
MODELING AND HIGH-PERFORMANCE CONTROL OF ELECTRIC MACHINES

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MODELING AND HIGH-PERFORMANCE CONTROL OF ELECTRIC MACHINES

JOHN CHIASSON



IEEE Press Series on Power Engineering
Mohamed E. El-Hawary, *Series Editor*



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To My Parents

and to

Marc Bodson

Pour son soutien et son aide jour après jour durant une période
très difficile de ma carrière

Amro El-Jaroudi
Mahmoud El Nokali

إِلَى اصْدِقَائِي الَّذِينَ وَقَّفُوا بِجَانِبِي فِي تِلْكَ الْأَوْقَاتِ الْعَصِيبَةِ مِنْ حَيَاتِي

James B. Lieber

על שתמיד האמנת בי ועל עזרתך הרבה בשעתי הקשה בקריירה האקדמית שלי

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Contents

I	DC Machines, Controls, and Magnetics	1
1	The Physics of the DC Motor	3
1.1	Magnetic Force	3
1.2	Single-Loop Motor	5
1.2.1	Torque Production	6
1.2.2	Commutation of the Single-Loop Motor	9
1.3	Faraday's Law	11
1.3.1	The Surface Element Vector $d\vec{S}$	12
1.3.2	Interpreting the Sign of ξ	13
1.3.3	Back Emf in a Linear DC Machine	14
1.3.4	Back Emf in the Single-Loop Motor	15
1.3.5	Self-Induced Emf in the Single-Loop Motor	18
1.4	Dynamic Equations of the DC Motor	20
1.5	Microscopic Viewpoint	23
1.5.1	Microscopic Viewpoint of the Single-Loop DC Motor	26
1.5.2	Drift Speed	28
1.6	Tachometer for a DC Machine* ¹	29
1.6.1	Tachometer for the Linear DC Machine	29
1.6.2	Tachometer for the Single-Loop DC Motor	30
1.7	The Multiloop DC Motor*	31
1.7.1	Increased Torque Production	32
1.7.2	Commutation of the Armature Current	32
1.7.3	Armature Reaction	38
1.7.4	Field Flux Linkage and the Air Gap Magnetic Field	40
1.7.5	Armature Flux Due to the External Magnetic Field	41
1.7.6	Equations of the PM DC Motor	43
1.7.7	Equations of the Separately Excited DC Motor	44
	Appendices	47
	Rotational Dynamics	47
	Gears	52
	Problems	57
2	Feedback Control	71
2.1	Model of a DC Motor Servo System	71

¹Sections marked with an asterisk (*) may be skipped without loss of continuity.

2.2	Speed Estimation	77
2.2.1	Backward Difference Estimation of Speed	77
2.2.2	Estimation of Speed Using an Observer	79
2.3	Trajectory Generation	82
2.4	Design of a State Feedback Tracking Controller	86
2.5	Nested Loop Control Structure*	90
2.6	Identification of the DC Motor Parameters*	96
2.6.1	Least-Squares Approximation	99
2.6.2	Error Index	103
2.6.3	Parametric Error Indices	103
2.7	Filtering of Noisy Signals*	108
2.7.1	Filter Representations	111
2.7.2	Causality	112
2.7.3	Frequency Response	112
2.7.4	Low-Pass Filters with Linear Phase	113
2.7.5	Distortion	114
2.7.6	Low-Pass Filtering of High-Frequency Noise	114
2.7.7	Butterworth Filters	116
2.7.8	Implementation of the Filter	118
2.7.9	Discretization of Differential Equations	120
2.7.10	Digital Filtering	122
2.7.11	State-Space Representation	124
2.7.12	Noncausal Filtering	126
	Appendix - Classical Feedback Control	129
	Tracking and Disturbance Rejection	129
	General Theory of Tracking and Disturbance Rejection	144
	Internal Model Principle	149
	Problems	151
3	Magnetic Fields and Materials	177
3.1	Introduction	177
3.2	The Magnetic Field \vec{B} and Gauss's Law	183
3.2.1	Conservation of Flux	186
3.3	Modeling Magnetic Materials	190
3.3.1	Magnetic Dipole Moments	192
3.3.2	The Magnetization \vec{M} and Ampère's Law	194
3.3.3	Relating \vec{B} to \vec{M}	201
3.4	The Magnetic Intensity Field Vector \vec{H}	205
3.4.1	The $B-H$ Curve	207
3.4.2	Computing \vec{B} and \vec{H} in Magnetic Circuits	211
3.4.3	\vec{B} is Normal to the Surface of Soft Magnetic Material	217
3.5	Permanent Magnets*	219
3.5.1	Hysteresis Loss	223
3.5.2	Common Magnetic Materials	225
	Problems	226

II AC Machine Theory 235

4 Rotating Magnetic Fields	245
4.1 Distributed Windings	245
4.2 Approximate Sinusoidally Distributed \vec{B} Field	249
4.2.1 Conservation of Flux and $1/r$ Dependence	254
4.2.2 Magnetic Field Distribution Due to the Stator Currents	256
4.3 Sinusoidally Wound Phases	257
4.3.1 Sinusoidally Wound Rotor Phase	257
4.3.2 Sinusoidally Wound Stator Phases	258
4.4 Sinusoidally Distributed Magnetic Fields	259
4.4.1 Sinusoidally Distributed Rotating Magnetic Field .	262
4.5 Magnetomotive Force (mmf)	264
4.6 Flux Linkage	266
4.7 Azimuthal Magnetic Field in the Air Gap*	269
4.7.1 Electric Field \vec{E}_{Sa}	275
4.7.2 The Magnetic and Electric Fields $\vec{B}_{Sa}, \vec{E}_{Sa}, \vec{B}_{Sb}, \vec{E}_{Sb}$	276
Problems	277
 5 The Physics of AC Machines	 293
5.1 Rotating Magnetic Field	293
5.2 The Physics of the Induction Machine	296
5.2.1 Induced Emfs in the Rotor Loops	297
5.2.2 Magnetic Forces and Torques on the Rotor	299
5.2.3 Slip Speed	302
5.3 The Physics of the Synchronous Machine	302
5.3.1 Two-Phase Synchronous Motor with a Sinusoidally Wound Rotor	303
5.3.2 Emfs and Energy Conversion	309
5.3.3 Synchronous Motor with a Salient Rotor	313
5.3.4 Armature and Field Windings	315
5.4 Microscopic Viewpoint of AC Machines*	315
5.4.1 Rotating Axial Electric Field Due to the Stator Cur- rents	316
5.4.2 Induction Machine in the Stationary Coordinate Sys- tem	317
5.4.3 Faraday's Law and the Integral of the Force per Unit Charge	323
5.4.4 Induction Machine in the Synchronous Coordinate System	326
5.4.5 Synchronous Machine	334
5.5 Steady-State Analysis of a Squirrel Cage Induction Motor*	334
5.5.1 Rotor Fluxes, Emfs, and Currents	336
5.5.2 Rotor Torque	337
5.5.3 Rotor Magnetic Field	342

5.5.4	Comparison with a Sinusoidally Wound Rotor . . .	344
	Problems	346
6	Mathematical Models of AC Machines	363
6.1	The Magnetic Field $\vec{B}_R(i_{Ra}, i_{Rb}, r, \theta - \theta_R)$	364
6.2	Leakage	366
6.3	Flux Linkages in AC Machines	370
6.3.1	Flux Linkages in the Stator Phases	370
6.3.2	Flux Linkages in the Rotor Phases	375
6.4	Torque Production in AC Machines	380
6.5	Mathematical Model of a Sinusoidally Wound Induction Machine	383
6.6	Total Leakage Factor	385
6.7	The Squirrel Cage Rotor	386
6.8	Induction Machine With Multiple Pole Pairs	387
6.9	Mathematical Model of a Wound Rotor Synchronous Machine	388
6.10	Mathematical Model of a PM Synchronous Machine	390
6.11	The Stator and Rotor Magnetic Fields of an Induction Machine Rotate Synchronously*	391
6.12	Torque, Energy, and Co-energy*	393
6.12.1	Magnetic Field Energy	393
6.12.2	Computing Torque From the Field Energy	396
6.12.3	Computing Torque From the Co-energy	397
	Problems	401
7	Symmetric Balanced Three-Phase AC Machines	413
7.1	Mathematical Model of a Three-Phase Induction Motor . .	413
7.2	Steady-State Analysis of the Induction Motor	434
7.2.1	Steady-State Currents and Voltages	434
7.2.2	Steady-State Equivalent Circuit Model	436
7.2.3	Rated Conditions	440
7.2.4	Steady-State Torque	441
7.2.5	Steady-State Power Transfer in the Induction Motor . .	444
7.3	Mathematical Model of a Three-Phase PM Synchronous Motor	449
7.4	Three-Phase, Sinusoidal, 60-Hz Voltages*	458
7.4.1	Why Three-Phase?	458
7.4.2	Why AC?	471
7.4.3	Why Sinusoidal Voltages?	472
7.4.4	Why 60 Hz?	474
	Problems	475
8	Induction Motor Control	493
8.1	Dynamic Equations of the Induction Motor	493

8.1.1	The Control Problem	494
8.2	Field-Oriented and Input-Output Linearization Control of an Induction Motor	497
8.2.1	Current-Command Field-Oriented Control	501
8.2.2	Experimental Results Using a Field-Oriented Con- troller	506
8.2.3	Field Weakening	509
8.2.4	Input-Output Linearization	511
8.2.5	Experimental Results Using an Input-Output Con- troller	514
8.3	Observers	519
8.3.1	Flux Observer	519
8.3.2	Speed Observer	521
8.3.3	Verghese–Sanders Flux Observer*	524
8.4	Optimal Field Weakening*	528
8.4.1	Torque Optimization Under Current Constraints	529
8.4.2	Torque Optimization Under Voltage Constraints	530
8.4.3	Torque Optimization Under Voltage and Current Con- straints	537
8.5	Identification of the Induction Motor Parameters*	548
8.5.1	Linear Overparameterized Model	549
8.5.2	Nonlinear Least-Squares Identification	552
8.5.3	Calculating the Parametric Error Indices	556
8.5.4	Mechanical Parameters	557
8.5.5	Simulation Results	558
8.5.6	Experimental Results	560
Appendix	565
	Elimination Theory and Resultants	565
Problems	568
9	PM Synchronous Motor Control	591
9.1	Field-Oriented Control	591
9.1.1	Design of the Reference Trajectory and Inputs	592
9.1.2	State Feedback Controller	595
9.1.3	Speed Observer	597
9.1.4	Experimental Results	598
9.1.5	Current Command Control	606
9.2	Optimal Field Weakening*	608
9.2.1	Formulation of the Torque Maximization Problem	608
9.2.2	Speed Ranges and Transition Speeds	609
9.2.3	Two Examples	615
9.3	Identification of the PM Synchronous Motor Parameters*	624
9.3.1	Experimental Results	627
9.4	PM Stepper Motors*	632
9.4.1	Open-Loop Operation of the Stepper Motor	635

9.4.2	Mathematical Model of a PM Stepper Motor	639
9.4.3	High-Performance Control of a PM Stepper Motor .	641
Appendices	641
Two-Phase Equivalent Parameters	641
Current Plots	643
Problems	645
10	Trapezoidal Back-Emf PM Synchronous Motors (BLDC)	651
10.1	Construction	651
10.2	Stator Magnetic Field \vec{B}_S	654
10.3	Stator Flux Linkage Produced by \vec{B}_S	657
10.4	Stator Flux Linkage Produced by \vec{B}_R	661
10.5	Emf in the Stator Windings Produced by \vec{B}_R	666
10.6	Torque	668
10.7	Mathematical Model	671
10.8	Operation and Control	673
10.8.1	The Terminology “Brushless DC Motor”	677
10.9	Microscopic Viewpoint of BLDC Machines*	679
10.9.1	Axial Electric Field \vec{E}_R	679
10.9.2	Emf Induced in the Stator Phases	681
Problems	684
Trigonometric Table and Identities		687
Trigonometric Table	687
Trigonometric Identities	688
References		691
Index		701

Preface

This book is intended to be an exposition of the modeling and control of electric machines, specifically, the direct current (DC) machine and the alternating current (AC) machines consisting of the induction motor, the permanent magnet (PM) synchronous motor, and the brushless DC motor. The particular emphasis here is on techniques used for high-performance applications, that is, applications that require both rapid and precise control of position, speed, and/or torque. Traditionally, DC motors were reserved for high-performance applications (positioning systems, rolling mills, traction drives, etc.) because of their relative ease of control compared to AC machines. However, with the advances in control methods, computing capability, and power electronics, AC motors continue to replace DC motors in high-performance applications. The intent here is to carefully derive the mathematical models of the AC machines and show how these mathematical models are used to design control algorithms that achieve high performance.

Electric machines are a particularly fascinating application of basic electricity and magnetism. The presentation here relies heavily on these basic concepts from Physics to develop the models of the motors. Specifically, Faraday's law ($\xi = -d\phi/dt$, where $\phi = \int_S \vec{B} \cdot d\vec{S}$), the magnetic force law ($\vec{F} = i\vec{L} \times \vec{B}$ or, $\vec{F} = q\vec{v} \times \vec{B}$), Gauss's law ($\oint \vec{B} \cdot d\vec{S} = 0$), Ampère's law ($\oint \vec{H} \cdot d\vec{L} = i_{\text{free}}$), the relationship between \vec{B} and \vec{H} , properties of magnetic materials, and so on are reviewed in detail and used extensively to derive the currently accepted nonlinear differential equation models of the various AC motors. The author made his best attempt to make the modeling assumptions as clear as possible and to consistently show that the magnetic and electric fields satisfy Maxwell's equations (as, of course, they must). The classical approach to teaching electric machinery is to present their equivalent circuit models and to analyze these circuit models ad nauseam. Further, the use of the basic Physics of electricity and magnetism to explain their operation is minimized if not omitted. However, the equivalent circuit is a result of assuming constant-speed operation of the machine and computing the sinusoidal steady-state solution of the nonlinear differential equation model of the machine. Here, the emphasis is on explaining how the machines work using fundamental concepts from electricity and magnetism, and on the derivation of their nonlinear differential equation models. The derivation of the corresponding equivalent circuit assuming steady-state conditions is then straightforward.

Electric machines also provide fascinating examples to illustrate con-

cepts from electromagnetic field theory (in contrast to electricity and magnetism). In particular, the way the electric and magnetic fields change as one goes between reference frames that are in relative motion are vividly illustrated using AC machines. For this reason, optional sections are included to show how the electric and magnetic fields change as one goes between a coordinate system attached to the stator to a coordinate system that rotates with the rotating magnetic field produced by the stator currents or a frame attached to the rotor. Also given in an optional section is the derivation of the axial electric and azimuthal magnetic fields in the air gap.

This is also a book on the control of electric machines based on their differential equation models. With the notable exception of the sinusoidal steady-state analysis of the induction motor in Chapter 7, very little attention is given to the classical equivalent circuits as these models are valid only in steady state. Rather, the differential equation models are used as the basis to develop the notions of field-oriented control, input-output linearization, flux observers, least-squares identification methods, state feedback trajectory tracking, and so on. This is a natural result of the emphasis here on high-performance control methods (e.g., field-oriented control) as opposed to classical methods (e.g., V/f , slip control, etc.).

There are of course many good books in the area of electric machines and their control. The author owes a debt of gratitude to Professor W. Leonhard for his book [1] (see the most recent edition [2]), from which he was educated in the modeling and control of electric drives. The present book is narrower in focus with an emphasis on the modeling and operation of electric machines based on elementary classical physics and an emphasis on high-performance control methods using a state-space formulation. The books by P. C. Krause [3] and P. C. Krause et al. [4] are complete in their derivation of the mathematical models of electric machines while C. B. Gray [5] presents electromagnetic theory in the context of electric machines. A comprehensive treatment using SIMULINK to simulate electric machinery is given in C-M. Ong's book [6]. The graduate level books by D. W. Novotny and T. A. Lipo [7], P. Vas [8], J. M. D. Murphy and F. G. Turnbull [9], I. Boldea and S. A. Nasar [10], B. Adkins and R. G. Harley [11], A. M. Trzynadlowski [12], M. P. Kazmierkowski and H. Tunia [13], B. K. Bose [14], and R. Krishnan [15] all cover the modeling and control of electric machines while the books by R. Ortega et al. [16], D. M. Dawson et al. [17], and F. Khorrarni et al. [18] emphasize advanced control methods.

The introductory-level books by S. J. Chapman [19], H. Woodson and J. Melcher [20], L. W. Matsch and J. D. Morgan [21], G. McPherson and R. D. Laramore [22], D. V. Richardson [23], P. C. Krause and O. Wasynczuk [24], N. Mohan [25], G. R. Slemon and A. Straughn [26], J. Sokira and W. Jaffe [27], G. J. Thaler and M. L. Wilcox [28], V. Deltoro [29], M. El-Hawary [30], P. C. Sen [31], and G. R. Slemon [32] are among the many books on electric machines from which this author has benefited.

The beautifully written textbooks *PSSC Physics* by the Physical Science Curriculum Study [33], *Physics* by D. Halliday and R. Resnick [34], *Principles of Electrodynamics* by M. Schwartz [35], and *Electromagnetic Fields* by R. K. Wangness [36] are used as references for the theory of electricity and magnetism.

This book borrows from these above works and hopefully makes its own contribution to the literature on electric machines.

Part I of the book consists of the first three chapters. Chapters 1 and 2 present a detailed review of the basic concepts of electricity and magnetism in the context of DC machines and an introduction to control methods, respectively, which will be used extensively in the remaining chapters. The third chapter on magnetic fields and magnetic materials is intended to be a detailed introduction to the subject. For example, most textbooks assume that the reader understands Ampère's law in the form $\oint \vec{H} \cdot d\vec{\ell} = i_{\text{free}}$ and that $\vec{B} = \mu\vec{H}$ in (soft) magnetic materials, yet it is the experience of the author that students do not have a fundamental understanding of these concepts.

These first three chapters are elementary in nature and were written to be accessible to undergraduates. The reason for this is that often control engineers do not have any background in electric machinery while power/electric-machine engineers often do not have any background in basic state-space concepts of control theory. Consequently, it is hoped that these chapters can bring the reader "up to speed" in these areas.

Chapter 1 reviews the basic ideas of electricity and magnetism that are needed to model electric machines. In particular, the notions of magnetic fields, magnetic force and Faraday's law are reviewed by using them to derive the standard model of a DC motor.

Chapter 2 provides an elementary introduction to the control techniques required for the high-performance control of electric machines. This includes an elementary presentation of state feedback control, observers, and identification theory as applied to DC machines to prepare the reader for the subsequent chapters.

Chapter 3 goes into the modeling of magnetic materials in terms of their use in electric machines. The fundamental result of this chapter is the modification of Ampère's law $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 i$ so that it is valid in the presence of magnetic material. This introduces the magnetic intensity field \vec{H} and its relationship to magnetic induction field \vec{B} via the magnetization vector \vec{M} to obtain the more general version of Ampère's law $\oint \vec{H} \cdot d\vec{\ell} = i_{\text{free}}$. All of this requires a significant discussion of the modeling of magnetic materials. The approximation $\vec{H} = 0$ in magnetic materials is discussed, and then it is shown how this approximation along with Ampère's law can be used to find the radial component of \vec{B} in the air gap of electric machines. Also presented is Gauss's law for \vec{B} ; this leads to the notion of conservation of flux, as well as the fact that \vec{B} is normal to the surface of

soft magnetic materials. This chapter should be read, but the reader should not get “bogged down” in the chapter. Rather, the main results should be remembered.

Part II consists of Chapters 4 through 10 and presents the modeling and control of AC machines.

Chapter 4 uses the results of Chapters 1 and 3 to explain how a radially directed rotating magnetic field can be established in the air gap of AC machines. In particular, the notions of distributed windings and of sinusoidally wound turns (phase windings) are explained. Ampère’s law is then used to show that a sinusoidal (spatially) distributed radial magnetic field is established in the air gap by the currents in the phase windings. The concept of flux linkage in distributed windings is explained, and the chapter ends with an optional section on the azimuthal magnetic field in the air gap.

Chapter 5 explains the fundamental Physics behind the working of induction and synchronous machines. Specifically, this chapter uses a simplified model of the induction motor and shows how voltages and currents are induced in the rotor loops by the rotating magnetic field established by the stator currents. Then it is shown how torque is produced on these induced currents by the same stator rotating magnetic field that induced them introducing the idea of slip. Similarly, the synchronous machine is analyzed to show how the rotating radial magnetic field established by the stator currents produces torque on a rotor carrying constant current.

An optional section on the microscopic point of view of the Physics of the induction motor is also presented. This includes a discussion of how the electric and magnetic fields change as one goes between coordinate systems that are rotating with respect to each other and how one reinterprets the Physics of the machine’s operation. The chapter ends with another optional section of the steady-state behavior of an induction machine with a squirrel cage rotor.

Chapter 6 derives the systems of differential equations that mathematically model the two-phase induction and synchronous machines. The concept of leakage is presented and accounted for in the derived models. These models are the accepted models used throughout the literature and form the basis for high-performance control of these machines. In an optional section it is shown that the stator and rotor *magnetic fields* of an induction motor rotate synchronously together as they do in a synchronous machine. The chapter ends with another optional section on the concepts of field energy and co-energy, and how the expression for the torque of an electric machine can be derived using these notions.

Chapter 7 presents the derivation of the models of three-phase AC machines and their two-phase equivalent models. These derivations readily follow from the results of Chapter 6. The classical steady-state analysis of the induction motor is also presented including its equivalent circuit. The chapter ends with a discussion of why the standard power system is an AC

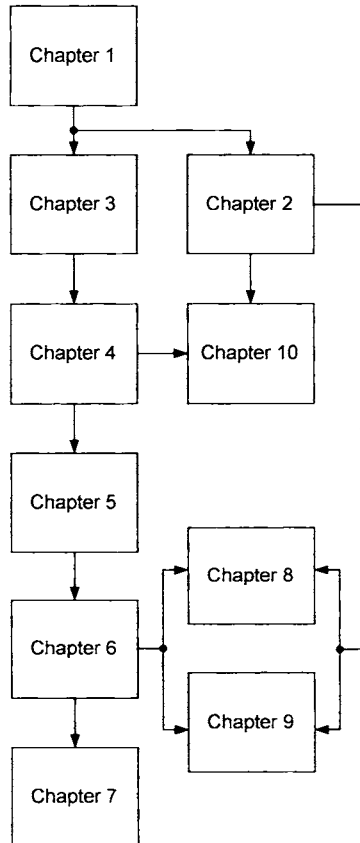
sinusoidal three-phase 60-Hz (or 50-Hz) system.

Chapter 8 covers the control of induction motors presenting both field-oriented control and input-output linearization control. Flux observers, field weakening, and speed observers are also presented along with experimental results. The chapter ends with an optional section on how to identify the induction motor parameters using a nonlinear least-squares technique.

Chapter 9 covers the control of synchronous motors describing field-oriented control, field weakening, speed observers and identification methods. The operation and modeling of permanent magnet stepping motors is also covered.

Chapter 10 covers the modeling and control of PM synchronous motors with trapezoidal back emf, which are also known as brushless DC (BLDC) motors.

The logical dependence of the chapters is shown in the block diagram below assuming that the optional sections are *not* covered.



Logical dependence of the chapters.

Finally, the author's intent for this book was for the reader to understand how electric machines are modeled and to understand the basic techniques in their control. The references at the end of the book are only those directly referenced in the book and are *not* representative of (nor give proper recognition to) the many important contributions made by researchers throughout the world. The reader is referred to Professor Leonhard's book [2] for a much more extensive reference list.

Comments on the Use of the Book

In using this book in a one-semester graduate-level course, the following material was usually covered:

Chapter 1, Sections 1.1–1.7
 Chapter 2, Sections 2.1–2.4
 Chapter 3, Sections 3.1–3.4
 Chapter 4, Sections 4.1–4.5
 Chapter 5, Sections 5.1–5.3
 Chapter 6, Sections 6.1–6.10
 Chapter 7, Sections 7.1–7.3
 Chapter 8, Sections 8.1–8.3
 Chapter 9, Section 9.1

Sections marked with an asterisk (*) may be omitted without loss of continuity. Some of these optional sections assume familiarity with Maxwell's equations in *differential* form.

Acknowledgments

There are many people that I would to acknowledge for their help in my work in the control of electric machines.

I did my Ph.D. thesis in Algebraic Systems Theory which is quite far from the area of Electric Machines. Nevertheless, I am most grateful to my advisor, Professor E. Bruce Lee, who has always shown me his kindness over the years.

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John Wiley & Sons maintains an ftp site at ftp://ftp.wiley.com/public/sci_tech_med/high_performance_control for downloading an errata sheet for the book. Instructors, upon obtaining password privileges, will also be able to download the simulation files that go with this textbook. A solutions manual is available to instructors by contacting their local Wiley representative.

Any comments, criticisms, and corrections are most welcome and may be sent to the author at chiasson@ieee.org.

John Chiasson

MODELING AND HIGH-PERFORMANCE CONTROL OF ELECTRIC MACHINES

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Part I

DC Machines, Controls, and Magnetics

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1

The Physics of the DC Motor

The principles of operation of a direct current (DC) motor are presented based on fundamental concepts from electricity and magnetism contained in any basic physics course. The DC motor is used as a concrete example for reviewing the concepts of magnetic fields, magnetic force, Faraday's law, and induced electromotive forces (emf) that will be used throughout the remainder of the book for the modeling of electric machines. All of the Physics concepts referred to in this chapter are contained in the book *Physics* by Halliday and Resnick [34].

1.1 Magnetic Force

Motors work on the basic principle that magnetic fields produce forces on wires carrying a current. In fact, this experimental phenomenon is what is used to define the magnetic field. If one places a current carrying wire between the poles of a magnet as in Figure 1.1, a force is exerted on the wire. Experimentally, the magnitude of this force is found to be proportional to both the amount of current in the wire and to the length of the wire that is between the poles of the magnet. That is, F_{magnetic} is proportional to ℓi . The direction of the magnetic field \vec{B} at any point is defined to be the direction that a small compass needle would point at that location. This direction is indicated by arrows in between the north and south poles in Figure 1.1.

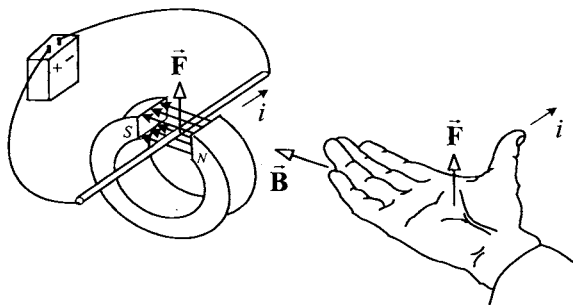


FIGURE 1.1. Magnetic force law. From *PSSC Physics*, 7th edition, by Haber-Schaim, Dodge, Gardner, and Shore, published by Kendall/Hunt, 1991.

With the direction of \vec{B} perpendicular to the wire, the strength (magnitude) of the *magnetic induction field* \vec{B} is defined to be

$$B = |\vec{B}| \triangleq \frac{F_{\text{magnetic}}}{\ell i}$$

where F_{magnetic} is the magnetic force, i is the current, and ℓ is the length of wire perpendicular to the magnetic field carrying the current. That is, B is the proportionality constant so that $F_{\text{magnetic}} = i\ell B$. As illustrated in Figure 1.1, the direction of the force can be determined using the right-hand rule. Specifically, using your right hand, point your fingers in the direction of the magnetic field and point your thumb in the direction of the current. Then the direction of the force is out of your palm.

Further experiments show that if the wire is parallel to the \vec{B} field rather than perpendicular as in Figure 1.1, then no force is exerted on the wire. If the wire is at some angle θ with respect to \vec{B} as in Figure 1.2, then the force is proportional to the *component* of \vec{B} perpendicular to the wire; that is, it is proportional to $B_{\perp} = B \sin(\theta)$. This is summarized in the *magnetic force law*: Let $\vec{\ell}$ denote a vector whose magnitude is the length ℓ of the wire in the magnetic field and whose direction is defined as the positive direction of current in the bar; then the magnetic force on the bar of length ℓ carrying the current i is given by

$$\vec{F}_{\text{magnetic}} = i\vec{\ell} \times \vec{B}$$

or, in scalar terms, $F_{\text{magnetic}} = i\ell B \sin(\theta) = i\ell B_{\perp}$. Again, $B_{\perp} \triangleq B \sin(\theta)$ is the component of \vec{B} perpendicular to the wire.¹

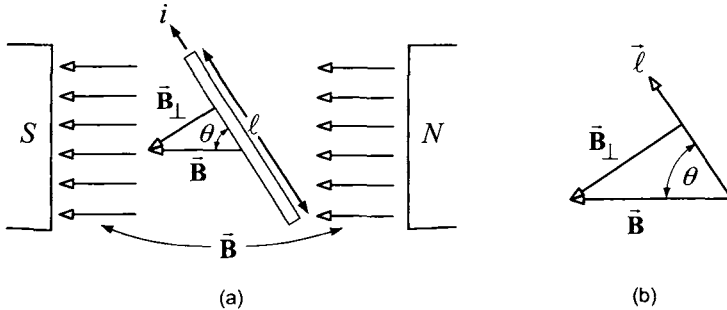


FIGURE 1.2. Only the component B_{\perp} of the magnetic field which is perpendicular to the wire produces a force on the current.

¹Motors are designed so that the conductors are perpendicular to the external magnetic field.

Example A Linear DC Machine [19]

Consider the simple linear DC machine in Figure 1.3 where a sliding bar rests on a simple circuit consisting of two rails. An external magnetic field is going through the loop of the circuit up out of the page indicated by the \otimes in the plane of the loop. Closing the switch results in a current flowing around the circuit and the external magnetic field produces a force on the bar which is free to move. The force on the bar is now computed.

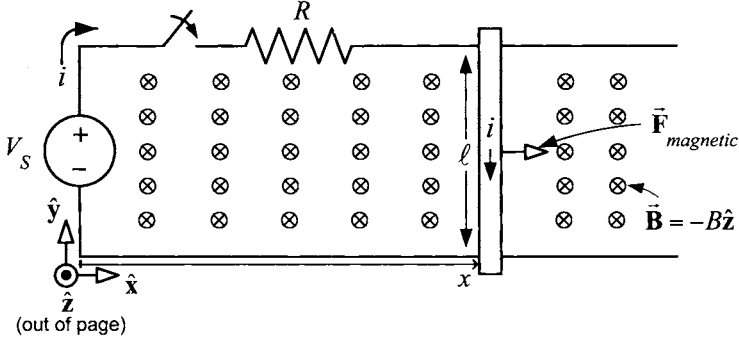


FIGURE 1.3. A linear DC motor.

The magnetic field is constant and points into the page (indicated by \otimes) so that written in vector notation, $\vec{\mathbf{B}} = -B\hat{\mathbf{z}}$ with $B > 0$. By the right hand rule, the magnetic force on the sliding bar points to the right. Explicitly, with $\vec{\ell} = -\ell\hat{\mathbf{y}}$, the force is given by

$$\begin{aligned}\vec{\mathbf{F}}_{\text{magnetic}} &= i\vec{\ell} \times \vec{\mathbf{B}} = i(-\ell\hat{\mathbf{y}}) \times (-B\hat{\mathbf{z}}) \\ &= i\ell B\hat{\mathbf{x}}.\end{aligned}$$

To find the equations of motion for the bar, let f be the coefficient of viscous (sliding) friction of the bar so that the friction force is given by $F_f = -f dx/dt$. Then, with m_ℓ denoting the mass of the bar, Newton's law gives

$$i\ell B - f dx/dt = m_\ell d^2x/dt^2.$$

Just after closing the switch at $t = 0$, but before the bar starts to move, the current is $i(0^+) = V_S(0^+)/R$. However, it turns out that as the bar moves the current does *not* stay at this value, but instead decreases due to electromagnetic induction. This will be explained later.

1.2 Single-Loop Motor

As a first step to modeling a DC motor, a simplistic single-loop motor is considered. It is first shown how torque is produced and then how the

current in the single loop can be reversed (commutated) every half turn to keep the torque constant.

1.2.1 Torque Production

Consider the magnetic system in Figure 1.4, where a cylindrical core is cut out of a block of a permanent magnet and replaced with a soft iron core. The term “soft” iron refers to the fact that material is easily magnetized (a permanent magnet is referred to as “hard” iron).

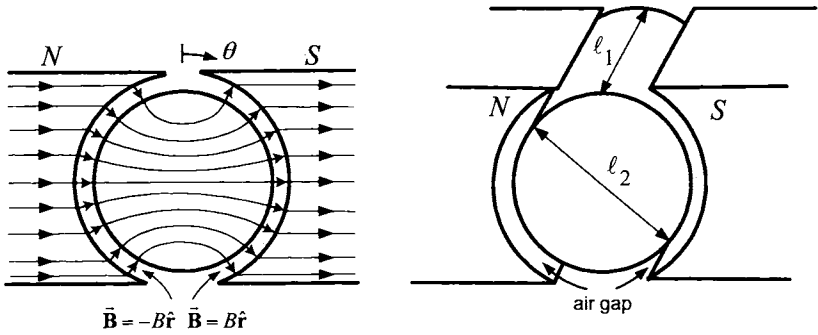


FIGURE 1.4. Soft iron cylindrical core placed inside a hollowed out permanent magnet to produce a radial magnetic field in the air gap.

An important property of soft magnetic materials is that the magnetic field at the surface of such materials tends to be normal (perpendicular) to the surface. Consequently, the cylindrical shape of the surfaces of the soft iron core and the stator permanent magnet has the effect of making the field in the air gap *radially* directed; furthermore, it is reasonably constant (uniform) in magnitude. A mathematical description of the magnetic field in the air gap due to the permanent magnet is simply

$$\vec{B} = \begin{cases} +B\hat{r} & \text{for } 0 < \theta < \pi \\ -B\hat{r} & \text{for } \pi < \theta < 2\pi \end{cases}$$

where $B > 0$ is the magnitude or strength of the magnetic field and θ is an arbitrary location in the air gap.²

Figure 1.5 shows a rotor loop wound around the iron core of Figure 1.4. The length of the rotor is ℓ_1 and its diameter is ℓ_2 . The torque on this rotor loop is now calculated by considering the magnetic forces on sides a and a' of the loop. On the other two sides of the loop, that is, the front and

²Actually it will be shown in a later chapter that the magnetic field must be of the form $\vec{B} = \pm B(r_0/r)\hat{r}$ in the air gap, that is, it varies as $1/r$ in the air gap. However, as the air gap is small, the \vec{B} field is essentially constant across the air gap.