Pairs Trading

Quantitative Methods and Analysis

GANAPATHY VIDYAMURTHY

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Contents

Preface

Most book readers are likely to concur with the idea that the least read portion of any book is the preface. With that in mind, and the fact that the reader has indeed taken the trouble to read up to this sentence, we promise to leave no stone unturned to make this preface as lively and entertaining as possible. For your reading pleasure, here is a nice story with a picture thrown in for good measure. Enjoy!

Once upon a time, there were six blind men. The blind men wished to know what an elephant looked like. They took a trip to the forest and with the help of their guide found a tame elephant. The first blind man walked into the broadside of the elephant and bumped his head. He declared that the elephant was like a wall. The second one grabbed the elephant's tusk and said it felt like a spear. The next blind man felt the trunk of the elephant and was sure that elephants were similar to snakes. The fourth blind man hugged the elephant's leg and declared the elephant was like a tree. The next one caught the ear and said this is definitely like a fan. The last blind man felt the tail and said this sure feels like a rope. Thus the six blind men all perceived one aspect of the elephant and were each right in their own way, but none of them knew what the whole elephant really looked like.

Oftentimes, the market poses itself as the elephant. There are people who say that predicting the market is like predicting the weather, because you can do well in the short term, but where the market will be in the long run is anybody's guess. We have also heard from others that predicting the market short term is a sure way to burn your fingers. "Invest for the long haul" is their mantra. Some will assert that the markets are efficient, and yet some others would tell you that it is possible to make extraordinary returns. While some swear by technical analysis, there are some others, the so-called fundamentalists, who staunchly claim it to be a voodoo science. Multiple valuation models for equities like the dividend discount model, relative valuation models, and the Merton model (treating equity as an option on firm value) all exist side by side, each being relevant at different times for different stocks. Deep theories from various disciplines like physics, statistics, control theory, graph theory, game theory, signal processing, probability, and geometry have all been applied to explain different aspects of market behavior.

It seems as if the market is willing to accommodate a wide range of sometimes opposing belief systems. If we are to make any sense of this smorgasbord of opinions on the market, we would be well advised to draw comfort from the story of the six blind men and the elephant. Under these circumstances, if the reader goes away with a few more perspectives on the market elephant, the author would consider his job well done.

Acknowledgments

All of what is in the book has resulted from the people who have touched my life, and I wish to acknowledge them. First, I thank my parents for raising me in an atmosphere of high expectations: my dad, for his keen interest in this project and for suggesting the term *stogistics*, and my mom, for her unwavering confidence in my abilities. I also thank my in-laws for being so gracious and generous with their support and for sharing in the excitement of the whole process.

I greatly thank friends Jaya Kannan and Kasturi Kannan for their thoughtful gestures and good cheer during the writing process. Thanks to my brother, brother-in-law, and their spouses—Chintu, Hema, Ganesh, and Annie—for their kind and timely enquiries on the status of the writing. It definitely served as a gentle reminder at times when I was lagging behind schedule.

I owe a deep debt of gratitude to my teachers not only for the gift of knowledge but also for inculcating a joy for the learning process, especially Professor Narasimha Murthy, Professor Earl Barnes, and Professor Robert V. Kohn, all of whom I have enjoyed the privilege of working with closely.

The contents of Chapters 11 and 12 are an outcome of the many discussions with Professor Robert V. Kohn (Courant Institute of Mathematics, New York University). The risk arbitrage data were provided by Jason Dahl. The cartoon illustrations done by Tom Kerr are better than I could ever imagine. I thank all of them.

Professors Marco Avellaneda (Courant Institute of Mathematics), Robert V. Kohn (Courant Institute of Mathematics), Kumar Venkataraman (Cox School of Business Southern Methodist University), and professionals Paul Crowley, Steve Evans, Brooke Allen, Jason Dahl, Bud Kroll, and Ajay Junnarkar agreed to review draft versions of the manuscript. Many thanks to all of them. All mistakes that have been overlooked are mine.

I thank my editor, Dave Pugh, for ensuring that the process of writing was a smooth and pleasurable one. Also, thanks to the staff at John Wiley, including Debra Englander for their assistance.

I apologize for any persons left out due to my absentmindedness. Please accept my unspoken thanks.

Last, but most importantly, I wish to thank my wife, Lalitha, for all the wonderful years, for teaching me regularization and being able to share in the excitement of new ideas. Also, thanks to Anjali and Sandhya without whose help the project would have concluded a lot sooner, but would have been no fun at all. You make it all worth it.

PART

One

Background Material

CHAPTER 1

Introduction

We start at the very beginning (a very good place to start). We begin with the CAPM model.

THE CAPM MODEL

CAPM is an acronym for the Capital Asset Pricing Model. It was originally proposed by William T. Sharpe. The impact that the model has made in the area of finance is readily evident in the prevalent use of the word *beta*. In contemporary finance vernacular, beta is not just a nondescript Greek letter, but its use carries with it all the import and implications of its CAPM definition.

Along with the idea of beta, CAPM also served to formalize the notion of a market portfolio. A market portfolio in CAPM terms is a portfolio of assets that acts as a proxy for the market. Although practical versions of market portfolios in the form of market averages were already prevalent at the time the theory was proposed, CAPM definitely served to underscore the significance of these market averages.

Armed with the twin ideas of market portfolio and beta, CAPM attempts to explain asset returns as an aggregate sum of component returns. In other words, the return on an asset in the CAPM framework can be separated into two components. One is the market or systematic component, and the other is the residual or nonsystematic component. More precisely, if r_p is the return on the asset, r_m is the return on the market portfolio, and the beta of the asset is denoted as β, the formula showing the relationship that achieves the separation of the returns is given as

$$r_p = \beta r_m + \theta_p \tag{1.1}$$

Equation 1.1 is also often referred to as the security market line (SML). Note that in the formula, βr_m is the market or systematic component of the return. β serves as a leverage number of the asset return over the market return. For instance, if the beta of the asset happens to be 3.0 and the market moves 1 percent, the systematic component of the asset return is now 3.0 percent. This idea is readily apparent when the SML is viewed in geometrical terms in Figure 1.1. It may also be deduced from the figure that β is indeed the slope of the SML.

θ_p in the CAPM equation is the residual component or residual return on the portfolio. It is the portion of the asset return that is not explainable by the market return. The consensus expectation on the residual component is assumed to be zero.

Having established the separation of asset returns into two components, CAPM then proceeds to elaborate on a key assumption made with respect to the relationship between them. The assertion of the model is that the market component and residual component are uncorrelated. Now, many a scholarly discussion on the import of these assumptions has been conducted and a lot of ink used up on the significance of the CAPM model since its introduction. Summaries of those discussions may be found in the references provided at the end of the chapter. However, for our purposes, the preceding introduction explaining the notion of beta and its role in the determination of asset returns will suffice.

Given that knowledge of the beta of an asset is greatly valuable in the CAPM context, let us discuss briefly how we can go about estimating its value. Notice that beta is actually the slope of the SML. Therefore, beta may be estimated as the slope of the regression line between market returns and the asset returns. Applying the standard regression formula for the estimation of the slope we have

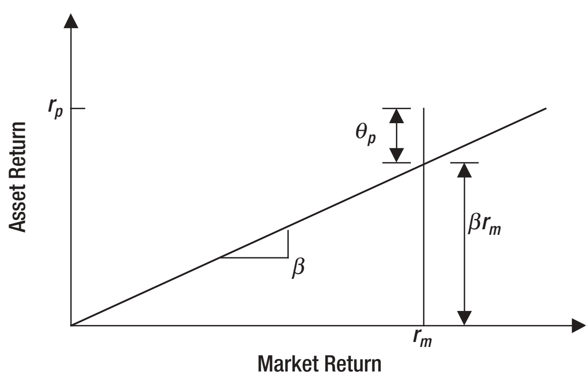


FIGURE 1.1 The Security Market Line.

$$\beta = \frac{\mathrm{cov}(r_p r_m)}{\mathrm{var}(r_m)} \tag{1.2}$$

that is, beta is the covariance between the asset and market returns divided by the variance of the market returns.

To see the typical range of values that the beta of an asset is likely to assume in practice, we remind ourselves of an oft-quoted adage about the markets, “A rising tide raises all boats.” The statement indicates that when the market goes up, we can typically expect the price of all securities to go up with it. Thus, a positive return for the market usually implies a positive return for the asset, that is, the sum of the market component and the residual component is positive. If the residual component of the asset return is small, as we expect it to be, then the positive return in the asset is explained almost completely by its market component. Therefore, a positive return in the market portfolio and the asset implies a positive market component of the return and, by implication, a positive value for beta. Therefore, we can expect all assets to typically have positive values for their betas.

MARKET NEUTRAL STRATEGY

Having discussed CAPM, we now have the required machinery to define *market neutral strategies*: They are strategies that are neutral to market returns, that is, the return from the strategy is uncorrelated with the market return. Regardless of whether the market goes up or down, in good times and bad the market neutral strategy performs in a steady manner, and results are typically achieved with a lower volatility. This desired outcome is achieved by trading market neutral portfolios. Let us therefore define what we mean by a market neutral portfolio.

In the CAPM context, *market neutral portfolios* may be defined as portfolios whose beta is zero. To examine the implications, let us apply a beta value of zero to the equation for the SML. It is easy to see that the return on the portfolio ceases to have a market component and is completely determined by θ_p, the residual component. The residual component by the CAPM assumption happens to be uncorrelated with market returns, and the portfolio return is therefore *neutral* to the market. Thus, a zero beta portfolio qualifies as a market neutral portfolio.

In working with market neutral portfolios, the trader can now focus on forecasting and trading the residual returns. Since the consensus expectation or mean on the residual return is zero, it is reasonable to expect a strong mean-reverting behavior (value oscillates back and forth about the mean value) of the residual time series.[1] This mean-reverting behavior can then be exploited in the process of return prediction, leading to trading signals that constitute the trading strategy.

Let us now examine how we can construct market neutral portfolios and what we should expect by way of the composition of such portfolios. Consider a portfolio that is composed of strictly long positions in assets. We expect that beta of the assets to be positive. Then positive returns in the market result in a positive return for the assets and thereby a positive return for the portfolio. This would, of course, imply a positive beta for the portfolio. By a similar argument it is easy to see that a portfolio composed of strictly short positions is likely to have a negative beta. So, how do we construct a zero beta portfolio, using securities with positive betas? This would not be possible without holding both long and short positions on different assets in the portfolio. We therefore conclude that one can typically expect a zero beta portfolio to comprise both long and short positions. For this reason, these portfolios are also called long–short portfolios. Another artifact of long–short portfolios is that the dollar proceeds from the short sale are used almost entirely to establish the long position, that is, the net dollar value of holdings is close to zero. Not surprisingly, zero beta portfolios are also sometimes referred to as *dollar neutral portfolios*.

Example

Let us consider two portfolios A and B, with positive betas β_A and β_B and with returns r_A and r_B

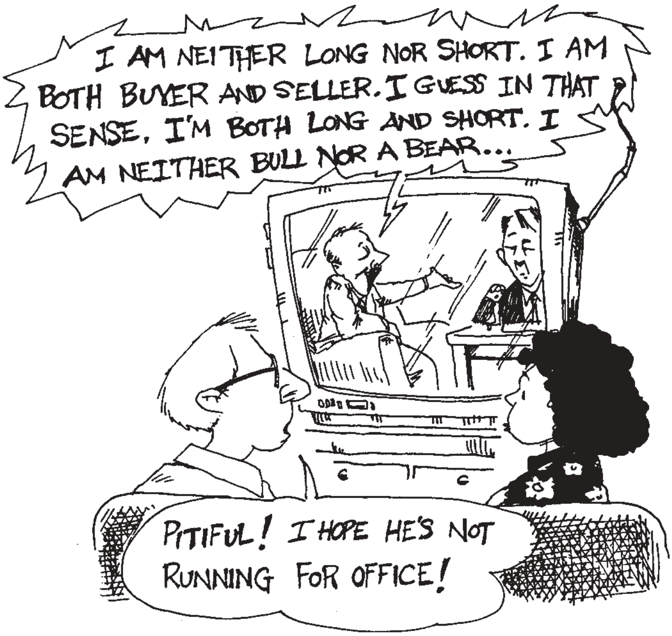
$$\begin{aligned} r_A &= \beta_A r_m + \theta_A \\ r_B &= \beta_B r_m + \theta_B \end{aligned} \tag{1.3}$$

We now construct a portfolio AB, by taking a short position on r units of portfolio A and a long position on one unit of portfolio B. The return on this portfolio is given as $r_{AB} = -r.r_A + r_B$. Substituting for the values of r_A and r_B, we have

$$r_{AB} = (-r\beta_A + \beta_B).r_m + (-r.\theta_A + \theta_B) \tag{1.4}$$

[1]The assertion of CAPM that the expected value of residual return is zero is rather strong. It has been discussed extensively in academic literature as to whether this prediction of CAPM is indeed observable. It is therefore recommended that we explicitly verify the mean-reverting behavior of the spread time series. In later chapters we will discuss methods to statistically check for mean-reverting behavior.

Thus, the combined portfolio has an effective beta of $-r\beta_A + \beta_B$. This value becomes zero, when $r = \beta_B/\beta_A$. Thus, by a judicious choice of the value of r in the long–short portfolio we have created a market neutral portfolio.



COCKTAIL CORNER

In cocktail situations involving investment professionals, it is fairly common to hear the terms *long–short*, *market neutral*, and *dollar neutral investing* bandied about. Very often they are assumed to mean the same thing. Actually, that need not be the case. You could be long–short and dollar neutral but still have a nonzero beta to the market. In which case you have a nonzero market component in the portfolio return and therefore are not market neutral.

If you ever encountered such a situation, you could smile to yourself. Tempting as it might be, I strongly urge that you restrain yourself. But, of course, if you are looking to be anointed the "resident nerd," you could go ahead and launch into an exhaustive explanation of the subtle differences to people with cocktails in hand not particularly looking for a lesson in precise terminology.

PAIRS TRADING

Pairs trading is a market neutral strategy in its most primitive form. The market neutral portfolios are constructed using just two securities, consisting of a long position in one security and a short position in the other, in a predetermined ratio. At any given time, the portfolio is associated with a quantity called the *spread*. This quantity is computed using the quoted prices of the two securities and forms a time series. The spread is in some ways related to the residual return component of the return already discussed. Pairs trading involves putting on positions when the spread is substantially away from its mean value, with the expectation that the spread will revert back. The positions are then reversed upon convergence. In this book, we will look at two versions of pairs trading in the equity markets; namely, statistical arbitrage pairs and risk arbitrage pairs.

Statistical arbitrage pairs trading is based on the idea of relative pricing. The underlying premise in relative pricing is that stocks with similar characteristics must be priced more or less the same. The spread in this case may be thought of as the degree of mutual mispricing. The greater the spread, the higher the magnitude of mispricing and greater the profit potential.

The strategy involves assuming a long–short position when the spread is substantially away from the mean. This is done with the expectation that the mispricing is likely to correct itself. The position is then reversed and profits made when the spread reverts back. This brings up several questions: How do we go about calculating the spread? How do we identify stock pairs for which such a strategy would work? What value do we use for the ratio in the construction of the pairs portfolio? When can we say that the spread has substantially diverged from the mean? We will address these questions and provide some quantitative tools to answer them.

Risk arbitrage pairs occur in the context of a merger between two companies. The terms of the merger agreement establish a strict parity relationship between the values of the stocks of the two firms involved. The spread in this case is the magnitude of the deviation from the defined parity relationship. If the merger between the two companies is deemed a certainty, then the stock prices of the two firms must satisfy the parity relationship, and the spread between them will be zero. However, there is usually a certain level of uncertainty on the successful completion of a merger after the announcement, because of various reasons like antitrust regulatory issues, proxy battles, competing bidders, and the like. This uncertainty is reflected in a nonzero value for the spread. Risk arbitrage involves taking on this uncertainty as risk and capturing the spread value as profits. Thus, unlike the case of statistical arbitrage pairs, which is based on valuation considerations, risk arbitrage trade is based strictly on a parity relationship between the prices of the two stocks.

The typical modus operandi is as follows. Let us call the acquiring firm the "bidder" and the acquired firm the "target." On the eve of merger announcement, the bidder shares are sold short and the target shares are bought. The position is then unwound on completion of the merger. The spread on merger completion is usually lower than when it was put on. The realized profit is the difference between the two spreads. In this book, we discuss how the ratio is determined based on the details of the merger agreement. We will develop a model for the spread dynamics that can be used to answer questions like, "What is the market expectation on the odds of merger completion?" We shall also demonstrate how the model may be used for risk management. Additionally, we will focus on trade timing and provide some quantitative tools for the process.

OUTLINE

The book provides an overview of two different versions of pairs trading in the equity markets. The first version is based on the idea of relative valuation and is called *statistical arbitrage pairs trading*. The second involves pairs trading that arises in the context of mergers and is called *risk arbitrage*. Even though they are commonly called arbitrage strategies in the industry, they are by no means risk-free. In this book we take an in-depth look at the various aspects of these strategies and provide quantitative tools to assist in their analysis.

I must also quickly point out at this juncture that the methodologies discussed in the book are not by any measure to be construed as the only way to trade pairs because, to put it proverbially, there is more than one way to skin a cat. We do, however, strive to present a compelling point of view attempting to integrate theory and practice. The book is by no means meant to be a guarantee for success in pairs trading. However, it provides a framework and insights on applying rigorous analysis to trading pairs in the equity markets.

The book is in three parts. In the first part, we present preliminary material on some key topics. We concede that there are books entirely devoted to each of the topics addressed, and the coverage of the topics here is not exhaustive. However, the discussion sets the context for the rest of the book and helps familiarize the reader with some important ideas. It also introduces some notation and definitions. The second part is devoted to statistical arbitrage pairs, and the third part is on risk arbitrage.

The book assumes some knowledge on the part of the reader of algebra, probability theory, and calculus. Nevertheless, we have strived to make the material accessible and the reader could choose to pick up the background along the way. As a refresher, the appendix at the end of this chapter lists the