MODERN ANTENNA DESIGN

Second Edition

THOMAS A. MILLIGAN

IEEE PRESS

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To Mary, Jane, and Margaret

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PREFACE

I wrote this book from my perspective as a designer in industry, primarily for other designers and users of antennas. On occasion I have prepared and taught antenna courses, for which I developed a systematic approach to the subject. For the last decade I have edited the "Antenna Designer's Notebook" column in the IEEE antenna magazine. This expanded edition adds a combination of my own design notebook and the many other ideas provided to me by others, leading to this collection of ideas that I think designers should know.

The book contains a systematic approach to the subject. Every author would like to be read from front to back, but my own career assignments would have caused to me to jump around in this book. Nevertheless, Chapter 1 covers those topics that every user and designer should know. Because I deal with complete antenna design, which includes mounting the antenna, included are the effects of nearby structures and how they can be used to enhance the response. We all study ideal antennas floating in free space to help us understand the basics, but the real world is a little different.

Instead of drawing single line graphs of common relationships between two parameters, I generated scales for calculations that I perform over and over. I did not supply a set of computer programs because I seldom use collections supplied by others, and younger engineers find my programs quaint, as each generation learns a different computer language. You'll learn by writing your own.

IEEE Antennas and Propagation Society's digital archive of all material published from 1952 to 2000 has changed our approach to research. I have not included extensive bibliographies, because I believe that it is no longer necessary. The search engine of the archive can supply an exhaustive list. I referred only to papers that I found particularly helpful. Complete sets of the transactions are available in libraries, whereas the wealth of information in the archive from conferences was not. I have started mining this information, which contains many useful design ideas, and have incorporated some of them in this book. In this field, 40-year-old publications are still useful and we should not reinvent methods. Many clever ideas from industry are usually published only once, if at all, and personally, I'll be returning to this material again and again, because all books have limited space.

As with the first edition, I enjoyed writing this book because I wanted to express my point of view of a rewarding field. Although the amount of information available is overwhelming and the mathematics describing it can cloud the ideas, I hope my explanations help you develop new products or use old ones.

I would like to thank all the authors who taught me this subject by sharing their ideas, especially those working in industry. On a personal note I thank the designers at Lockheed Martin, who encouraged me and reviewed material while I wrote: in particular, Jeannette McDonnell, Thomas Cencich, Donald Huebner, and Julie Huffman.

Thomas A. Milligan

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PROPERTIES OF ANTENNAS

One approach to an antenna book starts with a discussion of how antennas radiate. Beginning with Maxwell's equations, we derive electromagnetic waves. After that lengthy discussion, which contains a lot of mathematics, we discuss how these waves excite currents on conductors. The second half of the story is that currents radiate and produce electromagnetic waves. You may already have studied that subject, or if you wish to further your background, consult books on electromagnetics. The study of electromagnetics gives insight into the mathematics describing antenna radiation and provides the rigor to prevent mistakes. We skip the discussion of those equations and move directly to practical aspects.

It is important to realize that antennas radiate from currents. Design consists of controlling currents to produce the desired radiation distribution, called its *pattern*. In many situations the problem is how to prevent radiation from currents, such as in circuits. Whenever a current becomes separated in distance from its return current, it radiates. Simply stated, we design to keep the two currents close together, to reduce radiation. Some discussions will ignore the current distribution and instead, consider derived quantities, such as fields in an aperture or magnetic currents in a slot or around the edges of a microstrip patch. You will discover that we use any concept that provides insight or simplifies the mathematics.

An antenna converts bound circuit fields into propagating electromagnetic waves and, by reciprocity, collects power from passing electromagnetic waves. Maxwell's equations predict that any time-varying electric or magnetic field produces the opposite field and forms an electromagnetic wave. The wave has its two fields oriented orthogonally, and it propagates in the direction normal to the plane defined by the perpendicular electric and magnetic fields. The electric field, the magnetic field, and the direction of propagation form a right-handed coordinate system. The propagating wave field intensity decreases by $1/R$ away from the source, whereas a static field

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drops off by $1/R^2$. Any circuit with time-varying fields has the capability of radiating to some extent.

We consider only time-harmonic fields and use phasor notation with time dependence $e^{j\omega t}$. An outward-propagating wave is given by $e^{-j(kR-\omega t)}$, where k, the wave number, is given by $2\pi/\lambda$. λ is the wavelength of the wave given by c/f, where c is the velocity of light (3×10^8 m/s in free space) and f is the frequency. Increasing the distance from the source decreases the phase of the wave.

Consider a two-wire transmission line with fields bound to it. The currents on a single wire will radiate, but as long as the ground return path is near, its radiation will nearly cancel the other line's radiation because the two are 180° out of phase and the waves travel about the same distance. As the lines become farther and farther apart, in terms of wavelengths, the fields produced by the two currents will no longer cancel in all directions. In some directions the phase delay is different for radiation from the current on each line, and power escapes from the line. We keep circuits from radiating by providing close ground returns. Hence, high-speed logic requires ground planes to reduce radiation and its unwanted crosstalk.

1-1 ANTENNA RADIATION

Antennas radiate spherical waves that propagate in the radial direction for a coordinate system centered on the antenna. At large distances, spherical waves can be approximated by plane waves. Plane waves are useful because they simplify the problem. They are not physical, however, because they require infinite power.

The Poynting vector describes both the direction of propagation and the power density of the electromagnetic wave. It is found from the vector cross product of the electric and magnetic fields and is denoted **S**:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}^* \qquad \mathrm{W/m^2}$$

Root mean square (RMS) values are used to express the magnitude of the fields. $\mathbf{H}^*$ is the complex conjugate of the magnetic field phasor. The magnetic field is proportional to the electric field in the far field. The constant of proportion is η, the impedance of free space ($\eta = 376.73\,\Omega$):

$$|\mathbf{S}| = S = \frac{|\mathbf{E}|^2}{\eta} \qquad \mathrm{W/m^2} \tag{1-1}$$

Because the Poynting vector is the vector product of the two fields, it is orthogonal to both fields and the triplet defines a right-handed coordinate system: (**E, H, S**).

Consider a pair of concentric spheres centered on the antenna. The fields around the antenna decrease as $1/R$, $1/R^2$, $1/R^3$, and so on. Constant-order terms would require that the power radiated grow with distance and power would not be conserved. For field terms proportional to $1/R^2$, $1/R^3$, and higher, the power density decreases with distance faster than the area increases. The energy on the inner sphere is larger than that on the outer sphere. The energies are not radiated but are instead concentrated around the antenna; they are near-field terms. Only the $1/R^2$ term of the Poynting vector ($1/R$ field terms) represents radiated power because the sphere area grows as R^2 and gives a constant product. All the radiated power flowing through the inner sphere will propagate to the outer sphere. The sign of the input reactance depends on the near-field predominance of field type: electric (capacitive) or magnetic (inductive). At resonance (zero reactance) the stored energies due to the near fields are equal. Increasing the stored fields increases the circuit Q and narrows the impedance bandwidth.

Far from the antenna we consider only the radiated fields and power density. The power flow is the same through concentric spheres:

$$4\pi R_1^2 S_{1,\mathrm{avg}} = 4\pi R_2^2 S_{2,\mathrm{avg}}$$

The average power density is proportional to $1/R^2$. Consider differential areas on the two spheres at the same coordinate angles. The antenna radiates only in the radial direction; therefore, no power may travel in the θ or ϕ direction. Power travels in flux tubes between areas, and it follows that not only the average Poynting vector but also every part of the power density is proportional to $1/R^2$:

$$S_1 R_1^2 \sin\theta \, d\theta \, d\phi = S_2 R_2^2 \sin\theta \, d\theta \, d\phi$$

Since in a radiated wave S is proportional to $1/R^2$, E is proportional to $1/R$. It is convenient to define radiation intensity to remove the $1/R^2$ dependence:

$$U(\theta, \phi) = S(R, \theta, \phi) R^2 \qquad \text{W/solid angle}$$

Radiation intensity depends only on the direction of radiation and remains the same at all distances. A probe antenna measures the relative radiation intensity (pattern) by moving in a circle (constant R) around the antenna. Often, of course, the antenna rotates and the probe is stationary.

Some patterns have established names. Patterns along constant angles of the spherical coordinates are called either *conical* (constant θ) or *great circle* (constant ϕ). The great circle cuts when $\phi = 0^\circ$ or $\phi = 90^\circ$ are the principal plane patterns. Other named cuts are also used, but their names depend on the particular measurement positioner, and it is necessary to annotate these patterns carefully to avoid confusion between people measuring patterns on different positioners. Patterns are measured by using three scales: (1) linear (power), (2) square root (field intensity), and (3) decibels (dB). The dB scale is used the most because it reveals more of the low-level responses (sidelobes).

Figure 1-1 demonstrates many characteristics of patterns. The half-power beamwidth is sometimes called just the beamwidth. The tenth-power and null beamwidths are used in some applications. This pattern comes from a parabolic reflector whose feed is moved off the axis. The vestigial lobe occurs when the first sidelobe becomes joined to the main beam and forms a shoulder. For a feed located on the axis of the parabola, the first sidelobes are equal.

1-2 GAIN

Gain is a measure of the ability of the antenna to direct the input power into radiation in a particular direction and is measured at the peak radiation intensity. Consider the

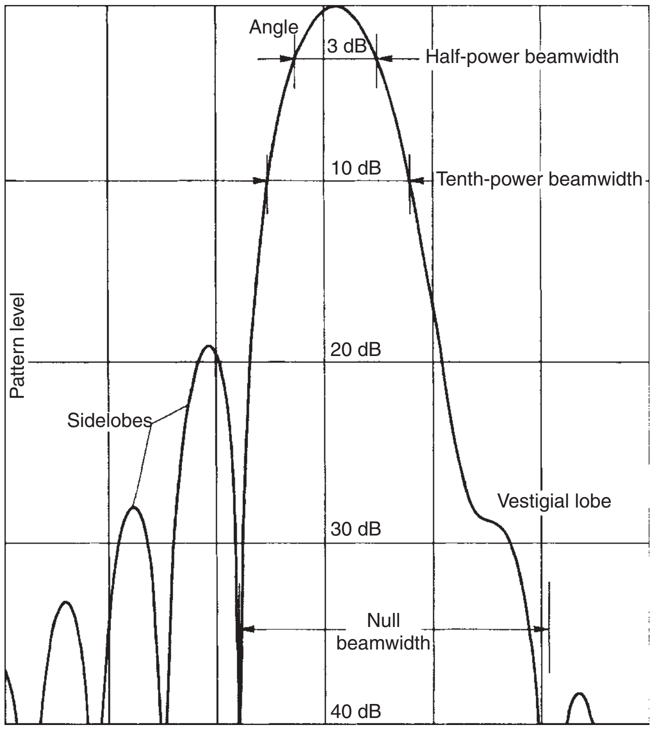


FIGURE 1-1 Antenna pattern characteristics.

power density radiated by an isotropic antenna with input power P_0 at a distance R: $S = P_0/4\pi R^2$. An isotropic antenna radiates equally in all directions, and its radiated power density S is found by dividing the radiated power by the area of the sphere $4\pi R^2$. The isotropic radiator is considered to be 100% efficient. The gain of an actual antenna increases the power density in the direction of the peak radiation:

$$S = \frac{P_0 G}{4\pi R^2} = \frac{|\mathbf{E}|^2}{\eta} \quad \text{or} \quad |\mathbf{E}| = \frac{1}{R}\sqrt{\frac{P_0 G \eta}{4\pi}} = \sqrt{S\eta} \tag{1-2}$$

Gain is achieved by directing the radiation away from other parts of the radiation sphere. In general, gain is defined as the gain-biased pattern of the antenna:

$$\begin{aligned} S(\theta, \phi) &= \frac{P_0 G(\theta, \phi)}{4\pi R^2} \qquad \text{power density} \\ U(\theta, \phi) &= \frac{P_0 G(\theta, \phi)}{4\pi} \qquad \text{radiation intensity} \end{aligned} \tag{1-3}$$

The surface integral of the radiation intensity over the radiation sphere divided by the input power P_0 is a measure of the relative power radiated by the antenna, or the antenna efficiency:

$$\frac{P_r}{P_0} = \int_0^{2\pi} \int_0^{\pi} \frac{G(\theta, \phi)}{4\pi} \sin\theta \, d\theta \, d\phi = \eta_e \qquad \text{efficiency}$$

where P_r is the radiated power. Material losses in the antenna or reflected power due to poor impedance match reduce the radiated power. In this book, integrals in the equation above and those that follow express concepts more than operations we perform during design. Only for theoretical simplifications of the real world can we find closed-form solutions that would call for actual integration. We solve most integrals by using numerical methods that involve breaking the integrand into small segments and performing a weighted sum. However, it is helpful that integrals using measured values reduce the random errors by averaging, which improves the result.

In a system the transmitter output impedance or the receiver input impedance may not match the antenna input impedance. Peak gain occurs for a receiver impedance conjugate matched to the antenna, which means that the resistive parts are the same and the reactive parts are the same magnitude but have opposite signs. Precision gain measurements require a tuner between the antenna and receiver to conjugate-match the two. Alternatively, the mismatch loss must be removed by calculation after the measurement. Either the effect of mismatches is considered separately for a given system, or the antennas are measured into the system impedance and mismatch loss is considered to be part of the efficiency.

Example Compute the peak power density at 10 km of an antenna with an input power of 3 W and a gain of 15 dB.

First convert dB gain to a ratio: $G = 10^{15/10} = 31.62$. The power spreads over the sphere area with radius 10 km or an area of $4\pi(10^4)^2$ m^2. The power density is

$$S = \frac{(3\,\mathrm{W})(31.62)}{4\pi \times 10^8\ \mathrm{m}^2} = 75.5\,\mathrm{nW/m}^2$$

We calculate the electric field intensity using Eq. (1-2):

$$|\mathbf{E}| = \sqrt{S\eta} = \sqrt{(75.5 \times 10^{-9})(376.7)} = 5333\,\mu\mathrm{V/m}$$

Although gain is usually relative to an isotropic antenna, some antenna gains are referred to a $\lambda/2$ dipole with an isotropic gain of 2.14 dB.

If we approximate the antenna as a point source, we compute the electric field radiated by using Eq. (1-2):

$$E(\theta,\phi) = \frac{e^{-jkR}}{R}\sqrt{\frac{P_0 G(\theta,\phi)\eta}{4\pi}} \tag{1-4}$$

This requires only that the antenna be small compared to the radial distance R. Equation (1-4) ignores the direction of the electric field, which we define as *polarization*. The units of the electric field are volts/meter. We determine the far-field pattern by multiplying Eq. (1-4) by R and removing the phase term e^{-jkR} since phase has meaning only when referred to another point in the far field. The far-field electric field E_{ff} unit is volts:

$$E_{\mathrm{ff}}(\theta,\phi) = \sqrt{\frac{P_0 G(\theta,\phi)\eta}{4\pi}} \quad \text{or} \quad G(\theta,\phi) = \frac{1}{P_0}\left[E_{\mathrm{ff}}(\theta,\phi)\sqrt{\frac{4\pi}{\eta}}\right]^2 \tag{1-5}$$

During analysis, we often normalize input power to 1 W and can compute gain easily from the electric field by multiplying by a constant $\sqrt{4\pi/\eta} = 0.1826374$.

1-3 EFFECTIVE AREA

Antennas capture power from passing waves and deliver some of it to the terminals. Given the power density of the incident wave and the effective area of the antenna, the power delivered to the terminals is the product

$$P_d = SA_{\text{eff}} \tag{1-6}$$

For an aperture antenna such as a horn, parabolic reflector, or flat-plate array, effective area is physical area multiplied by aperture efficiency. In general, losses due to material, distribution, and mismatch reduce the ratio of the effective area to the physical area. Typical estimated aperture efficiency for a parabolic reflector is 55%. Even antennas with infinitesimal physical areas, such as dipoles, have effective areas because they remove power from passing waves.

1-4 PATH LOSS [1, p. 183]

We combine the gain of the transmitting antenna with the effective area of the receiving antenna to determine delivered power and path loss. The power density at the receiving antenna is given by Eq. (1-3), and the received power is given by Eq. (1-6). By combining the two, we obtain the path loss:

$$\frac{P_d}{P_t} = \frac{A_2 G_1(\theta, \phi)}{4\pi R^2}$$

Antenna 1 transmits, and antenna 2 receives. If the materials in the antennas are linear and isotropic, the transmitting and receiving patterns are identical (reciprocal) [2, p. 116]. When we consider antenna 2 as the transmitting antenna and antenna 1 as the receiving antenna, the path loss is

$$\frac{P_d}{P_t} = \frac{A_1 G_2(\theta, \phi)}{4\pi R^2}$$

Since the responses are reciprocal, the path losses are equal and we can gather and eliminate terms:

$$\frac{G_1}{A_1} = \frac{G_2}{A_2} = \text{constant}$$

Because the antennas were arbitrary, this quotient must equal a constant. This constant was found by considering the radiation between two large apertures [3]:

$$\frac{G}{A} = \frac{4\pi}{\lambda^2} \tag{1-7}$$

We substitute this equation into path loss to express it in terms of the gains or effective areas:

$$\frac{P_d}{P_t} = G_1 G_2 \left(\frac{\lambda}{4\pi R}\right)^2 = \frac{A_1 A_2}{\lambda^2 R^2} \tag{1-8}$$

We make quick evaluations of path loss for various units of distance R and for frequency f in megahertz using the formula

$$\text{path loss(dB)} = K_U + 20\log(fR) - G_1(\text{dB}) - G_2(\text{dB}) \tag{1-9}$$

where K_U depends on the length units:

Unit	K_U
km	32.45
nm	37.80
miles	36.58
m	−27.55
ft	−37.87

Example Compute the gain of a 3-m-diameter parabolic reflector at 4 GHz assuming 55% aperture efficiency.

Gain is related to effective area by Eq. (1-7):

$$G = \frac{4\pi A}{\lambda^2}$$

We calculate the area of a circular aperture by $A = \pi(D/2)^2$. By combining these equations, we have

$$G = \left(\frac{\pi D}{\lambda}\right)^2 \eta_a = \left(\frac{\pi D f}{c}\right)^2 \eta_a \tag{1-10}$$

where D is the diameter and η_a is the aperture efficiency. On substituting the values above, we obtain the gain:

$$G = \left[\frac{3\pi(4 \times 10^9)}{0.3 \times 10^9}\right]^2 (0.55) = 8685 \quad (39.4\,\text{dB})$$

Example Calculate the path loss of a 50-km communication link at 2.2 GHz using a transmitter antenna with a gain of 25 dB and a receiver antenna with a gain of 20 dB.

$$\text{Path loss} = 32.45 + 20\log[2200(50)] - 25 - 20 = 88.3\,\text{dB}$$

What happens to transmission between two apertures as the frequency is increased? If we assume that the effective area remains constant, as in a parabolic reflector, the transmission increases as the square of frequency:

$$\frac{P_d}{P_t} = \frac{A_1 A_2}{R^2}\frac{1}{\lambda^2} = \frac{A_1 A_2}{R^2}\left(\frac{f}{c}\right)^2 = Bf^2$$

where B is a constant for a fixed range. The receiving aperture captures the same power regardless of frequency, but the gain of the transmitting antenna increases as the square of frequency. Hence, the received power also increases as frequency squared. Only for antennas, whose gain is a fixed value when frequency changes, does the path loss increase as the square of frequency.

1-5 RADAR RANGE EQUATION AND CROSS SECTION

Radar operates using a double path loss. The radar transmitting antenna radiates a field that illuminates a target. These incident fields excite surface currents that also radiate to produce a second field. These fields propagate to the receiving antenna, where they are collected. Most radars use the same antenna both to transmit the field and to collect the signal returned, called a *monostatic* system, whereas we use separate antennas for *bistatic* radar. The receiving system cannot be detected in a bistatic system because it does not transmit and has greater survivability in a military application.

We determine the power density illuminating the target at a range R_T by using Eq. (1-2):

$$S_{\text{inc}} = \frac{P_T G_T(\theta, \phi)}{4\pi R_T^2} \tag{1-11}$$

The target's radar cross section (RCS), the scattering area of the object, is expressed in square meters or dBm^2: 10 log(square meters). The RCS depends on both the incident and reflected wave directions. We multiply the power collected by the target with its receiving pattern by the gain of the effective antenna due to the currents induced:

$$\text{RCS} = \sigma = \frac{\text{power}_{\text{reflected}}}{\text{power density incident}} = \frac{P_s(\theta_r, \phi_r, \theta_i, \phi_i)}{P_T G_T / 4\pi R_T^2} \tag{1-12}$$

In a communication system we call P_s the *equivalent isotropic radiated power* (EIRP), which equals the product of the input power and the antenna gain. The target becomes the transmitting source and we apply Eq. (1-2) to find the power density at the receiving antenna at a range R_R from the target. Finally, the receiving antenna collects the power density with an effective area A_R. We combine these ideas to obtain the power delivered to the receiver:

$$P_{\text{rec}} = S_R A_R = \frac{A_R P_T G_T \sigma(\theta_r, \phi_r, \theta_i, \phi_i)}{(4\pi R_T^2)(4\pi R_R^2)}$$

We apply Eq. (1-7) to eliminate the effective area of the receiving antenna and gather terms to determine the bistatic radar range equation:

$$\frac{P_{\text{rec}}}{P_T} = \frac{G_T G_R \lambda^2 \sigma(\theta_r, \phi_r, \theta_i, \phi_i)}{(4\pi)^3 R_T^2 R_R^2} \tag{1-13}$$

We reduce Eq. (1-13) and collect terms for monostatic radar, where the same antenna is used for both transmitting and receiving:

$$\frac{P_{\text{rec}}}{P_T} = \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

Radar received power is proportional to $1/R^4$ and to G^2.

We find the approximate RCS of a flat plate by considering the plate as an antenna with an effective area. Equation (1-11) gives the power density incident on the plate that collects this power over an area A_R:

$$P_C = \frac{P_T G_T(\theta, \phi)}{4\pi R_T^2} A_R$$

The power scattered by the plate is the power collected, P_C, times the gain of the plate as an antenna, G_P:

$$P_s = P_C G_P = \frac{P_T G_T(\theta_i, \phi_i)}{4\pi R_T^2} A_R G_P(\theta_r, \phi_r)$$

This scattered power is the effective radiated power in a particular direction, which in an antenna is the product of the input power and the gain in a particular direction. We calculate the plate gain by using the effective area and find the scattered power in terms of area:

$$P_s = \frac{P_T G_T 4\pi A_R^2}{4\pi R_T^2 \lambda^2}$$

We determine the RCS σ by Eq. (1-12), the scattered power divided by the incident power density:

$$\sigma = \frac{P_s}{P_T G_T / 4\pi R_T^2} = \frac{4\pi A_R^2}{\lambda^2} = \frac{G_R(\theta_i, \phi_i) G_R(\theta_r, \phi_r) \lambda^2}{4\pi} \tag{1-14}$$

The right expression of Eq. (1-14) divides the gain into two pieces for bistatic scattering, where the scattered direction is different from the incident direction. Monostatic scattering uses the same incident and reflected directions. We can substitute any object for the flat plate and use the idea of an effective area and its associated antenna gain. An antenna is an object with a unique RCS characteristic because part of the power received will be delivered to the antenna terminals. If we provide a good impedance match to this signal, it will not reradiate and the RCS is reduced. When we illuminate an antenna from an arbitrary direction, some of the incident power density will be scattered by the structure and not delivered to the antenna terminals. This leads to the division of antenna RCS into the antenna mode of reradiated signals caused by terminal mismatch and the structural mode, the fields reflected off the structure for incident power density not delivered to the terminals.

1-6 WHY USE AN ANTENNA?

We use antennas to transfer signals when no other way is possible, such as communication with a missile or over rugged mountain terrain. Cables are expensive and take a long time to install. Are there times when we would use antennas over level ground? The large path losses of antenna systems lead us to believe that cable runs are better.

Example Suppose that we must choose between using a low-loss waveguide run and a pair of antennas at 3 GHz. Each antenna has 10 dB of gain. The low-loss waveguide has only 19.7 dB/km loss. Table 1-1 compares losses over various distances. The waveguide link starts out with lower loss, but the antenna system soon overtakes it. When the path length doubles, the cable link loss also doubles in decibels, but an antenna link

TABLE 1-1 Losses Over Distance

Distance (km)	Waveguide Loss (dB)	Antenna Path Loss (dB)
2	39.4	88
4	78.8	94
6	118.2	97.6
10	197	102

increases by only 6 dB. As the distance is increased, radiating between two antennas eventually has lower losses than in any cable.

Example A 200-m outside antenna range was set up to operate at 2 GHz using a 2-m-diameter reflector as a source. The receiver requires a sample of the transmitter signal to phase-lock the local oscillator and signal at a 45-MHz difference. It was proposed to run an RG/U 115 cable through the power and control cable conduit, since the run was short. The cable loss was 36 dB per 100 m, giving a total cable loss of 72 dB. A 10-dB coupler was used on the transmitter to pick off the reference signal, so the total loss was 82 dB. Since the source transmitted 100 mW (20 dBm), the signal was −62 dBm at the receiver, sufficient for phase lock.

A second proposed method was to place a standard-gain horn (15 dB of gain) within the beam of the source on a small stand out of the way of the measurement and next to the receiver. If we assume that the source antenna had only 30% aperture efficiency, we compute gain from Eq. (1-10) ($\lambda = 0.15$ m):

$$G = \left(\frac{2\pi}{0.15}\right)^2 (0.3) = 526 \quad (27.2\,\text{dB})$$

The path loss is found from Eq. (1-9) for a range of 0.2 km:

$$32.45 + 20\log[2000(0.2)] - 27.2 - 15 = 42.3\,\text{dB}$$

The power delivered out of the horn is 20 dBm − 42.3 dB = −22.3 dBm. A 20-dB attenuator must be put on the horn to prevent saturation of the receiver (−30 dBm). Even with a short run, it is sometimes better to transmit the signal between two antennas instead of using cables.

1-7 DIRECTIVITY

Directivity is a measure of the concentration of radiation in the direction of the maximum:

$$\text{directivity} = \frac{\text{maximum radiation intensity}}{\text{average radiation intensity}} = \frac{U_{\max}}{U_0} \tag{1-15}$$

Directivity and gain differ only by the efficiency, but directivity is easily estimated from patterns. Gain—directivity times efficiency—must be measured.

The average radiation intensity can be found from a surface integral over the radiation sphere of the radiation intensity divided by 4π, the area of the sphere in steradians:

$$\text{average radiation intensity} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi = U_0 \tag{1-16}$$

This is the radiated power divided by the area of a unit sphere. The radiation intensity $U(\theta,\phi)$ separates into a sum of co- and cross-polarization components:

$$U_0 = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} [U_C(\theta,\phi) + U_\times(\theta,\phi)]\sin\theta\, d\theta\, d\phi \tag{1-17}$$

Both co- and cross-polarization directivities can be defined:

$$\text{directivity}_C = \frac{U_{C,\max}}{U_0} \qquad \text{directivity}_\times = \frac{U_{\times,\max}}{U_0} \tag{1-18}$$

Directivity can also be defined for an arbitrary direction $D(\theta, \phi)$ as radiation intensity divided by the average radiation intensity, but when the coordinate angles are not specified, we calculate directivity at $U_{\max}$.

1-8 DIRECTIVITY ESTIMATES

Because a ratio of radiation intensities is used to calculate directivity, the pattern may be referred to any convenient level. The most accurate estimate is based on measurements at equal angle increments over the entire radiation sphere. The average may be found from coarse measurements by using numerical integration, but the directivity measured is affected directly by whether the maximum is found. The directivity of antennas with well-behaved patterns can be estimated from one or two patterns. Either the integral over the pattern is approximated or the pattern is approximated with a function whose integral is found exactly.

1-8.1 Pencil Beam

By estimating the integral, Kraus [4] devised a method for pencil beam patterns with its peak at $\theta = 0^\circ$. Given the half-power beamwidths of the principal plane patterns, the integral is approximately the product of the beamwidths. This idea comes from circuit theory, where the integral of a time pulse is approximately the pulse width (3-dB points) times the pulse peak: $U_0 = \theta_1\theta_2/4\pi$, where θ_1 and θ_2 are the 3-dB beamwidths, in radians, of the principal plane patterns:

$$\text{directivity} = \frac{4\pi}{\theta_1\theta_2}(\text{rad}) = \frac{41{,}253}{\theta_1\theta_2}(\text{deg}) \tag{1-19}$$

Example Estimate the directivity of an antenna with E- and H-plane (principal plane) pattern beamwidths of 24° and 36°.

$$\text{Directivity} = \frac{41{,}253}{24(36)} = 47.75 \quad (16.8\,\text{dB})$$

An analytical function, $\cos^{2N}(\theta/2)$, approximates a broad pattern centered on $\theta = 0^\circ$ with a null at $\theta = 180^\circ$:

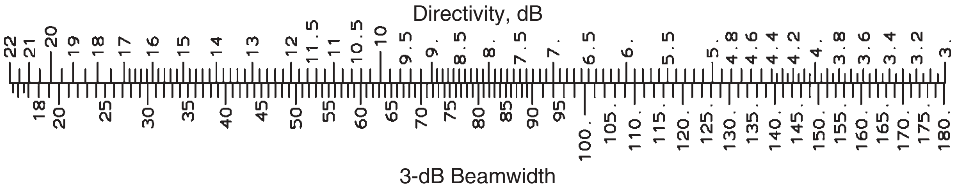
$$U(\theta) = \cos^{2N}(\theta/2) \quad \text{or} \quad E = \cos^N(\theta/2)$$

The directivity of this pattern can be computed exactly. The characteristics of the approximation are related to the beamwidth at a specified level, Lvl(dB):

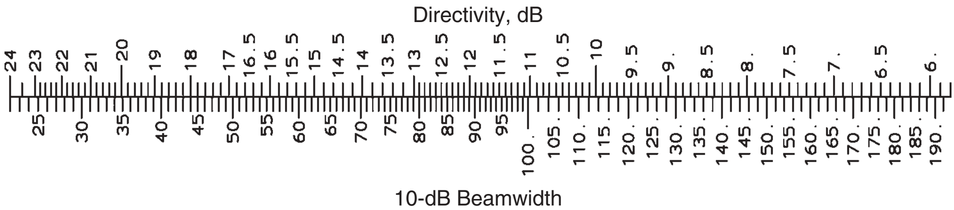
$$\text{beamwidth [Lvl(dB)]} = 4\cos^{-1}(10^{-\text{Lvl(dB)}/20N}) \tag{1-20a}$$

$$N = \frac{-\text{Lvl(dB)}}{20\log[\cos(\text{beamwidth}_{\text{Lvl(dB)}}/4)]} \tag{1-20b}$$

$$\text{directivity} = N + 1 \quad \text{(ratio)} \tag{1-20c}$$



SCALE 1-1 3-dB beamwidth and directivity relationship for $\cos^{2N}(\theta/2)$ pattern.



SCALE 1-2 10-dB beamwidth and directivity relationship for $\cos^{2N}(\theta/2)$ pattern.

Scales 1-1 and 1-2, which give the relationship between beamwidth and directivity using Eq. (1-20), are useful for quick conversion between the two properties. You can use the two scales to estimate the 10-dB beamwidth given the 3-dB beamwidth. For example, an antenna with a 90° 3-dB beamwidth has a directivity of about 7.3 dB. You read from the lower scale that an antenna with 7.3-dB directivity has a 159.5° 10-dB beamwidth. Another simple way to determine the beamwidths at different pattern levels is the square-root factor approximation:

$$\frac{\text{BW[Lvl 2(dB)]}}{\text{BW[Lvl 1(dB)]}} = \sqrt{\frac{\text{Lvl 2(dB)}}{\text{Lvl 1(dB)}}}$$

By this factor, $\text{beamwidth}_{10\,\text{dB}} = 1.826\ \text{beamwidth}_{3\,\text{dB}}$; an antenna with a 90° 3-dB beamwidth has a $(1.826)90^\circ = 164.3^\circ$ 10-dB beamwidth.

This pattern approximation requires equal principal plane beamwidths, but we use an elliptical approximation with unequal beamwidths:

$$U(\theta,\phi) = \cos^{2N_e}(\theta/2)\cos^2\phi + \cos^{2N_h}(\theta/2)\sin^2\phi \tag{1-21}$$

where N_e and N_h are found from the principal plane beamwidths. We combine the directivities calculated in the principal planes by the simple formula

$$\text{directivity (ratio)} = \frac{2\cdot \text{directivity}_e \cdot \text{directivity}_h}{\text{directivity}_e + \text{directivity}_h} \tag{1-22}$$

Example Estimate the directivity of an antenna with E- and H-plane pattern beamwidths of 98° and 140°.

From the scale we read a directivity of 6.6 dB in the E-plane and 4.37 dB in the H-plane. We convert these to ratios and apply Eq. (1-22):

$$\text{directivity (ratio)} = \frac{2(4.57)(2.74)}{4.57+2.74} = 3.426 \quad \text{or} \quad 10\log(3.426) = 5.35\,\text{dB}$$