Cavity Quantum Electrodynamics  
The Strange Theory of Light in a Box
Cavity Quantum Electrodynamics
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Cavity Quantum Electrodynamics
The Strange Theory of Light in a Box

Sergio M. Dutra

A JOHN WILEY & SONS, INC., PUBLICATION
To
Salvador P. Dutra,
from whom I inherited a fondness for books
My parents, who nurtured it
Kati, Dani, Anna, and Nani,
who have so patiently endured the writing of this book
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First, then, I will lay down some general rules, most of which, I believe, you have considered already; but if any of them be new to you, they may excuse the rest; if none at all, yet is my punishment more in writing than yours' in reading.

—Sir Isaac Newton's letter to a certain Francis Aston (May 18, 1669)

Cavity quantum electrodynamics is about what happens to light and, in particular to its interaction with matter when it is trapped inside a box. By light I mean not only visible light (i.e., the optical region of the spectrum) but also the infrared and microwave regions. This book has two main aims. The first is to serve as an introductory text for Ph.D. students, postdocs, and more experienced physicists wishing to enter this field. I have worked hard to write the book that I dreamed of reading when I started my Ph.D. The second aim is to serve as a supplementary text for an advanced undergraduate course in quantum mechanics.

Why would cavity quantum electrodynamics, which sounds like a highly specialized topic, be of any use as a supplement to an undergraduate lecture course in quantum mechanics? Because it provides a bridge between the bread-and-butter quantum mechanics that is usually taught to undergraduates and most of the exciting new physics and technology that is being done today. Quantum electrodynamics is the basis of all other quantum field theories, which together with general relativity, form the fundamental core of the whole of contemporary physics. Considering quantum electrodynamics in a cavity makes it more accessible both from a pedagogical point of view and from that of an experimentalist who wishes to observe the various quantum effects predicted by the theory. Many conceptual issues of elementary quantum mechanics, such as the EPR and Schrödinger cat paradoxes, have been investigated in cavity QED experiments that I wish to bring within reach of undergraduate students.
We also address many of the exciting questions that students often ask themselves when they are starting to learn modern physics, such as: What would happen if the photon had a rest mass? Does the photon have a wavefunction? Could it be that the electromagnetic field itself is the wavefunction of the photon? If in the microscopic world ruled by quantum mechanics both matter and light are made up of particles that sometimes behave as waves, why is it that in the macroscopic world ruled by classical mechanics, matter behaves only as particles and light only as waves? The main theme of this book is a question that even laypeople sometimes ask: What happens to light if we trap it inside a box with mirrored walls? Connected with this question, this book also deals with a more subtle question that is normally not addressed in quantum optics textbooks: What happens when the walls move?

Some popular science books touch on a few of these questions but only superficially. Serious introductory textbooks tend to avoid them, leaving the students rather frustrated. The persistent student can find these questions discussed in more advanced books and papers. But these discussions are all scattered in the literature and are often difficult for the uninitiated to follow. So one of the subsidiary aims has been to gather all these interesting issues together in a form that can be understood by someone who is just starting to think about them with fresh and genuine excitement. This is a serious physics textbook written in the spirit of those popular science books that often excite the imagination of would-be scientists.

The core structure of this book came out of a lecture course on cavity QED I gave at the University of Campinas (SP, Brazil) and at the Instituto Nacional de Astrofísica, Óptica y Electónica (Puebla, Mexico). But as the book began to mature, a number of extra topics were added, not to mention some key advances of this field that happened while the book was being written and had to be included (such as the realization of the first single-atom laser in the strong-coupling regime).

There are several appendixes. These are intended to help our occasional “top-down” and “just-in-time” approach to some of the subjects. Instead of the traditional approach, where students first have to go through a lot of apparently pointless learning of apparently unconnected mathematical techniques before they can hopefully “put everything back together again” at the end, whenever appropriate, this book aims at going straight to the point, introducing the math along the way as we need it. Most of the details are in the appendixes for later study, after the student is already motivated by the main text. I believe that this style of presentation stands a better chance of keeping a student’s enthusiasm alive than the traditional approach does.

Another feature of this book is the attention that has been paid to historical context. It is sad to see students learn a particular technique, such as canonical quantization, without knowing who proposed it first, why, and in which context. So I tried to write a little about the historical origins of some of the ideas and techniques discussed in this book, and to give references not only to the more recent literature but also to some of the historical breakthroughs. Nowadays, some references are freely available on the World Wide Web. So to make life easier for the reader, whenever I knew of such availability, I mention not only the printed source but also the URL where the free online version can be found. I am well aware that URLs change quite often, and even though I have checked them all prior to publication, there is a good chance that many
will be out of date by the time you read this [149]. Still, I think it is worth providing them, as they can help you find the updated URLs more quickly.

Last but not least, I would like to mention specifically some of the topics this book covers that are not usually found in quantum optics textbooks. These topics include, for instance:

1. A deduction of a first-quantized theory of the photon from relativistic invariance alone, with Maxwell's equations being deduced at the end rather than being used as the starting point.

2. A discussion about photon mass.

3. A brief introduction to the vacuum catastrophe, a very important open problem where quantum field theory diverges from experiment by more than 100 orders of magnitude. This problem might turn out to hide the key to the long-sought unification of gravity and quantum mechanics. Here it is seen under the new insight brought by the discovery in 1998 that the expansion of the universe is accelerating rather than slowing down.

4. A discussion about cavities with movable walls (the dynamic Casimir effect).

5. Quantization of the radiation field in material media from first principles.

6. Quantization of radiation with matter without using electromagnetic potentials in the dipole approximation (this is all that you need for most of quantum optics).

7. A pedagogical account of some mathematical subtleties and techniques, including, multiplication of distributions (e.g., the product of a delta function and a step function), and quaternions.

8. A first-principles discussion about cavity QED in leaky cavities using Fox–Li modes.

It is also worth pointing out that even though this book deals primarily with fundamental ideas in modern physics, many of the mathematical techniques presented here can be quite useful to someone who is not going to follow an academic career. For instance, quaternions, which are seldom taught to undergraduates, but to which I devote an entire appendix, are very important in the computer graphics and animation industries. Langevin equations, or stochastic differential equations as they are often referred to by mathematicians, are very important in quantitative finance and insurance.

Sergio M. Dutra
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The scientific profession is a bit of a craft and it is still best learned from a master craftsman as in the Middle Ages. The ultimate origin of this book is perhaps my M.Sc. and Ph.D. theses, which were supervised by Luiz Davidovich and Peter L. Knight, respectively. I was lucky to have been able to learn many of our "trade secrets" from these two highly skilled masters. Without them this book would not have been possible and I am forever grateful to them. I am also thankful to Han Woerdman, Moysés Nussenzveig, Amir Caldeira, Gerard Nienhuis, and Nicim Zagury, from whom I have learned many things.

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S. M. D.

¹Gilles has an eye-catching homepage: http://www.oyonale.com.
²"POV-Ray" is a trademark of Persistence of Vision Raytracer Pty. Ltd. All other trademarks are acknowledged as the property of their respective owners.
³The cover picture shows a lamppost in a box with mirrored walls, seen from inside through a fisheye lens of 100°.
An author ought to consider himself, not as a gentleman who gives a private or eleemosynary treat, but rather as one who keeps a public ordinary, at which all persons are welcome for their money. In the former case, it is well known that the entertainer provides what fare he pleases; and though this should be very indifferent, and utterly disagreeable to the taste of his company, they must not find any fault; ... good breeding forces them outwardly to approve and to commend whatever is set before them. Now the contrary of this happens to the master of an ordinary. Men who pay for what they eat will insist on gratifying their palates, however nice and whimsical these may prove; and if everything is not agreeable to their taste, will challenge a right to censure, to abuse, and to d—n their dinner without control.

To prevent, therefore, giving offence to their customers by any such disappointment, it hath been usual with the honest and well-meaning host to provide a bill of fare which all persons may peruse at their first entrance into the house ... we ... shall prefix not only a general bill of fare to our whole entertainment, but shall likewise give the reader particular bills to every course which is to be served up in this and the ensuing volumes.

—Henry Fielding [208]

In this introduction we give you the "general bill of fare" for the entire book. The best way to start is to say a few words about the main aim of the book, its raison d'être. This is twofold. First, it is to provide an introduction to the field of cavity quantum electrodynamics for postgraduates and any researcher in general who wishes to enter this field or make use of some of its techniques. If you fit this description,
you will find that this book deals with some topics often avoided by quantum optics textbooks. There is, however, a less obvious raison d'être for this book.

In his autobiographical notes [184], Einstein wrote that one of the reasons he chose to work in physics rather than mathematics was that in physics he could clearly differentiate what was fundamentally important and basic from "the rest of the more or less dispensable erudition." Mathematics, he wrote, "was split up into numerous specialities, each of which could easily absorb the short lifetime granted to us. . . . True enough, physics also was divided into separate fields, each of which was capable of devouring a short lifetime of work without having satisfied the hunger for deeper knowledge." However, in physics, he soon learned to scent out the problems that were capable of leading to fundamentals and to avoid "the multitude of things which clutter up the mind and divert it from the essential." Well, today, physics is much more fragmented into a multitude of specialities than it was in Einstein's time. This makes it even more difficult today than it was then to pass to students an idea of the fundamental issues in physics.

Despite being only one of many specialities within physics, cavity quantum electrodynamics can serve as a window to the fundamental issues of physics from which undergraduates can greatly profit. Light was a key ingredient of the second great unification in physics, Maxwell's unification of electricity, magnetism, and optics with his theory of classical electrodynamics. Light also held the secret to the two great fundamental discoveries in physics in the twentieth century: relativity and quantum mechanics. As we see in Chapter 3, the photon is an intrinsically relativistic particle that, as far as we know, always travels at the speed of light and has no rest mass; it is never in a nonrelativistic regime. Quantum electrodynamics is the basis of all other field theories, which together with general relativity form the fundamental core of contemporary physics. Cavity quantum electrodynamics is a "tamed" form of quantum electrodynamics that undergraduates can easily tackle after seeing electromagnetism and basic quantum mechanics.

So this book is also aimed at undergraduates. If you are an undergraduate, I think you will find this book quite different from the usual physics textbooks that you have come across. I hope you find it inspiring. There are several pointers to the literature in each chapter, mainly in the sections on recommended reading. Some of the primary historical papers are also mentioned there. You will find it very instructive, and hopefully entertaining, to read at least some of these remarkable papers. And please, remember that even though my main aim has been to show you how fascinating and

---

1 For a brief list of some of these topics, see the preface.
2 In Einstein's time, it was still possible for a single person to make contributions to the whole of physics, whereas nowadays this is very rare.
3 Already in his day, Einstein complained [184] about having to cram into one's mind for examinations all this "multitude of things which clutter up the mind and divert it from the essential."
4 The first being Newton's unification of celestial and earth mechanics with his theory of classical mechanics and gravity.
interesting fundamental issues in physics really are, I have also included material that
will be useful for those who do not wish to work in scientific research.5

Next, we will take a “panoramic view” of the entire book, beginning by asking a
simple, yet quite fundamental question: What is light? There is no straight or easy
answer, but on pursuing this question we introduce you to most of what we now know
about light. We review briefly how our understanding of what light is has evolved
from simple everyday concepts to the more subtle ones brought about by quantum
mechanics. We consider some deep questions concerning the nature of light. Why is
the speed of light always the same even when the source of light is moving? Why in
the classical mechanics regime is light a wave, whereas electrons are particles? We
briefly review the history of cavity QED. Last but not least, I explain what the various
chapters are about and suggest some ways in which the book can be read.

1.1 WHAT IS LIGHT? THE DEVELOPMENT OF OUR MENTAL PICTURE
OF LIGHT

Light is a funny thing. You cannot catch it with your hands. It appears to be very
light (please forgive the pun), even weightless. Light looks completely different from
the substances (matter) that make up the world around us, such as air, sand, water,
and so on. It is almost as if it is not material at all—as if it is something celestial,
even divine. For centuries light has caught the attention and imagination of children,
poets, artists, philosophers, and scientists.

Apart from being such a curious phenomenon, there is another, stronger reason
why studying light is both interesting and important. Light holds the key to some of
nature’s deepest secrets. In this section we give an overview of how we got to know,
to a better and better approximation,6 what light is. On the way, we will see how
this better understanding of the nature of light helped to unlock some of the strangest
hidden features of our world, now described by the two great theories of modern
physics: relativity and quantum mechanics.

1.1.1 The crudest picture: Geometrical optics

Our limited everyday experience of light is explained almost entirely by the theory
of geometrical optics.7 Its origins lie in antiquity, but most of it was developed in the
seventeenth century by a number of people, including René Descartes, Willebrord

5The preface mentions some of this material.
6Our commonsense picture of light is a very crude caricature. Nevertheless, modern scientific knowledge
of light is nothing but an extension of this caricature. Instead of our limited everyday experience, it accesses
a much broader one made possible by instruments. Instead of our imprecise language and reasoning, it
uses the precision and rigor of mathematics. The end result, however, is just a model of reality; a picture
much more accurate than what common sense can ever aspire to be, but still only a picture.
7The image of a lamppost on the cover of this book, for example, was generated by a computer using the
principles of geometrical optics.
Snell, Christian Huygens, and Isaac Newton. The modern mathematical theory of geometrical optics was developed mainly in the nineteenth century by William R. Hamilton. For a thorough historical account, see Mach’s book [422].

As readers might recall from their basic physics lectures, geometrical optics considers light as being composed of rays whose path is determined by Fermat’s principle of least time. So in vacuum, these rays travel along straight lines. But when they meet a medium with a different refractive index, such as a lens, they bend (refraction). Extended objects can be decomposed into an infinite number of points, each emitting rays in all directions. Geometrical optics can explain many everyday phenomena, including the reflection of light off mirrors, the workings of lenses, refraction, and the formation of images. Still, however, it tells us nothing about the nature of rays or why they always take the path of least time.

1.1.2 An improvement: Wave optics

In the seventeenth century there were two rival theories about the nature of light. Newton proposed that light rays were made out of particles that followed his laws of mechanics. Huygens, on the other hand, proposed that they were made out of waves. Geometrical optics does not allow us to decide between these two theories. As Newton carried much more authority than Huygens, the particle view prevailed until the nineteenth century. It is curious, however, that at the time these two theories were proposed, there was a discovery that could have settled the dispute in favor of Huygens. Hooke and Grimaldi had already observed diffraction. Diffraction signals the breakdown of the geometrical optics approximation and indicates that, more accurately, light must be regarded as a wave. The breakdown happens when the relevant dimensions (size of an aperture, for example) are so small that they become comparable to the wavelength of the light wave. But Huygens apparently did not know about this discovery and never attempted to describe it with his theory. It was only in the early part of the nineteenth century, with the discovery of interference by Young and with the work of Fresnel, Arago, and others, that the need for a wave theory of light was finally accepted. The transition was not made without a struggle, though.

There is an interesting anecdote that clearly describes the reluctance of even some of the most eminent scientists of the time to give up their old beliefs and adopt the wave theory of light. When Fresnel finished a talk at the French Academy of Sciences about his wave theory of diffraction, Poisson stood up and said that he could prove by reduction ad absurdum that Fresnel’s theory was wrong. Poisson claimed that Fresnel’s theory implied that the shadow cast by a circular object illuminated by a point source should have a bright spot in the center. This was, according to Poisson, absurd: Obviously, no shadow can have a bright spot in its center. Arago, however, was not so sure and did the experiment to find out. It turned out that there was indeed a bright spot in the center, which is now known as the spot of Arago.
1.1.3 Great guns and fresh insight into the nature of space and time: Classical electrodynamics and relativity

So if we look closer at light rays, we find that they are really waves. But what is the nature of these waves? In mechanics, waves are oscillations that propagate in a medium, such as water waves or sound in air. But light also propagates through a vacuum. Nineteenth-century physicists tried to solve this puzzle by postulating the existence of an elastic medium, the ether, that permeated everything, even the vacuum.

A big problem with the ether was that ordinary elastic media support both longitudinal and transverse waves. Since the early part of the nineteenth century, however, physicists were convinced that light waves were purely transverse, like the waves on a stretched string. This problem was solved by James MacCullagh. In 1839 he showed that if instead of a solid that resists compression and distortion, the ether is assumed to have a potential energy depending only on the rotation of its volume elements, it would support only transverse waves.

But the major breakthrough came from the apparently totally unrelated field of electricity and magnetism. In the second half of the nineteenth century, James Clerk Maxwell developed a theory of electromagnetism predicting that light is an electromagnetic wave. Classical electrodynamics was born and the nature of the waves in the wave theory of light was clarified. In a letter dated January 5, 1865, to his cousin Charles Cay, Maxwell wrote [87]: “I have also a paper afloat, containing an electromagnetic theory of light, which, till I am convinced of the contrary, I hold to be great guns.” And great guns it was, indeed! It took 20 years from Maxwell’s original paper to Hertz’s experimental confirmation; this is one of the greatest predictions of theoretical physics.

Maxwell’s theory not only unified electricity, magnetism, and optics, but also introduced the key idea of field in physics. In Newtonian mechanics there is the mysterious instantaneous action at a distance. A field, on the other hand, is a local property of space that can propagate from one point to another only at a finite speed. The idea of field was probably originated by Faraday’s lines of force, but it was Maxwell who put it in a sound mathematical form. With the development of the theory of relativity, we now know that there is no ether, just the field: It is the field, permeating even the vacuum, that oscillates, not a material medium.

A very important clue to relativity and the overthrow of the ether came from Maxwell’s puzzling prediction that electromagnetic waves propagate at the same speed, the speed of light, regardless of the speed of the source of the waves or of the observer. The fact that this speed is determined by the electric permittivity and magnetic permeability of the material was at first thought to be a signal that Maxwell’s theory was valid only in a special frame where the ether was at rest. But years later, Michaelson and Morley [445] showed that the speed of light is really the same in any reference frame. Relativity is built on this invariance of the speed of light and explains it by changing the Newtonian notions of absolute time and space. The invariance of the speed of light showed us that the true nature of space and time is completely different from what is naively apparent to us from our everyday experience: Time for
someone who is moving closer to the speed of light in relation to you passes much more slowly than it does for you, and his or her space (along the direction of motion) contracts in relation to your space. Electrodynamics is such a great theory that it had relativity already built into it many years before relativity itself was discovered by Einstein!

It is curious that even though in the long run, Maxwell's theory helped to revolutionize the Newtonian view of the world, Maxwell himself used theoretical mechanical models to build his theory. One of his very distinctive characteristics was his habit of working by analogy [87], where he recognized mathematical similarities between quite distinct physical problems and tried to apply the successes of a well-tested theory to the mathematical analogous but physically very different situation. He imagined, for instance, that the magnetic field was made up of vortex tubes filling space. Between these vortices, to prevent friction, there were ball bearings, which Maxwell identified with electric particles (see Figure 1.1).

![Figure 1.1](image_url)  
*Figure 1.1* Maxwell's mechanical model for the production of lines of force by an electric current. He imagined that the ether was filled with molecular vortices (represented by the hexagons). Between these vortices there were charged particles (the little balls). AB represents an electric current going from A to B. As this current starts flowing, the row of vortices gh above AB will rotate counterclockwise. Assuming that the row of vortices kl is still at rest, the layer of particles between these rows will be acted on by the row gh below, making them rotate clockwise and move opposite from the current (i.e., from right to left). These charges moving in the opposite direction to the initial current would then constitute an induced current. (Reproduced from [435].)
1.1.4 Quantum mechanics and quantum electrodynamics: The best picture so far

Classical electrodynamics is a great theory, but there are some problems with it, and these problems point the way to a new and more encompassing theory: quantum electrodynamics (QED). As the name says, this is the theory that joins electrodynamics with quantum mechanics.

To most physicists, quantum mechanics introduced an even greater departure from the old Newtonian ideas than relativity did. There were many puzzling phenomena that, left unexplained by classical physics, pointed the way to the brave new world of the quantum. These are reviewed in most elementary quantum mechanics textbooks and we do not wish to go into details here. Let us just say that a lot of them came from spectroscopy and can be well accounted for by a semiclassical theory where only matter is quantized, not light. Others, such as the Compton effect, however, called for a quantum description of light, too.

In quantum mechanics everything behaves sometimes as a particle, sometimes as a wave. So according to quantum electrodynamics, light is made up of particles called photons that sometimes behave as waves! As this entire book is about this best approximation that we have to what light really is, we will not say much more about QED in this introduction. We will say something about this funny wave–particle duality, though.

Strange as it is, the wave–particle duality revealed by quantum mechanics must be accepted as a fact of life. It just turns out that nature is that strange deep inside. The reason that we find it so weird is because we are used to our everyday macroscopic experience, where particles always behave as particles and waves always as waves. So instead of asking the pointless question why the photon sometimes behaves as a wave, what we should really ask is why in our macroscopic world photons only behave as waves, whereas electrons only behave as particles. This question was answered in 1933 by Pauli [477] (see also “Part VI, Final Remarks: The New Particles” by Peierls in Sec. 1.3 of [485], and [486], pp. 471–473). He showed that what determines the particle-only or wave-only behavior of a quantum particle in our macroscopic world is its rest mass and quantum statistics.

A classical particle always has a definite position and momentum. This introduces the well-known problem of complementarity, our inability to measure the position and momentum of a quantum mechanical particle simultaneously. In Chapter 4 we see how complementarity effectively disappears in our macroscopic world. But even more basic than this is the problem that to talk about position and momentum, both must be observables. As we see in Chapter 3, although the momentum of a quantum particle can be measured to any accuracy, its position cannot be determined more accurately than to locate it inside a volume the size of the cube of its Compton wavelength. Any attempt to determine the position more accurately than this involves an interaction energy high enough to generate particle–antiparticle pairs out of the vacuum. This then necessarily takes us from a single-particle description to a many-particle one, where the idea of position of an indistinguishable particle among many others of its kind makes no sense. So position is an observable only in the nonrelativistic
regime where the Compton wavelength is negligible in comparison with the de Broglie wavelength. Unfortunately, quantum particles with a vanishing rest mass do not have a nonrelativistic regime. That is why the photon does not behave as a classical particle in our macroscopic world.\(^8\)

What about the classical wave behavior? A wave is characterized by both a phase and an amplitude. Ideally, a phase operator $\hat{\phi}$ for the electromagnetic radiation field would be the canonical conjugate of the photon number operator $\hat{N}$, just as position is the canonical conjugate of momentum; that is, the commutator of $\hat{N}$ and $\hat{\phi}$ would be given by

$$[\hat{N}, \hat{\phi}] = -i. \quad (1.1)$$

Unlike position in nonrelativistic quantum mechanics, however, there are serious difficulties with the introduction of a phase operator in quantum mechanics (see [89], [484], Sec. 1.4 of [485], and [490]). One such difficulty arises because unlike the momentum operator, $\hat{N}$ has a discrete spectrum of eigenvalues. This can be seen by sandwiching (1.1) between a bra–ket pair of eigenstates of $\hat{N}$, $\langle n \rvert$ and $\lvert n' \rangle$:

$$\langle n - n' \rvert \langle n \rvert \hat{\phi} \mid n' \rangle = -i \delta_{nn'}.$$ \quad (1.2)

Now take $n' = n$ in (1.2). The left-hand side vanishes while the right-hand side becomes $-i$. Attempts to avoid this problem usually encounter another difficulty, this time associated with the lack of negative eigenvalues in the spectrum of $\hat{N}$ [i.e., it starts at $\hat{N} = 0$ rather than at $\hat{N} = -\infty$ (see Chapter 4)]. A further difficulty is connected with the fact that a phase of $\hat{\phi}$ and another of $\hat{\phi} + 2\pi$ should be equivalent. Fortunately, for our purposes here, we can use the well-defined operator $\exp(i\hat{\phi})$ rather than $\hat{\phi}$ itself.\(^9\) The commutator between $\hat{N}$ and $\exp(i\hat{\phi})$ is given by [89, 485]

$$[\hat{N}, e^{i\hat{\phi}}] = e^{i\hat{\phi}}. \quad (1.3)$$

Repetition of the steps that yielded (1.2) now yields

$$\langle n - n' \rvert \langle n \rvert e^{i\hat{\phi}} \mid n' \rangle = \langle n \rvert e^{i\hat{\phi}} \mid n' \rangle.$$ \quad (1.4)

Unlike (1.2), equation (1.4) does not lead to any contradiction. Its consequence is merely that $\langle n \rvert \exp(i\hat{\phi}) \mid n' \rangle$ can only be nonvanishing for $n' = n - 1$ [i.e., $\exp(i\hat{\phi})$ is similar to a raising or creation operator].

From (1.3) we can obtain the following uncertainty relation for the accuracy $\Delta N$ in determining $\hat{N}$ and the accuracy $|\Delta \exp(i\hat{\phi})|$ in $\exp(i\hat{\phi})$ [89]:

$$\Delta N \left| \Delta e^{i\hat{\phi}} \right| \geq \frac{1}{2} \left| \left\langle e^{i\hat{\phi}} \right\rangle \right|. \quad (1.5)$$

\(^8\) In the geometrical optics regime, however, we can regard photons as classical particles (Newton's view of the nature of rays). This is because the position of such a "classical" photon is never determined more accurately than a ray (i.e., in geometrical optics, a photon is at most localized to a volume on the order of the cube of its wavelength).

\(^9\) In terms of creation and annihilation operators for a single field mode or an harmonic oscillator, $\exp(i\hat{\phi})$ is given by $\hat{a}^\dagger (\hat{a}^\dagger \hat{a} + 1)^{-1/2}$ (see Carruthers and Nieto [89]).
If we call $\Delta \phi$ a measure of the phase uncertainty defined by\(^{10}\)

$$\Delta \phi \equiv \left| \frac{\Delta e^{i\phi}}{\langle e^{i\phi} \rangle} \right| = \sqrt{\frac{(\Delta \hat{C})^2 + (\Delta \hat{S})^2}{\langle \hat{C} \rangle^2 + \langle \hat{S} \rangle^2}}, \quad (1.6)$$

where $\hat{C}$ and $\hat{S}$ are the sine and cosine Hermitian operators\(^{11}\)

$$\hat{C} \equiv \frac{e^{i\phi} + e^{-i\phi}}{2} \quad \text{and} \quad \hat{S} \equiv \frac{e^{i\phi} - e^{-i\phi}}{i2}, \quad (1.7)$$

we have the following simple uncertainty relation for $\Delta N$ and $\Delta \phi$

$$\Delta N \Delta \phi \geq \frac{1}{2}. \quad (1.8)$$

Now it is clear from (1.8) that to measure the phase of a field with accuracy, we must change the number of particles. For light, this is not a big problem, as photons can be created easily because they have no rest mass. For a massive particle, however, this will require a large amount of energy. That is why massive particles do not behave as waves in our macroscopic world.

In a classical wave, however, it is not only the phase that must have negligible uncertainty, but also the amplitude. In other words, the uncertainty in the phase should be small compared to unity, but the uncertainty in $N$ should also be small compared to $N$ itself. From (1.8), we see that this requires

$$N \gg 1. \quad (1.9)$$

Photons can easily fulfill this requirement, as they are bosons. In fact, for a 1-kW transmitter at 1 MHz, it can be estimated that only at large distances exceeding $10^{10}$ km from the transmitter does the number of photons drop to about 1 [485]. Another example is a typical 10-mW HeNe laser, whose number of photons in the lasing mode can easily be estimated to be on the order\(^{12}\) of $10^{10}$. For fermions, however, the Pauli exclusion principle limits $N$ to 1. Thus, fermions cannot behave as classical waves in our macroscopic world.

### 1.2 A BRIEF HISTORY OF CAVITY QED

The first known reference to a cavity QED effect dates back to the 1940s, when the Physical Review published the abstract of a paper presented by Edward M. Purcell...
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at the 1946 Spring Meeting of the American Physical Society [502]. His paper was about the changes in the spontaneous emission rate for nuclear magnetic moment transitions at radio frequency, when a spin system is coupled to a resonant electrical circuit. At room temperature (300 K), for a frequency of 10 MHz and a \( \mu = 1 \) nuclear magneton, he found that the spontaneous emission relaxation time was about \( 5 \times 10^{21} \) seconds. If, however, small metallic particles with a diameter of \( 10^{-3} \) cm were mixed with the nuclear magnetic medium, he found that the spontaneous emission relaxation time dropped to only a few minutes.

The 1950s saw the realization of the first maser [242], which stimulated further research in the interaction between matter and the radiation field in cavities. During this decade, modification of spontaneous emission rates of electron spin transitions was predicted [115] and confirmed experimentally [191]. The mechanism of spontaneous emission rate enhancement deduced by Purcell was predicted to hold as well for collective spontaneous emission in magnetic resonance experiments [61]. Another significant development came in 1958, when Schawlow and Townes [536], Prokhorov [500], and Dicke [115] independently proposed use of the Fabry–Perot interferometer as a resonator for the realization of a maser that operated in the optical regime (i.e., a laser). This generated a lot of interest in the theory of open cavities.

The 1960s began with the realization of the first laser [424]. A number of theoretical papers were published on open resonators and their modes, with important contributions by Fox and Li [220, 221] and Vainshtein [605–607]. The 1960s also saw Jaynes and Cummings develop a fundamental model, which now bears their names, for the interaction between an atom/molecule and a single mode of the quantized radiation field [323, 559]. Last but not least were Drexhage’s beautiful experiments where the fluorescence of dye molecules placed at precisely controlled distances from a mirror was measured [100, 164].

In the 1970s, change in the spontaneous emission rate of an atom when it is inside a Fabry–Perot interferometer was calculated using full-blown quantum electrodynamics [39, 577]. These two papers marked the beginning of a great theoretical interest in cavity QED that has resulted in a large number of papers being published on this subject ever since.

After those primordial times, the field of cavity QED entered a period that one of its leading researchers has named the weak-coupling-regime age [503]. It was then that spontaneous emission enhancement [245] and inhibition [303] for an atom in an empty cavity was experimentally demonstrated.\(^{14}\)

Then there came what that researcher calls the strong-coupling-regime age. It was then that one-photon [440] and two-photon [81] single-atom masers (micromasers) were realized, and quantum Rabi oscillations [82] and vacuum Rabi splitting\(^{15}\) [592] were observed. Strong coupling can also be used to manipulate entanglement, and this has given rise to a number of applications in quantum information.

\(^{13}\)For a detailed review of the developments in resonator theory during this time, see [563].

\(^{14}\)Drexhage had worked with molecules in material media.

\(^{15}\)This is normal-mode splitting due to the oscillatory exchange of energy quanta between an atom and a quantized cavity mode.
At the time of writing, in parallel with the ongoing basic research, we are experiencing an "industrial age," where cavity QED ideas are being applied to optoelectronic devices. This is being done mainly through photonic crystals [326, 655], new materials whose dielectric permittivity varies in a regular way analogous to the spatial arrangement of atoms in a crystal. As in a crystal, these structures give rise to bands (allowed values of the wave vector) and gaps (forbidden values of the wave vector) that change the mode structure of free space. A single mode-cavity, for example, can be constructed in a photonic crystal by adding a defect in the regular variation of the dielectric permittivity. This defect acts as an impurity that when properly engineered will create a discrete level in the middle of a gap: a single mode where a radiation field can exist.

1.3 A MAP OF THE BOOK: AN OVERVIEW OF WHAT IS TO COME

This book is about theory, mostly theory connected with fundamental ideas in physics.\textsuperscript{16} There is almost no description of experiments. You can find these in experimental papers, review articles, and other books, some of which are listed in the References. Whenever possible, this theory has been presented using very simple models that emphasize only the key points we wish to focus our attention on, neglecting everything else. This model-building approach has been used by Maxwell and many other British physicists, often to the horror of their counterparts in continental Europe. Model building is not only a useful aid to highlight, develop, and illustrate theoretical ideas, but also a skill in high demand in the nonacademic job market. One of the aims of this book is to help train students, especially undergraduates, in this often neglected art.

The topics covered in this book were chosen using four basic criteria. First, there were those topics that were picked because they are in some way connected to fundamental ideas in physics. Second, there were topics which the author regards as important and which often rouse the curiosity of students but that somehow have not been addressed in other quantum optics textbooks (see the preface for a list of some of these topics). Third, there were topics which can also be found in other textbooks, but which this author addresses from a completely different point of view, with an original approach that cannot be found anywhere else. Fourth, everything had to fit in a coherent story, with one topic related to the others and following naturally in the sequence of our discussions (i.e., no nonsequiters). These criteria, of course, do not determine a list of topics uniquely. Ultimately, a large component of personal taste was involved in the choice.

Chapter 2 starts our incursion into cavity QED country with an introduction to field quantization. It goes through quantization in cavities as well as in free space. But before dealing with quantization, we discuss three issues that are essential for

\textsuperscript{16}This is also reflected on the choice of units. The CGS units adopted throughout the book are better suited for presenting theoretical ideas than SI units are.
most of what is to come. The first is canonical quantization (here we also review
Dirac's representation-free quantum mechanics). The second is why only the radiation
field is quantized. The third is what constitutes a resonator. Here, we discuss the
physics behind a cavity's resonator features rather than just presenting a calculation
of cavity modes. This is also where we introduce the model-building approach
that is used many times in the book. The chapter closes with a look at one of the
few consequences of field quantization that does not need interaction with matter
in its theoretical description: the Casimir force. The last subsection deals with a
very important problem related not only to quantum electrodynamics but also to
the whole of field theory and general relativity. This is the problem of the vacuum
catastrophe, which may well point the way to a new theory where gravitation and
quantum mechanics will be unified.

In Chapter 3 we present an alternative approach to quantum electrodynamics,
where rather than using canonical quantization, we follow the route usually taken in
elementary nonrelativistic quantum mechanics: going from a single-particle wave-
function description to a many-particle second-quantized theory. We address various
fundamental issues in quantum electrodynamics, including why the idea of a wave-
function of the photon in configuration space is plagued with problems, and the
connection between nonlocalizability and vanishing rest mass. Instead of adopting
the traditional abstract group-theoretical approach, these issues are discussed using
concrete examples. The chapter also offers insight into the way that first-order-in-
time relativistic wave equations such as the Dirac equation can be derived from the
relativistic energy–momentum relation.

In Chapter 4 we end our discussion on light without matter by looking at some of
the things that we can do with light by itself in a cavity. First we will see, in more detail
than was presented in this introduction, how our ordinary "classical" experience of
the radiation field as a wave can arise from quantum electrodynamics. This will take
us to coherent states and statistical mixtures. Statistical mixtures will naturally lead
us to introduce the density matrix. Then we will show how quantum mechanics can be
exploited to effectively reduce the noise of light below its fundamental quantum limit
experienced in coherent states. Rather than just postulating the form of the squeezing
operator, we deduce it from its intended properties. We discuss in great detail the
usual graphical representation of squeezed states, where they are depicted as phasors
with an "ellipse of noise" attached to the tip of the phasor. We review how squeezed
states can be detected using homodyne detection and how they can be generated (in
an unusual way) by suddenly changing the size of a cavity.

The two shortest chapters in the book are Chapters 5 and 6. The former starts on
our examination of the interaction between quantized field and matter by introducing
the coupling between the two. This is done in three different ways. First, we see how
the simple free-space quantization without electromagnetic potentials and associated
gauge complications can easily be extended to include interaction with matter, as
long as this interaction is only within the dipole approximation. Second, we will

17Such a calculation is left for Appendix A.