters is called the *ABCD* matrix. Sometimes, it may also be referred to as the transfer or chain matrix. The *ABCD* parameters have the following properties:

$$AD - BC = 1 \quad \text{For a reciprocal network} \tag{2.24}$$

$$A = D \quad \text{For a symmetrical network} \tag{2.25}$$

If the network is lossless, then A and D will be purely real and B and C will be purely imaginary.

If the network in Figure 2.1 is turned around, then the transfer matrix defined in (2.23) becomes

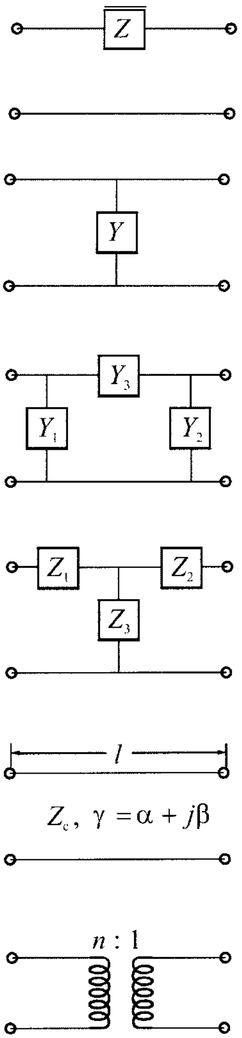
$$\begin{bmatrix} A_t & B_t \\ C_t & D_t \end{bmatrix} = \begin{bmatrix} D & B \\ C & A \end{bmatrix} \tag{2.26}$$

where the parameters with t subscripts are for the network after being turned around, and the parameters without subscripts are for the network before being turned around (with its original orientation). In both cases, V_1 and I_1 are at the left terminal and V_2 and I_2 are at the right terminal.

The *ABCD* parameters are very useful for analysis of a complex two-port network that may be divided into two or more cascaded subnetworks. Figure 2.2 gives the *ABCD* parameters of some useful two-port networks.

2.6 TRANSMISSION LINE NETWORKS

Since $V_2 = -I_2 Z_{02}$, the input impedance of the two-port network in Figure 2.1 is given by



$$A = 1 \quad B = Z$$
$$C = 0 \quad D = 1$$

$$A = 1 \quad B = 0$$
$$C = Y \quad D = 1$$

$$A = 1 + \frac{Y_2}{Y_3} \qquad B = \frac{1}{Y_3}$$
$$C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3} \qquad D = 1 + \frac{Y_1}{Y_3}$$

$$A = 1 + \frac{Z_1}{Z_3} \qquad B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$
$$C = \frac{1}{Z_3} \qquad D = 1 + \frac{Z_2}{Z_3}$$

$$A = \cosh \gamma l \qquad B = Z_c \sinh \gamma l$$
$$C = \frac{\sinh \gamma l}{Z_c} \qquad D = \cosh \gamma l$$

$$A = n \quad B = 0$$
$$C = 0 \quad D = \frac{1}{n}$$

FIGURE 2.2 Some useful two-port networks and their *ABCD* parameters.

$$Z_{in1} = \frac{V_1}{I_1} = \frac{Z_{02}A + B}{Z_{02}C + D} \tag{2.27}$$

Substituting the *ABCD* parameters for the transmission line network given in Figure 2.2 into (2.27) leads to a very useful equation

$$Z_{in1} = Z_c \frac{Z_{02} + Z_c \tanh \gamma l}{Z_c + Z_{02} \tanh \gamma l} \tag{2.28}$$

where Z_c, γ, and l are the characteristic impedance, the complex propagation constant, and the length of the transmission line, respectively. For a lossless line, $\gamma = j\beta$ and (2.28) becomes

$$Z_{in1} = Z_c \frac{Z_{02} + jZ_c \tan \beta l}{Z_c + jZ_{02} \tan \beta l} \tag{2.29}$$

Besides the two-port transmission line network, two types of one-port transmission networks are of equal significance in the design of microwave filters. These are formed by imposing an open circuit or a short circuit at one terminal of a two-port transmission line network. The input impedances of these one-port networks are readily found from (2.27) or (2.28):

$$Z_{in1}|_{Z_{02}=\infty} = \frac{A}{C} = \frac{Z_c}{\tanh \gamma l} \tag{2.30}$$

$$Z_{in1}|_{Z_{02}=0} = \frac{B}{D} = Z_c \tanh \gamma l \tag{2.31}$$

Assuming a lossless transmission, these expressions become

$$Z_{in1}|_{Z_{02}=\infty} = \frac{Z_c}{j \tan \beta l} \tag{2.32}$$

$$Z_{in1}|_{Z_{02}=0} = jZ_c \tan \beta l \tag{2.33}$$

We will further discuss the transmission line networks in the next chapter when we introduce Richards' transformation.

2.7 NETWORK CONNECTIONS

Often in the analysis of a filter network, it is convenient to treat one or more filter components or elements as individual subnetworks, and then connect them to determine the network parameters of the filter. The three basic types of connection that are usually encountered are:

1. Parallel
2. Series
3. Cascade

Suppose we wish to connect two networks N' and N'' in parallel, as shown in Figure 2.3(a). An easy way to do this type of connection is to use their Y matrices. This is because

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} + \begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix}$$

Therefore,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \left(\begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} + \begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} \right) \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{2.34a}$$

or the Y matrix of the combined network is

$$[Y] = [Y'] + [Y''] \tag{2.34b}$$

This type of connection can be extended to more than two two-port networks connected in parallel. In that case, the short-circuit admittance matrix of the composite network is given simply by the sum of the short-circuit admittance matrices of the individual networks.

Analogously, the networks of Figure 2.3(*b*) are connected in series at both their input and output terminals; consequently

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} + \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix}$$

This gives

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \left(\begin{bmatrix} Z_{11}' & Z_{12}' \\ Z_{21}' & Z_{22}' \end{bmatrix} + \begin{bmatrix} Z_{11}'' & Z_{12}'' \\ Z_{21}'' & Z_{22}'' \end{bmatrix} \right) \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{2.35a}$$

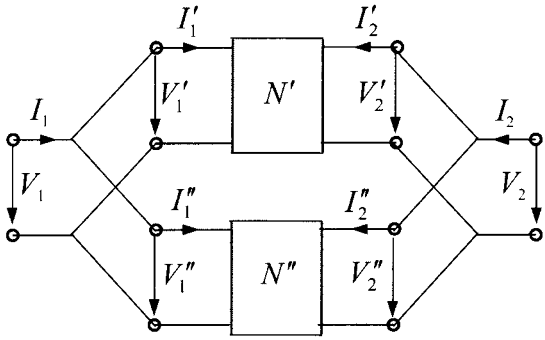
and thus the resultant Z matrix of the composite network is given by

$$[Z] = [Z'] + [Z''] \tag{2.35b}$$

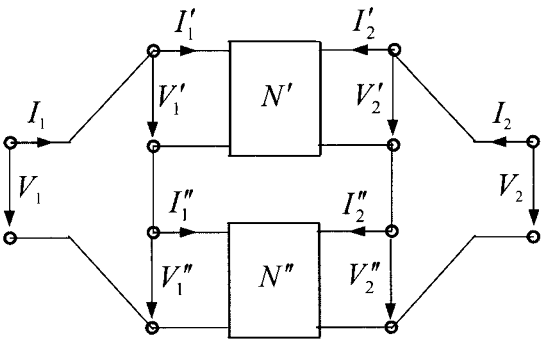
Similarly, if there are more than two two-port networks to be connected in series to form a composite network, the open-circuit impedance matrix of the composite network is equal to the sum of the individual open-circuit impedance matrices.

The cascade connection of two or more simpler networks appears to be used most frequently in analysis and design of filters. This is because most filters consist of cascaded two-port components. For simplicity, consider a network formed by the cascade connection of two subnetworks, as shown in Figure 2.3(c). The following terminal voltage and current relationships at the terminals of the composite network would be obvious:

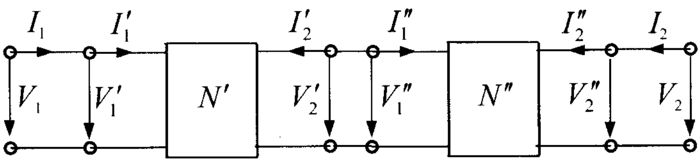
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_1' \\ I_1' \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_2'' \\ I_2'' \end{bmatrix}$$



(a)



(b)



(c)

FIGURE 2.3 Basic types of network connection: (a) parallel, (b) series, and (c) cascade.