Beyond Reason

Eight Great Problems That Reveal the Limits of Science

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Beyond Reason

· INTRODUCTION ·

Where Reason Cannot Go

We all live with limitations—some natural, some rule-based. We cannot fly unaided, nor can we beat the ace of spades with the deuce. This book is about the grander limitations that stand like granite walls around our scientific and technological enterprise. "This far and no farther," they seem to say, an ancient prohibition from sacred ground. I locate these barriers "beyond reason" because reason, even though it found them, sees no way around them.

Potential barriers might include a prohibition of time travel, especially into the past. However, I know of no established physical or mathematical theory that prohibits it. There may also be some reason why we can never solve the prime number problem, but I do not know it.

To speak of time travel and prime numbers in one breath expresses the dual nature of this book. Everyone knows what "time travel" means, but few know what a prime number is. Thus I must immediately explain that a prime number is one that cannot be divided evenly by any other number except 1. To solve the prime number problem means to arrive at what mathematicians call a closed-form expression involving a variable *n* that, when you substitute specific values of *n* into the expression, yields the *n*th prime number. I know of no theory that prohibits such a possibility, although the problem may well have no solution.

This book deals with two kinds of science: one inductive, the other deductive. One is vastly more popular than the other, but could not exist without it.

Although no current physical theory appears to prohibit time travel, the discovery of a method is sure to be cloaked in the language of mathematics. Conversely, any prohibition of the possibility will involve formulas and equations. It has become somewhat fashionable to speak of mathematics as a "language." While it has many features of a language—mainly notation that looks like Egyptian hieroglyphics to nonmathematicians—there must be more than mere language going on. We still have to account for the amazing success of mathematics as a description of physical reality. The precision of so many theories of physical reality may hint at a deeper truth, that mathematics is a major structural foundation of our universe. Nothing expresses the presence of structure better than barriers. There are some things we cannot know and some things we cannot do, all as a result of the internal logical structure of this field. The limitation does not depend in any way on the wishes or fears of individual scientists, much less their cultural backgrounds.

For the sake of names I would invoke the entire universe of mathematics, both known and unknown, by the term "holos," Greek for "whole." The word "cosmos" has essentially the same meaning in Greek but, in this context, means the physical universe in which we live, move, and have our being. With such terminology one can make mystical-sounding statements such as "The cosmos rests upon the holos." I have only the vaguest idea of what this sentence might actually mean, but it advertises the aim of this book: to discover how physical reality depends on mathematical reality, and to examine how mathematical reality manifests itself—at least to exploring minds that are still capable of curiosity.

The physical world as described by physics has a somewhat eerie mathematical configuration, taken together. A stone thrown in a vacuum will execute a parabola with a precision great enough to rule out any other polynomial function as a possible path. Did Galileo and Newton lay this fantasy upon us because they were Italian or English? Because they were expressing a post-Renaissance yearning for perfection? Or were Galileo and Newton merely traveling the mental landscape of pure reason? Would Galileo recognize the thoughts of Newton? If not, why not? Both were explorers of the holos, and neither had much choice about what he would find—or not find.

The discovery of barriers to knowledge has been accelerating somewhat over the past two centuries, keeping pace with science itself. We have known since the late nineteenth century that we cannot square the circle, but in the 1920s we learned that there exist theorems that we shall never prove. In the 1940s, even as computers emerged from the smoke and dust of a world war, we discovered noncomputable functions and the unsolvability of the halting problem. This meant, among other things, that we would never be able write a program that debugs (finds all the errors in) other programs. In the 1970s we learned that even computable functions could be a problem. There are literally hundreds of well-known and important problems lurking behind every conceivable nook and cranny of our technological infrastructure. There are instances of these problems that cannot be solved by any computer (no matter how fast) before the universe comes to an end.

The foregoing barriers were all discovered by reasoning in pure mathematics. Although physicists may object, saying that I am laying claim to their territory, it can be suggested that the other barriers were discovered by reasoning in applied mathematics (or physics, if you wish). We have known since the mid-nineteenth century that we cannot build a perpetual motion machine, but learned, in the first ten years of the twentieth century, that we shall never travel through space any faster than the speed of light. In the 1920s we were dismayed to learn that we would never be able to measure precisely the simplest properties of tiny particles like the photon and the electron. In the 1980s, chaos descended on the field of dynamical systems with the discovery that the behavior of some not-so-tiny systems suffered from another form of unpredictability. We cannot predict the weather with anything like complete accuracy, nor will we ever be able to.

There are people among us who will brook no barriers, bridling at every limitation, as though it were their God-given right to be, well, God. Theology aside, there are others who, like me, will find marvels in these barriers to thought and action. A barrier gives shape, after all.

Like a landscape, science has spaces that we may freely roam, investigating phenomena and making progress. But we rebound from the walls that reason has discovered. “Adamantine” is too soft a word. Unlike cliffs or chasms, these cannot be penetrated by the intellectual equivalents of sledgehammer, bulldozer, or dynamite. They are insurmountable, impenetrable barriers. Reason brought these barriers to our attention, yet reason cannot penetrate them.

TWO OLD CHESTNUTS

For several centuries, it was thought that someone clever enough might just succeed in squaring the circle. Given a circle on a sheet of paper, construct a square with exactly the same area. The construction must be Euclidean, of course: the only instruments allowed are an unmarked ruler and a compass. Such means may seem unduly restrictive, yet with a ruler and a compass alone, we can construct a square equal in area to that of a triangle, to any square (of course), and to any pentagon, hexagon, or any regular polygon. The figures in this sequence come ever closer to resembling a circle. Who can distinguish between a circle and a thousand-sided regular polygon? Yet we cannot square the circle.

Although “squaring the circle” has become synonymous with all hopeless projects, the task was not known to be impossible until late in the nineteenth century, when the German mathematician Ferdinand Lindemann proved it so, once and for all. Before Lindemann’s proof some of the finest mathematical minds in the world made attempts at the construction. All of them failed.

In the physical world similar barriers await us. For several centuries, some of the cleverest people around tried to construct perpetual motion machines. The rewards were, of course, enormous. Not only would the person who constructed the first perpetual motion machine become the most revered person in all of history (certain spiritual leaders aside), he or she also would stand a good chance of becoming the wealthiest person who ever lived. Mills and factories would all be run by such machines, and everyone who used them would, of course, pay the holder of the patent. With such huge rewards waiting, it is not surprising that more than a few had faked the results. But all of them failed.

Their devices were often wheels with rolling weights, with weights on pivots, running water, stationary or moving magnets, rods, belts, pulleys, and a variety of other attachments. Some of the devices made Rube Goldberg look like a rank amateur.

We may construct a machine that, once set in motion, will run for an arbitrarily long time. For example, we may build a succession of wheels that, once spun, continue to revolve for an hour, a day, or even a year.

What brings every wheel to a state of immobility, sooner or later, is friction. Even as our would-be perpetual wheel began to spin, we would hear a faint noise coming from it. It takes energy to make noise, the energy of friction, slowly bleeding away the wheel's momentum.

Of course, if we set a wheel revolving out in space, it will continue to revolve virtually forever, but that is not the precise meaning of perpetual motion. The device, besides continuing to run forever, must be capable of useful work. That is what all those unsuccessful inventors were aiming at.

Again, it was not until the mid-nineteenth century that we finally understood that the project was doomed. The newly emerging theory of thermodynamics dictated that the total energy of an isolated system was the sum of two components: potential energy (the energy of position) and kinetic energy (the energy of motion). In many of the proposed designs, potential energy was continually being converted into kinetic energy and back again, as though either process might, by some magic, increase the total energy of the system. Not so. Moreover, the new thermodynamics also stated that energy of an isolated system was a conserved quantity, neither created nor destroyed.

However, none of the proposed machines was an isolated system. The friction encountered by the devices also took part in the equation. The energy of motion included not only the moving parts of the proposed device but also the movements of molecules affected by it, those in the device and those in the surrounding environment. Although the total energy of the system + environment remained constant, more and more of it would be found in the agitation of molecules of metal, wood, and air. Slowly or quickly, the kinetic energy available for moving parts would decline.

More recently, we have encountered other barriers to what can be achieved by technology. As Einstein showed, neither we nor any signal we send can travel faster than the speed of light. Likewise, quantum mechanics, the most successful physical theory of all time, tells us that we cannot predict the detailed behavior of fundamental particles such as electrons and photons. We cannot even predict the weather, according to the theory of chaotic dynamical systems.

The latter barriers are not imposed by mathematics per se, but by well-established physical theories with a mathematical form that extends from physical axioms to physical theorems. Along the way, there are mathematical steps that invoke the grander edifice lurking in the background. We must, for example, use the Pythagorean theorem to get from Einstein's basic assumption (that light appears to travel at the same speed relative to all frames of reference) to the formulas of special relativity. Mathematics will have its way.

This book has accordingly been divided into two halves, the first concerning physical impossibilities, the second devoted to mathematical ones. This division reflects not only the cosmos and the holos but also the two major parts of science. Inductive science includes physics, chemistry, astronomy, and the other so-called experimental sciences. Deductive science includes pure mathematics, applied mathematics, the theory of computation, and related fields.

The process of induction involves inferring the rules governing the behavior of physical systems from particular instances. Knowledge accretes in a succession of increasingly general layers. For example, the properties of chemical compounds, once a confusing mélange of specific observations, became perfectly comprehensible from a unified point of view with the wide adoption of Dalton's theory of the atom.

The process of deduction, on the other hand, operates on abstract systems which, because they are perfectly defined in full generality, can be subjected directly to deductive operations. We already know everything we need to know about such systems in order to find out more. Here, knowledge builds outward from the axioms of a system toward all the specific theorems that are ultimately obtainable. And here, too, knowledge develops increasingly general layers. Seemingly unrelated things such as number systems, symmetries of crystals, and permutations of letters all came to be viewed as examples of groups, for example.

Scientific knowledge is about generality. One could say that the more general a successful theory is, the more "scientific" it has become. The most general fields of inductive science are heavily mathematical—in proportion to their generality, not surprisingly. Thus quantum physics and relativity theory are practically *all* mathematics—with an interpretive framework grounded in observations. In contrast, biology has relatively little mathematics in it, but a host of observational data that ecologists and biologists are still trying to make sense of. The two greatest discoveries of biology—the Darwin-Wallace theory of evolution and the Watson-Crick discovery of DNA—are among the most general (and mathematical) parts of biology. The Darwin-Wallace theory can be viewed as a simple deduction from axioms about reproduction, the tendency for populations to grow without limit, and survival. The Watson-Crick theory, on the other hand, describes a mathematical code that is the basic structure of every living creature. As pure mathematics, neither the deduction nor the code are of great interest, but their significances for science and humanity are immense.

All of this is not to say that at least one of these barriers may be found to be illusory, but we may expect most of them to be with us for a long, long time. Such a view, with its built-in back door, is only partially cowardly.

Theories are overturned, as Thomas Kuhn has rightly observed, from time to time. That much was known long before Kuhn, who introduced the term "paradigm shift" to signal more than a mere overturn. It was Kuhn who argued that scientific revolutions have their roots not so much in data, but in how we interpret them; theories are "social constructions." This is undoubtedly true in that social preferences surely favor one direction of research over another or one conceptual framework over a different one. Cultural bias may cause a scientist to miss a result, but someone else will probably find it.

But to claim that science is one big social construction amounts to throwing the baby out with the bathwater. In fact, Kuhn's interpretation of his main example, the Copernican revolution, is incorrect. The Ptolemaic notion of planetary motions was superseded by the Copernican model for a very good reason. The Ptolemaic theory, with its complicated system of epicycles, was simply wrong. Long before Copernicus, few astronomers were very happy with epicycles. Even in the post-Ptolemaic period, several Arab astronomers were certain it was incorrect. Anyone who thinks that a social construction lurks in the labors of Kepler (who put the theory on a solid footing with his discovery of elliptical orbits) should read of research driven purely by the observational data of Tycho Brahe, and learn of Kepler's frustration when he discovered that the one social construction he did attempt, the "mysterium cosmographicum," was a dismal failure. It did not fit the data.

Today, no one doubts the Copernican theory. It is the correct theory. The extremely slight changes induced by general relativity do not change the fact that the Earth goes around the Sun and not vice versa.

It has been said, "Never say never." Will the barriers described in this book never be penetrated, leaped over, or gotten around? At the end of each chapter, I examine what appear to be exceptions to the claim or discuss potential breakthroughs that would undermine it. For example, to do away with million-year space voyages that are limited by the speed of light, science fiction writers have invented the warp drive, a device that folds space. Two parts of the space-time continuum, previously far apart, are brought into proximity, and the starship *Enterprise* leaps halfway across the galaxy. Well, be careful! The limitations here are sometimes subtle. Even if we had such a spaceship now, it would not actually violate the cosmic speed limit. After all, it wouldn't travel *through* space, but *around* it, so to speak.

Limitations on what we can know or do, whether real or only apparent, have the salutary effect of driving the scientific and technological process forward. Based on the apparent acceleration of impossible findings, it seems safe to predict that science will increasingly become entangled with things unknowable and undoable. Perhaps all the barriers, taken together, will more sharply outline this place we find ourselves in.

Math in the Cosmos

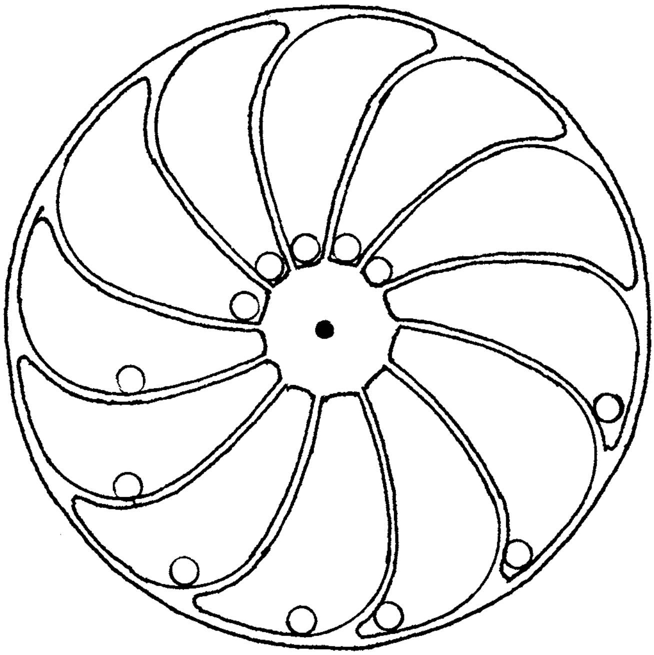
· 1 ·

The Energy Drain

Impossible Machines

> IT IS NOT POSSIBLE TO BUILD A MACHINE THAT RUNS FOREVER WITH NO SOURCE OF ENERGY, YET PRODUCES USABLE ENERGY.

LIKE SQUARING THE CIRCLE, the problem of making a machine that would run forever has probably absorbed more man-hours than the building of the Egyptian pyramids. The idea of the "perpetuum mobile" is nearly as old as the pyramids. It is so seductive that I run a certain risk even writing about the subject. I mean, why shouldn't it be possible to build a machine that runs forever? I'll even throw in the requirement that it produces a little usable energy. In my mind's eye I see a simple but wonderful contrivance that will (in my mind) run forever. It is nothing more than a wheel with beautifully curved spokes, each engraved with a track that carries a steel ball.



A perpetual wheel

Clearly, the balls on the right-hand side of the wheel shown here are going to weigh more heavily, in turning the wheel, than those on the left. Hence, with barely a nudge, the wheel will begin to rotate. As each new spoke comes into position, another steel ball rolls to the right. In fact, because of the continuing downward acceleration of the right half of the wheel, it will spin with increasing velocity. One might imagine that it will spin faster and faster, without limit, until it literally flies into pieces. But at some point during this potentially fatal acceleration, the centrifugal force on the balls, particularly those on the left side, will prevent them from rolling toward the hub, as they would normally do. At such a pass, the wheel will continue to spin, gradually slowing, owing to frictional forces. As soon as the wheel slows enough to allow the balls on the left half of the wheel to roll to the right once more, it will, of course, begin to speed up again.

The wheel will therefore seek and find, ultimately, the narrowest range of speeds wherein it will remain spinning like the governor of a steam engine. Unlike a governor, however, this wheel has no external source of power! It will spin at exactly this rate until the parts begin to wear out. Sooner or later, something will give way and the wheel will roll to a clattering halt.

Nevertheless, it has been a valid demonstration of perpetual motion, at least conceptually. We could construct the wheel and its parts of modern space-age alloys, with Teflon bearings and silicon lubricants. It might be guaranteed to run for a thousand years. Or a million. Put enough care into it, and the latest version of the wheel will run for a billion years. We are limited only by the lifetime of the universe, whatever that might turn out to be.

The time requirement of the problem—that the device operate perpetually—is clearly unrealistic. As I just hinted, there is rather strong evidence that the universe will one day cease to exist altogether, taking our wonderful machine with it. But in a purely theoretical sense, the machine is potentially capable. It would, if it could, spin forever.

Now, if it were possible to eliminate all friction from an ordinary wheel, it, too, would spin forever, thanks to a law first discovered by the great British natural scientist Isaac Newton. This motion, while perpetual within any practical meaning of the word, is not the sort of motion we have just been discussing, as it produces no new energy. I will therefore call it type one perpetual motion. Earth satellites are essentially type one perpetual motion machines.

In type two perpetual motion we expect the device not only to exhibit potentially eternal motion but also to produce energy while doing so. In my opinion, type two perpetual motion is the more exciting of the two kinds.

Can the wheel in my mind's eye do actual work? Think of my wonderful wheel again, now writ large.

It will be found inside a secret government building somewhere in the desert. The wheel will be forty stories tall and made of more than a million tons of steel, Kevlar, diamond, and other awesome materials. We'll take an elevator to the fifteenth floor of the building, near the hub. We'll stare in stupefaction from our observation window as, one by one, the great spokes swing by, each with a great steel ball rolling along it with a terrifying rumbling noise. The secret installation sounds like a cosmic bowling alley.

Now, just before the great wheel reaches that critical speed where centrifugal force begins to eat away noticeably at its acceleration, a giant dynamo engages gears with the wheel, and outside the building power lines surge with millions of kilowatts of power, all of it apparently free.

That's what I mean by energy. Even a type two machine that produces the tiniest excess of energy may nevertheless be scaled up to almost any size—or multiplied a thousandfold by mass production. Indeed, any macroscopic machine that runs "forever" under ordinary circumstances must be overcoming frictional forces and must be type two.

When scientists say that perpetual motion is impossible, they mean only that a machine that produces more energy than it consumes (typically none) is impossible. Type two machines are impossible. The reason for the impossibility lies with a fundamental tenet of modern physics, the law of conservation of energy. I will come back to this law later, subjecting it to a scrutiny it rarely receives. (Perhaps there is a flaw somewhere.)

In the meantime, we have been examining "machines" in the ordinary sense of the word, macroscopic systems of metal, plastic, even wood. But the microscopic world is inhabited by other systems: atoms in a state of apparent perpetual motion, electrons that whirl incessantly about nuclei, albeit in an unmanifest state, even at absolute zero. They are not producing any new energy, but their motion appears to be truly perpetual. After all, there is nothing to "wear out."

From this point of view, the closest we have yet come to a type one perpetual motion machine, here on Earth, is a doughnut-shaped ring of some hi-tech superconducting material in which electrons have been set circulating. Back in the 1980s, newspapers and science magazines frequently showed pictures of a superconducting ring floating above a magnet. How did it work?

The doughnut or torus is made from one of the latest metallo-ceramic hybrid materials that "superconducts" electrons at a temperature that is low, but not too low to be achievable in a laboratory. Superconduction works like ordinary conduction except that electrons move through the material without the production of heat (and subsequent loss of energy) that accompanies normal conduction.

Electrons in such a ring are set in motion by placing the ring in a strong magnetic field. The apparent levitation of the ring is not part of