
Fundamentals of Global Positioning System Receivers A Software Approach

SECOND EDITION

JAMES BAO-YEN TSUI

 WILEY-
INTERSCIENCE

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Fundamentals of Global Positioning System Receivers

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Preface

In this new edition of the book, only minor changes were made to the original nine chapters but three new chapters treat topics of increasing interest to GPS users and equipment developers. One topic, improving the GPS receiver sensitivity may extend their operations into buildings, which is becoming important for emergency rescue and urban warfare. Thus, Chapters 10 and 11 are devoted to the processing of weak signals, as well as the limitations of autonomous GPS receivers. These same approaches are also applicable to GPS receivers in noisy environments and under interference conditions. Other subjects new to this edition, such as using the almanac data to simplify signal acquisition; determining the number of analog-to-digital converter bits required for the GPS receiver to work under strong interference; and, using GPS signals reflected from the ground as an altimeter are covered in Chapter 12.

I constantly discuss technical subjects with Mr. D. Lin and Dr. L. L. Liou, my colleagues at AFRL, and Dr. Y. T. Morton of Miami University. They worked closely with me and made tremendous contributions in this edition. I very much appreciate their help. I would especially like to thank Drs. J. Morton and T. Y. Morton of Miami University and Dr. J. Garrison of Purdue University for reviewing my manuscripts.

The management in AFRL/SNR as usual provided excellent guidance and support. Special thanks to W. Moore, K. Loree, M. Longbrake, B. Holsapple, and Dr. S. Hary. I also would like to thank my new colleagues, M. Berarducci, J. Buck, J. Coker, J. C. Ha, Dr. M. Miller, S. Moore, T. Nguyen, H. Noffke, N. Wilkins, J. McCartney, T. Niedzwiecki, M. Thompson, and C. Tolle for their help.

Preface to the First Edition

The purpose of this book is to present detailed fundamental information on a global positioning system (GPS) receiver. Although GPS receivers are popularly used in every-day life, their operation principles cannot be easily found in one book. Most other types of receivers process the input signals to obtain the necessary information easily, such as in amplitude modulation (AM) and frequency modulation (FM) radios. In a GPS receiver the signal is processed to obtain the required information, which in turn is used to calculate the user position. Therefore, at least two areas of discipline, receiver technology and navigation scheme, are employed in a GPS receiver. This book covers both areas.

In the case of GPS signals, there are two sets of information: the civilian code, referred to as the coarse/acquisition (C/A) code, and the classified military code, referred to as the P(Y) code. This book concentrates only on the civilian C/A code. This is the information used by commercial GPS receivers to obtain the user position.

The material in this book is presented from the software receiver viewpoint for two reasons. First, it is likely that narrow band receivers, such as the GPS receiver, will be implemented in software in the future. Second, a software receiver approach may explain the operation better. A few key computer programs can be used to further illustrate some points.

This book is written for engineers and scientists who intend to study and understand the detailed operation principles of GPS receivers. The book is at the senior or graduate school level. A few computer programs written in Matlab are listed at the end of several chapters to help the reader understand some of the ideas presented.

I would like to acknowledge the following persons. My sincere appreciation to three engineers: Dr. D. M. Akos from Stanford University, M. Stockmaster from Rockwell Collins, and J. Schamus from Veridian. They worked with me at the Air Force Research Laboratory, Wright Patterson Air Force Base on the

design of a software GPS receiver. This work made this book possible. Dr. Akos also reviewed my manuscripts. I used information from several courses on GPS receivers given at the Air Force Institute of Technology by Lt. Col. B. Riggins, Ph.D. and Capt. J. Requet, Ph.D. Valuable discussion with Drs. F. VanGraas and M. Braasch from Ohio University helped me as well. I am constantly discussing GPS subjects with my coworkers, D. M. Lin and V. D. Chakravarthy.

The management in the Sensor Division of the Air Force Research Laboratory provided excellent guidance and support in GPS receiver research. Special thanks are extended to Dr. P. S. Hadorn, E. R. Martinsek, A. W. White, and N. A. Pequignot. I would also like to thank my colleagues, R. L. Davis, S. M. Rodrigue, K. M. Graves, J. R. McCall, J. A. Tenbarger, Dr. S. W. Schneider, J. N. Hedge Jr., J. Caschera, J. Mudd, J. P. Stephens, Capt. R. S. Parks, P. G. Howe, D. L. Howell, Dr. L. L. Liou, D. R. Meeks, and D. Jones, for their consultation and assistance.

Last, but not least, I would like to thank my wife, Susan, for her encouragement and understanding.

Introduction

1.1 INTRODUCTION⁽¹⁻¹³⁾

This book presents detailed information in a compact form about the global positioning system (GPS) coarse/acquisition (C/A) code receiver. Using the C/A code to find the user location is referred to as the standard position service (SPS). Most of the information can be found in references 1 through 13. However, there is much more information in the references than the basics required to understand a GPS receiver. Therefore, one must study the proper subjects and put them together. This is a tedious and cumbersome task. This book does this job for the reader.

This book not only introduces the information available from the references, it emphasizes its applications. Software programs are provided to help understand some of the concepts. These programs are also useful in designing GPS receivers. In addition, various techniques to perform acquisition and tracking on the GPS signals are included.

This book concentrates only on the very basic concepts of the C/A code GPS receiver. Any subject not directly related to the basic receiver (even if it is of general interest, i.e., differential GPS receiver and GPS receiver with carrier-aided tracking capacity) will not be included in this book. These other subjects can be found in reference 1.

1.2 HISTORY OF GPS DEVELOPMENT^(1,5,12)

The discovery of navigation seems to have occurred early in human history. According to Chinese storytelling, the compass was discovered and used in wars

during foggy weather before recorded history. There have been many different navigation techniques to support ocean and air transportation. Satellite-based navigation started in the early 1970s. Three satellite systems were explored before the GPS programs: the U.S. Navy Navigation Satellite System (also referred to as the Transit), the U.S. Navy's Timation (TIME navigationION), and U.S. Air Force project 621B. The Transit project used a continuous wave (cw) signal. The closest approach of the satellite can be found by measuring the maximum rate of Doppler shift. The Timation program used an atomic clock that improves the prediction of satellite orbits and reduces the ground control update rate. The Air Force 621B project used the pseudorandom noise (PRN) signal to modulate the carrier frequency.

The GPS program was approved in December 1973. The first satellite was launched in 1978. In August 1993, GPS had 24 satellites in orbit and in December of the same year the initial operational capability was established. In February 1994, the Federal Aviation Agency (FAA) declared GPS ready for aviation use.

1.3 A BASIC GPS RECEIVER

The basic GPS receiver discussed in this book is shown in Figure 1.1. The signals transmitted from the GPS satellites are received from the antenna. Through the radio frequency (RF) chain the input signal is amplified to a proper amplitude and the frequency is converted to a desired output frequency. An analog-to-digital converter (ADC) is used to digitize the output signal. The antenna, RF chain, and ADC are the hardware used in the receiver.

After the signal is digitized, software is used to process it, and that is why this book has taken a software approach. Acquisition means to find the signal of a certain satellite. The tracking program is used to find the phase transition of the navigation data. In a conventional receiver, the acquisition and tracking are

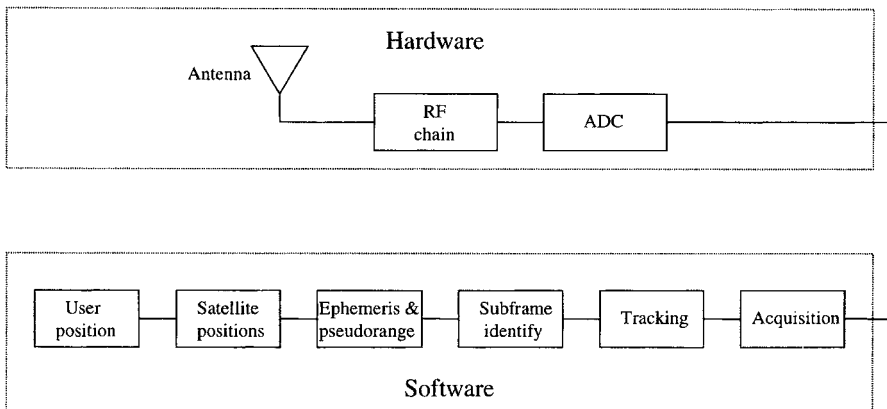


FIGURE 1.1 A fundamental GPS receiver.

performed by hardware. From the navigation data phase transition the subframes and navigation data can be obtained. Ephemeris data and pseudoranges can be obtained from the navigation data. The ephemeris data are used to obtain the satellite positions. Finally, the user position can be calculated for the satellite positions and the pseudoranges. Both the hardware used to collect digitized data and the software used to find the user position will be discussed in this book.

1.4 APPROACHES OF PRESENTATION

There are two possible approaches to writing this book. One is a straightforward way to follow the signal flow shown in Figure 1.1. In this approach the book would start with the signal structure of the GPS system and the methods to process the signal to obtain the necessary the information. This information would be used to calculate the positions of the satellites and the pseudoranges. By using the positions of the satellites and the pseudoranges the user position can be found. In this approach, the flow of discussion would be smooth, from one subject to another. However, the disadvantage of this approach is that readers might not have a clear idea why these steps are needed. They could understand the concept of the GPS operation only after reading the entire book.

The other approach is to start with the basic concept of the GPS from a system designers' point of view. This approach would start with the basic concept of finding the user position from the satellite positions. The description of the satellite constellation would be presented. The detailed information of the satellite orbit is contained in the GPS data. In order to obtain these data, the GPS signal must be tracked. The *C/A* code of the GPS signals would then be presented. Each satellite has an unique *C/A* code. A receiver can perform acquisition on the *C/A* code to find the signal. Once the *C/A* code of a certain satellite is found, the signal can be tracked. The tracking program can produce the navigation data. From these data, the position of the satellite can be found. The relative pseudorange can be obtained by comparing the time a certain data point arrived at the receiver. The user position can be calculated from the satellite positions and pseudoranges of several satellites.

This book takes this second approach to present the material because it should give a clearer idea of the GPS function from the very beginning. The final chapter describes the overall functions of the GPS receiver and can be considered as taking the first approach for digitizing the signal, performing acquisition and tracking, extracting the navigation data, and calculating the user position.

1.5 SOFTWARE APPROACH

This book uses the concept of software radio to present the subject. The software radio idea is to use an analog-to-digital converter (ADC) to change the input signal into digital data at the earliest possible stage in the receiver. In other words,

the input signal is digitized as close to the antenna as possible. Once the signal is digitized, digital signal processing will be used to obtain the necessary information. The primary goal of the software radio is minimum hardware use in a radio. Conceptually, one can tune the radio through software or even change the function of the radio such as from amplitude modulation (AM) to frequency modulation (FM) by changing the software; therefore great flexibility can be achieved.

The main purpose of using the software radio concept to present this subject is to illustrate the idea of signal acquisition and tracking. Although using hardware to perform signal acquisition and tracking can also describe GPS receiver function, it appears that using software may provide a clearer idea of the signal acquisition and tracking. In addition, a software approach should provide a better understanding of the receiver function because some of the calculations can be illustrated with programs. Once the software concept is well understood, the readers should be able to introduce new solutions to problems such as various acquisition and tracking methods to improve efficiency and performance. At the time (December 1997) this chapter was being written, a software GPS receiver using a 200 MHz personal computer (PC) could not track one satellite in real time. When this chapter was revised in December 1998, the software had been modified to track two satellites in real time with a new PC operating at 400 MHz. Although it is still impossible to implement a software GPS receiver operating in real time, with the improvement in PC operating speed and software modification it is likely that by the time this book is published a software GPS receiver will be a reality. Of course, using a digital signal processing (DSP) chip is another viable way to build the receiver. When this second edition was prepared, software receivers could already operate at real time. In Section 12.10, some of the results will be presented.

Only the fundamentals of a GPS receiver are presented in this book. In order to improve the performance of a receiver, fine tuning of some of the operations might be necessary. Once readers understand the basic operation principles of the receiver, they can make the necessary improvement.

1.6 POTENTIAL ADVANTAGES OF THE SOFTWARE APPROACH

An important aspect of using the software approach to build a GPS receiver is that the approach can drastically deviate from the conventional hardware approach. For example, the user may take a snapshot of data and process them to find the location rather than continuously tracking the signal. Theoretically, 30 seconds of data are enough to find the user location. This is especially useful when data cannot be collected in a continuous manner. Since the software approach is in the infant stage, one can explore many potential methods.

The software approach is very flexible. It can process data collected from various types of hardware. For example, one system may collect complex data referred to as the inphase and quadrature-phase (I and Q) channels. Another system may collect real data from one channel. The data can easily be changed from one form to another. One can also generate programs to process complex

signals from programs processing real signals or vice versa with some simple modifications. A program can be used to process signals digitized with various sampling frequencies. Therefore, a software approach can almost be considered as hardware independent.

New algorithms can easily be developed without changing the design of the hardware. This is especially useful for studying some new problems. For example, in order to study the antijamming problem one can collect a set of digitized signals with jamming signals present and use different algorithms to analyze it.

1.7 ORGANIZATION OF THE BOOK

This book contains twelve chapters. Chapter 2 introduces the user position requirements, which lead to the GPS parameters. Also included in Chapter 2 is the basic concept of how to find the user position if the satellite positions are known. Chapter 3 discusses the satellite constellation and its impact on the GPS signals, which in turn affects the design of the GPS receiver. Chapter 4 discusses the earth-centered, earth-fixed system. Using this coordinate system, the user position can be calculated to match the position on every-day maps. The GPS signal structure is discussed in detail in Chapter 5. Chapter 6 discusses the hardware to collect data, which is equivalent to the front end of a conventional GPS receiver. Changing the format of data is also presented. Chapter 7 presents several acquisition methods. Some of them can be used in hardware design and others are suitable for software applications. Chapter 8 discusses two tracking methods. One uses the conventional phase-locked loop approach and the other one is more suitable for the software radio approach. Chapter 9 is a summary of the previous chapters. It takes all the information in the first eight chapters and presents in it an order following the signal flow in a GPS receiver. Chapters 10 and 11 are devoted to weak GPS signal processing. Not only the processing but the limitation of an autonomous GPS receiver is also defined. Chapter 12 includes various subjects related to GPS receivers.

Computer programs written in Matlab are listed at the end of several chapters. Some of the programs are used only to illustrate ideas. Others can be used in the receiver design. In the final chapter all of the programs related to designing a receiver will be listed. These programs are by no means optimized and they are used only for demonstration purposes.

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Basic GPS Concept

2.1 INTRODUCTION

This chapter will introduce the basic concept of how a GPS receiver determines its position. In order to better understand the concept, GPS performance requirements will be discussed first. These requirements determine the arrangement of the satellite constellation. From the satellite constellation, the user position can be solved. However, the equations required for solving the user position turn out to be nonlinear simultaneous equations. In addition, some practical considerations (i.e., the inaccuracy of the user clock) will be included in these equations. These equations are solved through a linearization and iteration method. The solution is in a Cartesian coordinate system and the result will be converted into a spherical coordinate system. However, the earth is not a perfect sphere; therefore, once the user position is found, the shape of the earth must be taken into consideration. The user position is then translated into the earth-based coordinate system. Finally, the selection of satellites to obtain better user position accuracy and the dilution of precision will be discussed.

2.2 GPS PERFORMANCE REQUIREMENTS⁽¹⁾

Some of the performance requirements are listed below:

1. The user position root mean square (rms) error should be 10–30 m.
2. It should be applicable to real-time navigation for all users including the high-dynamics user, such as in high-speed aircraft with flexible maneuverability.

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3. It should have worldwide coverage. Thus, in order to cover the polar regions the satellites must be in inclined orbits.
4. The transmitted signals should tolerate, to some degree, intentional and unintentional interference. For example, the harmonics from some narrow-band signals should not disturb its operation. Intentional jamming of GPS signals is a serious concern for military applications.
5. It cannot require that every GPS receiver utilize a highly accurate clock such as those based on atomic standards.
6. When the receiver is first turned on, it should take minutes rather than hours to find the user position.
7. The size of the receiving antenna should be small. The signal attenuation through space should be kept reasonably small.

These requirements combining with the availability of the frequency band allocation determines the carrier frequency of the GPS to be in the L band (1–2 GHz) of the microwave range.

2.3 BASIC GPS CONCEPT

The position of a certain point in space can be found from distance measured from this point to some known positions in space. Let us use some examples to illustrate this point. In Figure 2.1, the user position is on the x -axis; this is a one-dimensional case. If the satellite position S_1 and the distance to the satellite x_1 are both known, the user position can be at two places, either to the left or right of S_1 . In order to determine the user position, the distance to another satellite with known position must be measured. In this figure, the positions of S_2 and x_2 uniquely determine the user position U .

Figure 2.2 shows a two-dimensional case. In order to determine the user position, three satellites and three distances are required. The trace of a point with constant distance to a fixed point is a circle in the two-dimensional case. Two satellites and two distances give two possible solutions because two circles intersect at two points. A third circle is needed to uniquely determine the user position.

For similar reasons one might decide that in a three-dimensional case four satellites and four distances are needed. The equal-distance trace to a fixed point is a sphere in a three-dimensional case. Two spheres intersect to make a circle. This circle intersects another sphere to produce two points. In order to determine which point is the user position, one more satellite is needed.

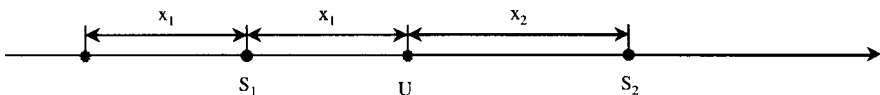


FIGURE 2.1 One-dimensional user position.

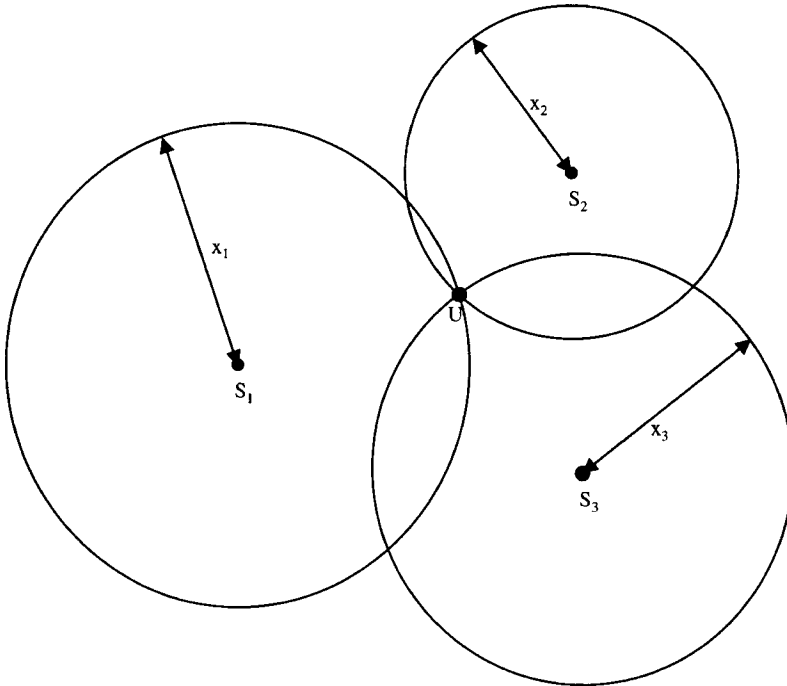


FIGURE 2.2 Two-dimensional user position.

In GPS the position of the satellite is known from the ephemeris data transmitted by the satellite. One can measure the distance from the receiver to the satellite. Therefore, the position of the receiver can be determined.

In the above discussion, the distance measured from the user to the satellite is assumed to be very accurate and there is no bias error. However, the distance measured between the receiver and the satellite has a constant unknown bias, because the user clock usually is different from the GPS clock. In order to resolve this bias error one more satellite is required. Therefore, in order to find the user position five satellites are needed.

If one uses four satellites and the measured distance with bias error to measure a user position, two possible solutions can be obtained. Theoretically, one cannot determine the user position. However, one of the solutions is close to the earth's surface and the other one is in space. Since the user position is usually close to the surface of the earth, it can be uniquely determined. Therefore, the general statement is that four satellites can be used to determine a user position, even though the distance measured has a bias error.

The method of solving the user position discussed in Sections 2.5 and 2.6 is through iteration. The initial position is often selected at the center of the earth. The iteration method will converge on the correct solution rather than the one in space. In the following discussion four satellites are considered the minimum number required in finding the user position.

2.4 BASIC EQUATIONS FOR FINDING USER POSITION

In this section the basic equations for determining the user position will be presented. Assume that the distance measured is accurate and under this condition three satellites are sufficient. In Figure 2.3, there are three known points at locations r_1 or (x_1, y_1, z_1) , r_2 or (x_2, y_2, z_2) , and r_3 or (x_3, y_3, z_3) , and an unknown point at r_u or (x_u, y_u, z_u) . If the distances between the three known points to the unknown point can be measured as ρ_1 , ρ_2 , and ρ_3 , these distances can be written as

$$\begin{aligned}\rho_1 &= \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} \\ \rho_2 &= \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} \\ \rho_3 &= \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2}\end{aligned}\quad (2.1)$$

Because there are three unknowns and three equations, the values of x_u , y_u , and z_u can be determined from these equations. Theoretically, there should be two sets of solutions as they are second-order equations. These equations can be solved relatively easily with linearization and an iterative approach. The solution of these equations will be discussed later in Section 2.6.

In GPS operation, the positions of the satellites are given. This information can be obtained from the data transmitted from the satellites and will be discussed in Chapter 5. The distances from the user (the unknown position) to the

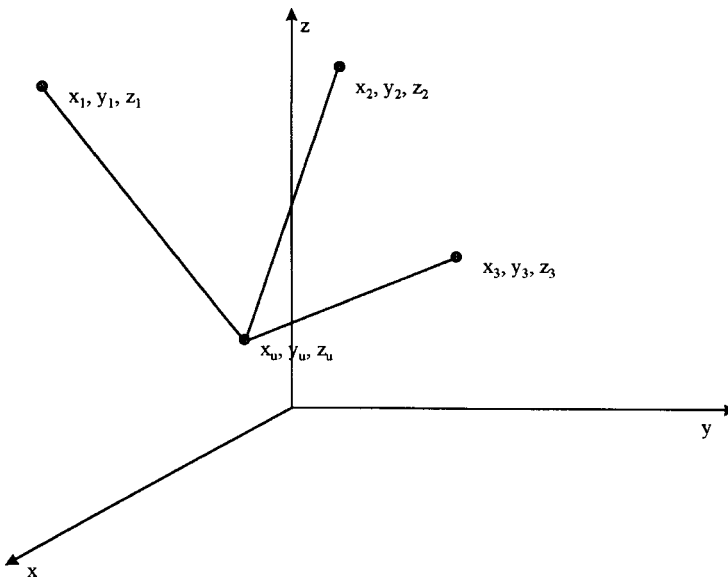


FIGURE 2.3 Use three known positions to find one unknown position.

satellites must be measured simultaneously at a certain time instance. Each satellite transmits a signal with a time reference associated with it. By measuring the time of the signal traveling from the satellite to the user the distance between the user and the satellite can be found. The distance measurement is discussed in the next section.

2.5 MEASUREMENT OF PSEUDORANGE⁽²⁾

Every satellite sends a signal at a certain time t_{si} . The receiver will receive the signal at a later time t_u . The distance between the user and the satellite i is

$$\rho_{iT} = c(t_u - t_{si}) \quad (2.2)$$

where c is the speed of light, ρ_{iT} is often referred to as the true value of pseudorange from user to satellite i , t_{si} is referred to as the true time of transmission from satellite i , t_u is the true time of reception.

From a practical point of view it is difficult, if not impossible, to obtain the correct time from the satellite or the user. The actual satellite clock time t'_{si} and actual user clock time t'_u are related to the true time as

$$\begin{aligned} t'_{si} &= t_{si} + \Delta b_i \\ t'_u &= t_u + b_{ut} \end{aligned} \quad (2.3)$$

where Δb_i is the satellite clock error, b_{ut} is the user clock bias error. Besides the clock error, there are other factors affecting the pseudorange measurement. The measured pseudorange ρ_i can be written as⁽²⁾

$$\rho_i = \rho_{iT} + \Delta D_i - c(\Delta b_i - b_{ut}) + c(\Delta T_i + \Delta I_i + v_i + \Delta v_i) \quad (2.4)$$

where ΔD_i is the satellite position error effect on range, ΔT_i is the tropospheric delay error, ΔI_i is the ionospheric delay error, v_i is the receiver measurement noise error, Δv_i is the relativistic time correction.

Some of these errors can be corrected; for example, the tropospheric delay can be modeled and the ionospheric error can be corrected in a two-frequency receiver. The errors will cause inaccuracy of the user position. However, the user clock error cannot be corrected through received information. Thus, it will remain as an unknown. As a result, Equation (2.1) must be modified as

$$\begin{aligned} \rho_1 &= \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} + b_u \\ \rho_2 &= \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} + b_u \\ \rho_3 &= \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} + b_u \\ \rho_4 &= \sqrt{(x_4 - x_u)^2 + (y_4 - y_u)^2 + (z_4 - z_u)^2} + b_u \end{aligned} \quad (2.5)$$

where b_u is the user clock bias error expressed in distance, which is related to the quantity b_{ut} by $b_u = cb_{ut}$. In Equation (2.5), four equations are needed to solve for four unknowns x_u , y_u , z_u , and b_u . Thus, in a GPS receiver, a minimum of four satellites is required to solve for the user position. The actual measurement of the pseudorange will be discussed in Chapter 9.

2.6 SOLUTION OF USER POSITION FROM PSEUDORANGES

One common way to solve Equation (2.5) is to linearize them. The above equations can be written in a simplified form as

$$\rho_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2} + b_u \quad (2.6)$$

where $i = 1, 2, 3,$ and 4 , and $x_u, y_u, z_u,$ and b_u are the unknowns. The pseudorange ρ_i and the positions of the satellites x_i, y_i, z_i are known.

Differentiate this equation, and the result is

$$\begin{aligned} \delta\rho_i &= \frac{(x_i - x_u)\delta x_u + (y_i - y_u)\delta y_u + (z_i - z_u)\delta z_u}{\sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2}} + \delta b_u \\ &= \frac{(x_i - x_u)\delta x_u + (y_i - y_u)\delta y_u + (z_i - z_u)\delta z_u}{\rho_i - b_u} + \delta b_u \end{aligned} \quad (2.7)$$

In this equation, $\delta x_u, \delta y_u, \delta z_u,$ and δb_u can be considered as the only unknowns. The quantities $x_u, y_u, z_u,$ and b_u are treated as known values because one can assume some initial values for these quantities. From these initial values a new set of $\delta x_u, \delta y_u, \delta z_u,$ and δb_u can be calculated. These values are used to modify the original $x_u, y_u, z_u,$ and b_u to find another new set of solutions. This new set of $x_u, y_u, z_u,$ and b_u can be considered again as known quantities. This process continues until the absolute values of $\delta x_u, \delta y_u, \delta z_u,$ and δb_u are very small and within a certain predetermined limit. The final values of $x_u, y_u, z_u,$ and b_u are the desired solution. This method is often referred to as the iteration method.

With $\delta x_u, \delta y_u, \delta z_u,$ and δb_u as unknowns, the above equation becomes a set of linear equations. This procedure is often referred to as linearization. The above equation can be written in matrix form as

$$\begin{bmatrix} \delta\rho_1 \\ \delta\rho_2 \\ \delta\rho_3 \\ \delta\rho_4 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & 1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & 1 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & 1 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & 1 \end{bmatrix} \begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ \delta b_u \end{bmatrix} \quad (2.8)$$

where

$$\alpha_{i1} = \frac{x_i - x_u}{\rho_i - b_u} \quad \alpha_{i2} = \frac{y_i - y_u}{\rho_i - b_u} \quad \alpha_{i3} = \frac{z_i - z_u}{\rho_i - b_u} \quad (2.9)$$