THEORY OF CODE DIVISION MULTIPLE ACCESS COMMUNICATION

Kamil Sh. Zigangirov

IEEE SERIES ON COMMUNICATION

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PREFACE

The objective of this book is to provide an introduction to code division multipleaccess (CDMA) communications. Our motivation for emphasizing CDMA communication is a result of the technological developments that have occurred during the past decade. We are currently witnessing an explosive growth in wireless communication and cellular mobile radio systems, which are based on different multiple-access techniques. We anticipate that, in the near future, we will see a replacement of the current time- and frequency division methods in wireless communication and mobile radio by CDMA.

This textbook originates as an adaptation for undergraduate study of the wellknown book *CDMA*, *Principles of Spread Spectrum Communication* by A.J. Viterbi and is based on courses which I taught several years at Lund University in Sweden. The reader can see an indubitable influence of Viterbi's book on the content of this book. In particular, our treatment of direct-sequence CDMA follows the ideas and methods of Viterbi's book, but for completeness we also include in the book a consideration of frequency hopping CDMA and pulse position hopping ("time hopping") CDMA. We have studied also in more detail forward transmission in the direct-sequence CDMA system. Furthermore, we consider it necessary to include in our textbook information-theoretical analysis of CDMA communication.

My understanding of the field, and hence the content of this text, has been influenced by a number of books on the topic of digital and spread spectrum communications. In addition to the pioneering book by Viterbi I have to mention *Digital Communication* by J.G. Proakis and *Introduction to Spread Spectrum Communication* by R.L. Peterson, R.E. Ziemer, and D.E. Borth. Readers familiar

with these books will recognize their influence here. Numerous other important books and papers are mentioned in the comments to the chapters.

I am grateful for the warm support of the Department of Information Technology of Lund University while this book was being written. I am particularly indebted to my friend Rolf Johannesson, who supported my work on the manuscript of this book. I would like to express appreciation to my colleagues in the department, especially to John Anderson and Göran Lindell, for discussions of related problems of communication theory. Being Series Editor, John Anderson carefully read the original manuscript and made many corrections. Many thanks are also due to the reviewer, Roger Ziemer, for the substantial work he did in improving the text of the book.

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1

INTRODUCTION TO CELLULAR MOBILE RADIO COMMUNICATION

The subject of this book is code division multiple access (CDMA) communications. A major application of CDMA is wireless communication including mobile radio. In this chapter we introduce the basic concepts of mobile radio systems, including cellular concepts, consider the general structure of a cellular system, and study different principles of multiple-access (time, frequency, and code division) and spread spectrum concepts.

This chapter begins with an overview of the principles of cellular radio systems. Next, given the focus on simultaneous wideband transmission of all users over a common frequency spectrum, we consider direct-sequence CDMA systems, frequency-hopped CDMA systems, and pulse position-hopped CDMA systems. The chapter concludes with a description of this book. The book is devoted to the analysis of different aspects of CDMA communication. Given the rapid and continuing growth of cellular radio systems throughout the world, CDMA digital cellular radio systems will be the widest-deployed form of spread spectrum systems for voice and data communication. It is a major technology of the twenty-first century.

1.1 CELLULAR MOBILE RADIO SYSTEMS

A cellular radio system provides a wireless connection to the public telephone network for any user location within the radio range of the system. The term *mobile* has traditionally been used to classify a radio terminal that can be moved during



Figure 1.1. An illustration of a cellular system.

communication. Cellular systems accommodate a large number of mobile units over a large area within a limited frequency spectrum. There are several types of radio transmission systems. We consider only *full duplex systems*. These are communication systems that allow simultaneous two-way communication. Transmission and reception for a full duplex system are typically on two different channels, so the user may constantly transmit while receiving signals from another user.

Figure 1.1 shows a basic cellular system that consists of *mobiles*, *base stations*, and a *switching center*. Each mobile communicates via radio with one or more base stations. A call from a user can be transferred from one base station to another during the call. The process of transferring is called *handoff*.

Each mobile contains a *transceiver* (transmitter and receiver), an antenna, and control circuitry. The base stations consist of several transmitters and receivers, which simultaneously handle full duplex communications and generally have towers that support several transmitting and receiving antennas. The base station connects the simultaneous mobile calls via telephone lines, microwave links, or fiber-optic cables to the switching center. The switching center coordinates the activity of all of the base stations and connects the entire cellular system to the public telephone network.

The channels used for transmission from the base station to the mobiles are called *forward* or *downlink channels*, and the channels used for transmission from the mobiles to the base station are called *reverse* or *uplink channels*. The two channels responsible for call initiation and service request are the *forward control channel* and *reverse control channel*.

Once a call is in progress, the switching center adjusts the transmitted power of the mobile (this process is called *power control*¹) and changes the channel of the mobile and base station (handoff) to maintain call quality as the mobile moves in and out of range of a given base station.

¹Sometimes the mobile adjusts the transmitted power by measuring the power of the received signal (so-called *open-loop power control*).



Figure 1.2. An illustration of the cellular frequency reuse concept.

The cellular concept was a major breakthrough in solving the problem of spectral congestion. It offered high system capacity with a limited spectrum allocation. In a modern conventional mobile radio communication system, each base station is allocated a portion of the total number of channels available to the entire system and nearby base stations are assigned different groups of channels so that all the available channels are assigned to a relatively small number of neighboring base stations. Neighboring base stations are assigned different groups of channels so that interference between the users in different cells is small.

The idealized allocation of cellular channels is illustrated in Figure 1.2, in which the cells are shown as contiguous hexagons. Cells labeled with the same number use the same group of channels. The same channels are never reused in contiguous cells but may be reused by noncontiguous cells. The κ cells that collectively use the complete set of available frequencies is called a *cluster*. In Figure 1.2, a cell cluster is outlined in bold and replicated over the coverage area. Two cells that employ the same allocation, and hence can interfere with each other, are separated by more than one cell diameter.

The factor κ is called the *cluster size* and is typically equal to 3, 4, 7, or 12. To maximize the capacity over a given coverage area we have to choose the smallest possible value of κ . The factor $1/\kappa$ is called the *frequency reuse factor* of a cellular system. In Figure 1.2 the cluster size is equal to 7, and the frequency reuse factor is equal to 1/7.

EXAMPLE 1.1

The American analog technology standard, known as Advanced Mobile Phone Service (AMPS), employs frequency modulation and occupies a 30-kHz frequency slot for each voice channel [47]. Suppose that a total of 25-MHz bandwidth is allocated to a particular cellular radio communication system with cluster size 7. How many channels per cell does the system provide?

Solution

Allocation of 12.5 MHz each for forward and reverse links provides a little more than 400 channels in each direction for the total system, and correspondingly a little less than 60 per cell.

The other-cell interference can be reduced by employing sectored antennas at the base station, with each sector using different frequency bands. However, using sectored antennas does not increase the number of slots and consequently the frequency reuse factor is not increased.

A multiple access system that is more tolerant to interference can be designed by using digital modulation techniques at the transmitter (including both source coding and channel error-correcting coding) and the corresponding signal processing techniques at the receiver.

1.2 FREQUENCY DIVISION AND TIME DIVISION MULTIPLE ACCESS

Multiple access schemes are used to allow many mobile users to share simultaneously a common bandwidth. As mentioned above, a full duplex communication system typically provides two distinct bands of frequencies (channels) for every user. The forward band provides traffic from the base station to the mobile, and the reverse band provides traffic from the mobile to the base station. Therefore, any duplex channel actually consists of two simplex channels.

Frequency division multiple access (FDMA) and *time division multiple access (TDMA)* are the two major access techniques used to share the available bandwidth in a conventional mobile radio communication systems.

Frequency division multiple access assigns individual channels (frequency bands) to individual users. It can be seen from Figure 1.3 that each user is allocated a unique frequency band. These bands are assigned on demand to users who request service. During the period of the call, no other user can share the same frequency band. The bandwidths of FDMA channels are relatively narrow (25–30 kHz) as each channel supports only one call per carrier. That is, FDMA is usually implemented in narrowband systems. If an FDMA channel is not in use (for example, during pauses in telephone conversation) it sits idle and cannot be used by other users to increase the system capacity.



Figure 1.3. FDMA scheme in which different users are assigned different frequency bands.



Figure 1.4. TDMA scheme in which each user occupies a cyclically repeating time slot.

Time division multiple access systems divide the transmission time into time slots, and in each slot only one user is allowed to either transmit or receive. It can be seen from Figure 1.4 that each user occupies cyclically repeating wording, so a channel may be thought of as a particular time slot that reoccurs at slot locations in every frame. Unlike in FDMA systems, which can accommodate analog frequency modulation (FM), digital data and digital modulation must be used with TDMA.

TDMA shares a single carrier frequency with several users, where each user makes use of nonoverlapping time slots. Analogously to FDMA, if a channel is not in use, then the corresponding time slots sit idle and cannot be used by other users. Data transmission for users of a TDMA system is not continuous but occurs in bursts. Because of burst transmission, synchronization overhead is required in TDMA systems. In addition, guard slots are necessary to separate users. Generally, the complexity of TDMA mobile systems is higher compared with FDMA systems.

EXAMPLE 1.2

The global system for mobile communications (GSM) utilizes the frequency band 935–960 MHz for the forward link and frequency range 890–915 MHz for the reverse link. Each 25-MHz band is broken into radio channels of 200 kHz. Each radio channel consists of eight time slots. If no guard band is assumed, find the number of simultaneous users that can be accommodated in GSM. How many users can be accommodated if a guard band of 100 kHz is provided at the upper and the lower end of the GSM spectrum?

Solution

The number of simultaneous users that can be accommodated in GSM in the first case is equal to

$$\frac{25 \cdot 10^6}{(200 \cdot 10^3)/8} = 1000$$

In the second case the number of simultaneous users is equal to 992.

Each user of a conventional multiple access system, based on the FDMA or the TDMA principle, is supplied with certain resources, such as frequency or time slots, or both, which are disjoint from those of any other user. In this system, the multiple access channel reduces to a multiplicity of single point-to-point channels. The transmission rate in each channel is limited only by the bandwidth and time allocated to it, the channel degradation caused by background noise, multipath fading, and shadowing effects.

Viterbi [47] pointed out that this solution suffers from three weaknesses. The first weakness is that it assumes that all users transmit continuously. However, in a two-person conversation, the percentage of time that a speaker is active, that is, talking, ranges from 35% to 50%. In TDMA or FDMA systems, reallocation of the channel for such brief periods requires rapid circuit switching between the two users, which is practically impossible.

The second weakness is the relatively low frequency reuse factor of FDMA and TDMA. As we can see from Example 1.1 the frequency reuse factor 1/7 reduces the number of channels per cell in AMPS from 400 to less than 60.

Using antenna sectorization (Fig. 1.5) for reducing interference does not increase system capacity. As an example, a cell site with a three-sectored antenna has an interference that is approximately one-third of the interference received by an omnidirectional antenna. Even with this technique, the interference power received at a given base station from reused channels in other cells is only about 18 dB below the signal power received from the desired user of the same channel in the given cell. Reuse factors as large as 1/4 and even 1/3 have been considered and even used, but decreasing the distance between interfering cells increases the other-cell interference to the point of unacceptable signal quality.



Figure 1.5. A three-sectored antenna in a single isolated cell.

A third source of performance degradation, which is common to all multiple access systems, particularly in terrestrial environments, is fading. Fading is caused by interference between two or more versions of the transmitted signal that arrive at the receiver at slightly different time. This phenomenon is particularly severe when each channel is allocated a narrow bandwidth, as for FDMA systems.

1.3 DIRECT SEQUENCE CDMA

A completely different approach, realized in CDMA systems, does not attempt to allocate disjoint frequency or time resources to each user. Instead the system allocates all resources to all active users.

In *direct sequence* (DS) CDMA systems, the narrowband message signal is multiplied by a very large-bandwidth signal called the *spreading signal*. All users in a DS CDMA system use the same carrier frequency and may transmit simultaneously. Each user has its own spreading signal, which is approximately orthogonal to the spreading signals of all other users. The receiver performs a correlation operation to detect the message addressed to a given user. The signals from other users appear as noise due to decorrelation. For detecting the message signal, the receiver requires the spreading signal used by the transmitter. Each user operates independently with no knowledge of the other users (*uncoordinated transmission*).

Potentially, CDMA systems provide a larger *radio channel capacity* than FDMA and TDMA systems. The radio channel capacity (not to be confused with *Shannon's channel capacity*, see Chapter 8) can be defined as the maximum number K_0 of simultaneous users that can be provided in a fixed frequency band. Radio channel capacity is a measure of the *spectrum efficiency* of a wireless system. This parameter is determined by the *required signal-to-noise ratio* at the input of the receiver and by the channel bandwidth W.

To explain the principle of DS CDMA let us consider a simple example. Suppose that two users, user 1 and user 2, located the same distance from the base station, wish to send the *information* (or *data*) sequences² $u^{(1)} = u_0^{(1)}, u_1^{(1)}, u_2^{(1)}, u_3^{(1)} = 1, -1, -1, 1$ and $u^{(2)} = u_0^{(2)}, u_1^{(2)}, u_2^{(2)}, u_3^{(2)} = -1, 1, -1, -1,$ respectively, to the base station. First, user 1 maps the data sequence $u^{(1)}$ into the data signal $u^{(1)}(t)$, and user 2 maps $u^{(2)}$ into the data signal $u^{(2)}(t)$, such that the real number 1 corresponds to a positive rectangular pulse of unit amplitude and duration *T*, and the real number -1 corresponds to a negative rectangular pulse of the same amplitude and same duration (Fig. 1.6a). Then both users synchronously transmit the data signals over the *multiple access adding channel*. Because each pulse corresponds to the transmission of one bit, the transmission rate R = 1/T (bit/s) for each user and the overall rate is 2/T (bit/s).

²In information-theoretic literature, binary sequences consist of symbols from the binary logical alphabet {0, 1}. In CDMA applications it is more convenient to use the binary real number alphabet {1, -1}. The mapping $0 \rightleftharpoons 1$, $1 \rightleftharpoons -1$ establishes a one-to-one correspondence between sequences of binary logical symbols and sequences of binary real numbers (see also Chapter 4).





If the *propagation delay* and the *attenuation* in the channel for both signals are the same, the output of the adding channel, that is, the input of the base station receiver, is the sum of identically attenuated transmitted signals. In our example the received signal is nonzero only in the third interval (Fig. 1.6b). Then the receiver cannot decide which pulses were sent by the users in the first, second, and fourth intervals, but it knows that in the third interval both of the users have sent negative pulses, and correspondingly $u_2^{(1)} = -1$, $u_2^{(2)} = -1$.

Suppose now that instead of sending the data signals $u^{(1)}(t)$ and $u^{(2)}(t)$ directly over the multiple access adding channel, the users first *spread* them, that is, multiply them by the *spreading signals* $a^{(1)}(t)$ and $a^{(2)}(t)$, respectively. The signals $a^{(1)}(t)$ and $a^{(2)}(t)$, presented in Figure 1.6c, are sequences of positive and negative unit amplitude rectangular pulses of duration T_c , $T_c < T$ (in our example $T_c = T/4$). These pulses are called *chips*, and T_c is called the *chip duration*. We will always consider the case when the ratio $T/T_c = N$ is an integer. The spread signals $u^{(1)}(t) \cdot a^{(1)}(t)$ and $u^{(2)}(t) \cdot a^{(2)}(t)$ (Fig. 1.6d) are sent over the adding channel. The received signal $r(t) = u^{(1)}(t) \cdot a^{(1)}(t) + u^{(2)}(t) \cdot a^{(2)}(t)$ is presented in Figure 1.6e.

As we will see in Chapter 2, the bandwidth of the signal formed by the sequence of positive and negative pulses of duration *T* is proportional to 1/T. Therefore, the bandwidth of the signals $u^{(k)}(t)$, k = 1, 2, is proportional to the transmission rate *R* and the bandwidth *W* of the spread signals is proportional to $1/T_c$. The ratio $T/T_c \approx W/R$ that characterizes the increase of the bandwidth by spreading is called the *spreading factor* or *processing gain*.

The base station receiver *despreads* the received signal r(t), that is, multiplies r(t) by the spreading signals $a^{(1)}(t)$ and $a^{(2)}(t)$. The results of despreading are given in Figure 1.6f. It is obvious that the receiver can correctly decide which data sequences were transmitted by the users in each of the four intervals.

The spreading signal $a^{(k)}(t)$, k = 1, 2, can be generated by mapping the *spreading sequences* $a^{(k)} = a_0^{(k)}, a_1^{(k)}, \ldots, a_n^{(k)}, \ldots, a_n^{(k)} \in \{1, -1\}$ into sequences of positive and negative pulses, analogous to mapping the data sequence $u^{(k)}$, k = 1, 2, into the data signal $u^{(k)}(t)$. Suppose now that we repeat each symbol $u_n^{(k)}$ of the data sequence $u^{(k)}$ N times, $N = T/T_c = W/R$, to get a sequence $v^{(k)} = v_0^{(k)}, v_1^{(k)}, \ldots, v_n^{(k)} \ldots$ where $v_n^{(k)} = u_{\lfloor n/N \rfloor}^{(k)}$. Here $\lfloor x \rfloor$ means the largest integer that is less or equal to x. (For example, in Fig. 1.6 we have N = 4.) Then we multiply symbols of the sequence $v^{(k)}$ by symbols of the sequence $a^{(k)}$. We get the sequence

$$\mathbf{v}^{(k)} * \mathbf{a}^{(k)} \stackrel{\text{def}}{=} v_0^{(k)} a_0^{(k)}, v_1^{(k)} a_1^{(k)}, \dots, v_n^{(k)} a_n^{(k)}, \dots$$
(1.1)

If we map the symbols of the sequence $v^{(k)} * a^{(k)}$ into a sequence of positive and negative pulses, as we did before, we get the spread signals $u^{(k)}(t) \cdot a^{(k)}(t)$, k = 1, 2. This is an alternative way of spreading.

The operation of repeating the symbol $u_n^{(k)}N$ times can be considered as *encoding*. The code is called the *repetition code*³; it consists of two *codewords*: N is the *block length* and r = 1/N (bit/symbol) is the *code rate*. In the general, we will consider more complicated code constructions. Obviously, for rectangular pulses the operations of mapping sequences into signals and multiplication of signals/sequences are permutable, but for nonrectangular pulses these operations are, generally speaking, not permutable. Below we will consider both ways of generating spread signals.

Figure 1.6 corresponds to the *synchronous model* of the transmission, when the received signals from both transmitters are in the same phase. But the situation would not differ significantly in the *asynchronous case* (Fig. 1.7), when the received signals are in different phases. Using the same procedure of despreading as in the synchronous case, the receiver can even more easily recover both transmitted sequences $u^{(1)}$ and $u^{(2)}$. The necessary condition of the correct despreading is the knowledge of the phases of both transmitted signals $u^{(1)}(t) \cdot a^{(1)}(t)$ and $u^{(2)}(t) \cdot a^{(2)}(t)$. In other words, although the transmitters of the different users can be unsynchronized, the transmitter and the receiver corresponding to a particular user should be *synchronized*.

In general, we do not have two but K simultaneous active users and they operate asynchronously. A realistic model of the received signal should also include additive white Gaussian noise (AWGN) $\xi(t)$. The received (baseband) signal is

$$r(t) = \sum_{k=1}^{K} \sqrt{P^{(k)}} u^{(k)}(t - \delta^{(k)}) a^{(k)}(t - \delta^{(k)}) + \xi(t)$$
(1.2)



Figure 1.7. Example of the transmission over an adding channel, asynchronous case.

³In the literature, repetition coding is sometimes not considered as a coding and the transmission is called *uncoded transmission*.

where $P^{(k)}$ is the power of the signal from the *k*th user at the base station and $\delta^{(k)}$ is the *k*th user's *time offset*. The time offset values $\delta^{(k)}$ characterize asynchronism between different users, propagation delay, etc. If we are interested in the reception of the information from the *k*th user, we will present the received signal (1.2) as

$$r(t) = \sqrt{P^{(k)}} u^{(k)}(t - \delta^{(k)}) a^{(k)}(t - \delta^{(k)}) + \xi^{(k)}(t)$$
(1.3)

where the total noise

$$\xi^{(k)}(t) = \sum_{k' \neq k} \sqrt{P^{(k')}} u^{(k')}(t - \delta^{(k')}) a^{(k')}(t - \delta^{(k')}) + \xi(t)$$
(1.4)

includes the interference from the (K - 1) other active users and additive noise. If the receiver is synchronized with the *k*th user, that is, $\delta^{(k)}$ is known, the despreading of the signal, that is, multiplication by $a^{(k)}(t - \delta^{(k)})$, reduces the problem in the case of repetition coding to detection of the known signal in noise (see Chapter 3) or, in the case of more complicated codes, to the decoding problem (see Chapter 4).

We emphasize that the model of uplink communication in the DS CDMA system considered here is the *information-theoretic* model. The model that is studied in *communication theory* describes processes in the transmitter-receiver, particularly the processes of *modulation-demodulation*, in more detail.

The receiver for binary DS CDMA signaling schemes can have one of two equivalently performing structures, a correlator implementation and a matched-filter implementation (see Chapters 2 and 3). The correlator receiver performs a correlation operation with all possible signals sampling at the end of each T-second signaling interval and comparing the outputs of the correlators. In the matched-filter receiver, correlators are replaced by matched filters.

The model of uplink DS CDMA communication with *K* users is presented in Figures 1.8 and 1.9. The base station receiver includes *K* demodulators synchronized with the modulators of the *K* transmitters. Assuming perfect synchronization, the output of the *k*th demodulator, k = 1, 2, ..., K, is the sequence $\{v_n^{(k)}a_n^{(k)} + \xi_n^{(k)}\}$, where the noise components $\xi_n^{(k)}$ are contributions of all other active users and AWGN. Despreading consists of multiplication by the spreading sequence $\{a_n^{(k)}\}$. The input of the *k*th decoder is the sequence

$$\{v_n^{(k)} + \xi_n^{(k)} a_n^{(k)}\}$$
(1.5)

The output of the *k*th decoder is the decoded information sequence $\{\hat{u}_n^{(k)}\}$.

Using power control, the switching center can adjust the powers of transmitted signals such that the powers of the received signals would be approximately the same. If the power control is perfect the power of the received signal equals P independently from the user, that is, $P^{(k)} = P$, k = 1, 2, ..., K. Each receiver at the base station of a single-cell communication system receives a



Figure 1.8. The model of uplink transmission in the DS CDMA system.



Figure 1.9. The model of the base station receiver of the DS CDMA system.

composite waveform containing the desired signal of power P, the component due to background AWGN $\xi(t)$, and the other-user interference component of power P(K-1). Then the average one-sided total noise power spectral density⁴ becomes

$$I_0 = (K-1) \frac{P}{W} + N_0 \tag{1.6}$$

⁴In this book we will later use only two-sided power spectral density, which for modulated signals is equal to half of the one-sided power spectral density.

where N_0 is the one-sided power spectral density of the AWGN and W is the signal bandwidth.

As we will see later, the important parameter that is the figure merit of the digital modem is *bit energy-to-noise density ratio* (for brevity we will call this parameter *signal-to-noise ratio*, *SNR*)

$$\frac{E_{\rm b}}{I_0} = \frac{P}{I_0 R},\tag{1.7}$$

where $E_{\rm b} = P/R$ is the received energy per bit. Combining (1.6) and (1.7) we get

$$\frac{E_{\rm b}}{I_0} = \frac{P/R}{(K-1)\frac{P}{W} + N_0} = \frac{W/R}{(K-1) + \frac{N_0W}{P}}$$
(1.8)

From (1.8) follows the next formula for the radio channel capacity K_0 of a single-cell CDMA system:

$$K_0 = 1 + \frac{W/R}{E_b/I_0} - \frac{N_0W}{P} = 1 + \frac{W/R}{E_b/I_0} - \frac{W/R}{E_b/N_0}$$
(1.9)

or, because we usually can neglect the influence of the background AWGN,

$$K_0 \approx 1 + \frac{W/R}{E_{\rm b}/I_0}$$
 (1.10)

The ratio W/R (in Hz/bit/s) was defined above as the *spreading factor* or the *processing gain*. Typical values of W/R range from one hundred (20 dB) to one million (60 dB). The required signal-to-noise ratio depends on the type of error-correcting coding used, the type of noise, and the limitations on the output probability of error. Under the condition that the number of active users K is large, we may consider the total noise as Gaussian noise of one-sided power spectral density I_0 . Then, if the trivial repetition code is used, the bit error probability is the same as for uncoded transmission, that is,

$$P_{\rm b} = Q\left(\sqrt{\frac{2E_{\rm b}}{I_0}}\right) \tag{1.11}$$

where the Q function defined by the integral

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp(-y^{2}/2) \, \mathrm{d}y$$
 (1.12)

can be upperbounded by the inequality (Problem 1.5)

$$Q(x) \le \frac{1}{2} \exp(-x^2/2), \ x \ge 0$$
 (1.13)

If the repetition code is used, we get from Formulas (1.10)-(1.13)

$$K_0 \approx 1 - \frac{W/R}{\ln 2P_{\rm b}} \tag{1.14}$$

For the voice channel the required bit error rate is in the range $10^{-3}-10^{-4}$, and the required signal-to-noise ratio E_b/I_0 is in the interval 4–8 dB, depending on the error correction code.

EXAMPLE 1.3

If the repetition code is used in the communication system and the required bit error rate is 10^{-4} , what is the required signal-to-noise ratio? What is the required E_b/I_0 if $P_b = 10^{-5}$?

Solution

Using Formula (1.11) we get that to $P_{\rm b} = 10^{-4}$ corresponds $E_{\rm b}/I_0 = 6.92$ (8.40 dB) and to $P_{\rm b} = 10^{-5}$ corresponds $E_{\rm b}/I_0 = 9.09$ (9.59 dB). If we use the upper bound (1.13) we get that if $E_{\rm b}/I_0 = 6.12$, then $P_{\rm b} < 4.96 \cdot 10^{-4}$, and if $E_{\rm b}/I_0 = 9.09$, then $P_{\rm b} < 5.6 \cdot 10^{-5}$.

Suppose further that two more processing features are added to the CDMA system to diminish interference. The first involves the monitoring of users' activity such that each transmitter is switched off or reduces its power during the periods of no user activity. In a two-way telephone conversation, the activity of each of the speakers is $1/\gamma_v$, where $\gamma_v \approx 8/3 \approx 2.67$; therefore, we can, in principle, reduce the interference noise in Formulas (1.6)–(1.10) by the factor γ_v . The parameter γ_v is called the *voice activity gain*.

Similarly, if we assume that the population of mobiles is uniformly distributed in the area of the single isolated cell, employing a sectored antenna reduces the interference noise by the *antenna gain* γ_a . For a three-sectored antenna, this gain is less than 3 and can be estimated as $\gamma_a \approx 2.4$.

To calculate the capacity of the entire CDMA system, not only of a single isolated cell, we have to include in I_0 the one-sided power spectral density N_{oc} of the other-cell interference noise. Let us suppose that the frequency reuse factor of the CDMA system is equal to 1, that is, all users in all cells employ the common spectral allocation of W Hz. It was shown previously [47] that the total interference from the users in all the other cells equals approximately 0.6 of that caused by all the users in the given cell (other-cell relative interference factor f is equal to 0.6), that is, $N_{oc} = 0.6(K - 1)P/W$. Thus, in consideration of the total system capacity, the interference term of I_0 should be increased by the factor 1.6. Finally, introducing the voice activity and antenna gain factors,

 γ_v and γ_a , and the other-cell relative interference factor, f, into the total noise power spectral density expression yields

$$I_0 = (K - 1)\frac{P}{W} \frac{1 + f}{\gamma_v \gamma_a} + N_0$$
(1.15)

Thus, analogously to Formula (1.9), we get the following expression for the total radio channel capacity of the CDMA system [47]:

$$K_{0} = 1 + \frac{W/R}{E_{b}/I_{0}} \frac{\gamma_{v}\gamma_{a}}{1+f} - \frac{N_{0}W}{P} \frac{\gamma_{v}\gamma_{a}}{1+f} = 1 + \frac{W/R}{E_{b}/I_{0}} \frac{\gamma_{v}\gamma_{a}}{1+f} - \frac{W/R}{E_{b}/N_{0}} \frac{\gamma_{v}\gamma_{a}}{1+f}$$
(1.16)

or, if the AWGN is negligible,

$$K_0 \approx 1 + \frac{W/R}{E_b/I_0} \frac{\gamma_v \gamma_a}{1+f}$$
 (1.17)

EXAMPLE 1.4 [47]

Consider a cellular system with voice activity gain $\gamma_v = 2.67$, antenna gain $\gamma_a = 2.4$, required signal-to-noise ratio $E_b/I_0 = 4$ (6 dB), and other-cell relative interference factor f = 0.6. What is the radio channel capacity of this system?

Solution

Using Formula (1.17), we get

$$K_0 \approx W/R$$

The radio channel capacity is approximately equal to the spreading factor.

In Example 1.1 and Example 1.2 we mentioned two standards, AMPS and GSM. They standardize non-CDMA systems. The first DS CDMA system standardized as Interim Standard 95 (IS-95) [44] was adopted in 1993. IS-95 is specified for uplink operation in 824–849 MHz and for downlink in 869–894 MHz.

EXAMPLE 1.5

Each channel of the CDMA system IS-95 occupies 1.25 MHz of the spectrum on each one-way link. Bands of 25 MHz are available in each direction. The maximum user rate is R = 9.6 kb/s. If a minimum acceptable E_b/I_0 is 6 dB, determine the capacity of a CDMA system using

- a) Omnidirectional base station antennas and no voice activity detection and
- b) Three-sectored antennas at the base station with $\gamma_a = 2.4$ and voice activity detection with $\gamma_v = 2.67$

The received signal power P is 10^{-11} W, the one-sided AWGN power spectral density $N_0 = 10^{-17}$ W/Hz, and the other-cell relative interference factor f = 0.6.

Solution

From Formula (1.17) we have for each channel

- a) $K_0 = 1 + \frac{1.25 \cdot 10^6 / 9.6 \cdot 10^3}{4 \cdot 1.6} \frac{10^{-17} \cdot 1.25 \cdot 10^6}{1.6 \cdot 10^{-11}} \approx 1 + 18.8 0.8 = 19$ b) $K_0 \approx 1 + 18.8 \cdot 2.4 \cdot 2.67 - 0.8 \cdot 2.4 \cdot 2.67 =$
 - $1 + 120.3 5.1 \approx 115$

Because the system has 25/1.25 = 20 channels in each link, the total capacity is equal to 380 in the first case and 2300 in the second case.

Our last example of this section concerns the third-generation (3G) mobile communication systems, based on wideband CDMA (WCDMA) [55]. For WCDMA there are available bands 1920–1980 MHz in reverse direction and 2110– 2170 MHz in forward direction, that is, 60 MHz in each direction. The speech codec in WCDMA employs the Adaptive Multi-Rate (AMR) technique standardized in 1999. It has eight source rates, from 4.75 kb/s up to 12.2 kb/s.

EXAMPLE 1.6

Each channel of the WCDMA system occupies 5 MHz of the spectrum on each link. Assume that the user rate 12.2 kb/s. The other parameters are the same as in Example 1.5. Find the capacity of the WCDMA system under the given conditions.

Solution

From Formula (1.17) we get for each channel

$$K_0 = 1 + \frac{5 \cdot 10^6 / 12.2 \cdot 10^3}{4 \cdot 1.6} \cdot 2.4 \cdot 2.6 - \frac{10^{-17} \cdot 5 \cdot 10^6}{1.6 \cdot 10^{-11}} \cdot 2.4 \cdot 2.6$$

= 1 + 409 - 20 = 390

Under the given conditions, the total capacity of the WCDMA system equals $60 \cdot 390/5 = 4680$.

The mathematical model of the CDMA system considered above is a model of many-to-one transmission. Strictly speaking, it describes only reverse link transmission. In Chapter 3 we show that Formula (1.17) for the radio channel capacity is valid also for forward link. The forward link transmission that is one-to-many transmission has some advantages in comparison to many-to-one transmission. First, the signals transmitted to different users can be synchronized and accommodated by a pilot signal, such that the users can use coherent receivers. For the reverse link, a pilot signal is not always used because of power limitations. Second, because the transmitter knows the transmitted information sequences of all the users, it can in principle use this information in the encoding process, and

improve the performance of the overall system. In this case we can talk about *coordinated transmission* or *partially coordinated transmission*. We consider this problem in Chapter 9.

In the DS CDMA system each of the active users occupies in each time instance all wideband channels. In the next section we consider a system in which the wideband channel is divided into narrow frequency bands. Each of the active users occupies in each time instance only one band and periodically changes this band.

1.4 FREQUENCY-HOPPED CDMA

Conventional frequency-hopped (FH) CDMA is a digital multiple access system in which individual users select one of Q frequencies within a wideband channel as carrier frequency. The pseudorandom changes of the carrier frequencies randomize the occupancy of a specific band at any given time, thereby allowing for multiple access over a wide range of frequencies. In a conventional FH CDMA system, the total hopping bandwidth W is divided into Q narrow bands each of bandwidth B, where B = W/Q. Each of the Q bands is defined as a spectral region with a central frequency called the *carrier frequency*. The set of possible carrier frequencies is called the *hopset*. The bandwidth B of a band used in a hopset is called the *instantaneous bandwidth*. The bandwidth of the spectrum W over which the hopping occurs is called the total hopping bandwidth. Information is sent by hopping the carrier frequency according to the pseudorandom law, which is known to the desired receiver. In each hop, a small set of code symbols is sent with conventional narrowband modulation before the carrier frequency hops again. The time duration between hops is called *hop duration* or *hopping period* and is denoted by T_c . The time duration between transmission of two consecutive symbols is T.

Usually in FH CDMA frequency shift-keying (FSK) is used. If in FH CDMA system q-FSK is used, then each of the Q bands is divided into q subbands and during each hop one or several of the central frequencies of the subbands within the band can be sent. We will also call each frequency subband the *transmission channel*. We denote the total number Qq of transmission channels by M. If binary FSK (BFSK) is used, M = 2Q and the pair of possible instantaneous frequencies changes with each hop.

At the receiver side, after the frequency hopping has been removed from the received signal, the resulting signal is said to be *dehopped*. Before demodulation, the dehopped signal is applied to a conventional receiver. If another user transmits in the same band at the same time in a FH CDMA system, a *collision* can occur.

Frequency hopping can be classified as slow or fast. *Slow frequency hopping* occurs if one or more *q*-ary symbols are transmitted in the interval between frequency hops. Thus slow frequency hopping implies that the symbol rate 1/T exceeds the hopping rate $1/T_c$. *Fast frequency hopping* occurs if there is more than one frequency hop during one symbol transmission time. If other users

occupied the same frequency band in the same time, the probability of incorrect transmission of the corresponding information symbols would become high. Therefore, it is advisable to combine frequency hopping with *interleaving* and *coding*.

Figure 1.10a illustrates slow frequency hopping if FSK is used in the system and Q = 4, q = 4, and M = 16. In this figure the instantaneous frequency subbands (transmission channels) are shown as a function of the time. The 4-ary symbol transmission time T is equal to $T_c/3$, where T_c is the hop duration. Two bits are collected each T second, and one of four frequencies is generated by the modulator. This frequency is translated to one of Q = 4 frequency hop bands by the FH modulator. In this example, a frequency hop occurs after each group of 3 symbols or when 6 bits have been transmitted. The dehopped signal is shown in Figure 1.10b.

A representation of a transmitted signal for a fast frequency-hopped system is illustrated in Figure 1.11. The output of the data modulator is one of the tones as before, but now time T of the transmission of one group of 2 bits is subdivided into $T/T_c = 4$ chips (hops). In this example, each pair of bits is transmitted during 4 carrier frequency hops.



Figure 1.10. Illustration of FSK slow-frequency-hopped spread spectrum system. (a) transmitted signal; (b) dehopped signal. (4-FSK modulation)