A History of Probability and Statistics and Their Applications before 1750

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Preface

Until recently a book on the history of statistics in the 19th century was badly needed. When I retired six years ago, I decided to write such a book, feeling that I had a good background in my statistical education in the 1930s, when the curriculum in statistics was influenced mainly by the writings of Laplace, Gauss, and Karl Pearson. Studying the original works of these authors I found no difficulty in understanding Gauss and Pearson, but I soon encountered difficulties with Laplace. The reason is of course that Gauss and Pearson are truly 19th century figures, whereas Laplace has his roots in the 18th century.

I then turned to the classical authors and worked my way back to Cardano through de Moivre, Montmort, Nicholas and James Bernoulli, Huygens, Fermat, and Pascal. Comparing my notes with Todhunter's *History*, I found to my surprise that his exposition of the topics in probability theory that I found most important was incomplete, and I therefore decided to write my own account.

The present book, covering the period before 1750, is an introduction to the one I had in mind. It describes the contemporaneous development and interaction of three topics: probability theory and games of chance; statistics in astronomy and demography; and life insurance mathematics.

Besides the story of the life and works of the great natural philosophers who contributed to the development of probability theory and statistics, I have told the story of important problems and methods, in this way exhibiting the gradual advance of solving these problems. I hope to have achieved a better balance than had been achieved before in evaluating the contributions of the various authors; in particular, I have stressed the importance of the works of John Graunt, Montmort, and Nicholas Bernoulli.

The contents of the book depend heavily on research carried out by many authors during the past 40 years. I have drawn freely on these sources and acknowledged my debt in the references. The manuscript was written during the years 1985–1987, so works published in 1986 and 1987 are not fully integrated in the text. Some important books and papers from 1988 are briefly mentioned.

With hesitation, I have also included some background material on the history of mathematics and the natural and social sciences because I have always felt that my students needed such knowledge. I realize of course that my qualifications for doing so are rather poor since I am no historian of science. These sections and also the biographies are based on secondary sources.

The plan of the book is described in Section 1.2.

I am grateful to Richard Gill for advice on my English in Chapters 2 and 3, to Steffen L. Lauritzen for translating some Russian papers, and to Olaf Schmidt for a discussion of Chapter 10. In particular, I want to thank Søren Johansen for discussions on the problem of the duration of play.

I am grateful to two anonymous reviewers from the publisher for valuable comments on the manuscript and for advice resulting in considerable reduction of the background material. I thank the copy editor for improving my English and transforming it into American.

I thank the Institute of Mathematical Statistics, University of Copenhagen, for placing working facilities at my disposal.

I thank the Almqvist & Wiksell Periodical Company for permission to use material in my paper published in *Scandinavian Actuarial Journal*, 1987; the International Statistical Institute for permission to use material from three papers of mine published in *International Statistical Review*, 1983, 1984, and 1986; and Springer-Verlag for permission to use material from my paper published in *Archive for History of Exact Sciences*, 1988.

I am grateful to the Department of Statistics, Harvard University, for permission to quote from Bing Sung's *Translations from James Bernoulli*, Technical Report No. 2, 1966, and to Thomas Drucker for permission to quote from his (unpublished) translation of Nicholas Bernoulli's *De Usu Artis Conjectandi in Jure.*

My first book on statistics, written fifty years ago, was dedicated to G. K., so is this one.

ANDERS HALD

September 1988

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CHAPTER 1

The Book and Its Relation to Other Works

1.1 PRINCIPLES OF EXPOSITION

This book contains an exposition of the history of probability theory and statistics and their applications before 1750 together with some background material. A history should of course give an account of the time and place of important events and their interpretations. However, opinions differ greatly on where to put the main emphasis of interpretation.

We have attempted to cover three aspects of the history: problems, methods, and persons. We describe probabilistic and statistical problems and their social and scientific background; we discuss the mathematical methods of solution and the statistical methods of analysis; and we include the background and general scientific contributions of the persons involved, not only their contributions to probability and statistics.

Since history consists of facts and their interpretation, history continually changes because new facts are found in letters, archives, and books, and new interpretations are offered in the light of deeper understanding based, in this case, on the latest developments in probability theory, statistics, and the history of science.

In the 17th and 18th centuries many problems were formulated as challenge problems, and answers were given without proofs. Some books on probability were written for the educated public and therefore contained statements without proofs. In such cases we have tried to follow the author's hints and construct a proof which we believe represents the author's intentions.

The material has been ordered more according to problems and methods than according to persons in an attempt to treat the achievements of the various authors as contributions to a general framework.

A leading principle of the exposition of probability theory and life insurance mathematics has been to rewrite the classics in uniform modern terminology and notation. It is clear that this principle may be criticized for distorting the facts. Many authors prefer to recount the old proofs with the original notation to convey the flavor of the past to the reader. There are two essential steps in modernization that we have made here. The first is to use a single letter, p, say, to denote a probability instead of the ratio of the number of favorable cases to the total number of cases, $a/(a+b)$, say, where a and b are positive integers. This change of notation conceals the fact that nearly all the probabilities discussed were constrained to rational fractions. The advantage of this notation was noted by de Moivre (1738, p. 29) who writes, "Before I make an end of this Introduction, it will not be improper to shew how some operations may often be contracted by barely introducing one single Letter, instead of two or three, to denote the Probability of the happening of one Event" and, further (on p. 30), that "innumerable cases of the same nature, belonging to any number of Events, may be solved without any manner of trouble to the imagination, by the mere force of a proper Notation." However, de Moivre did not rewrite the *Doctrine of Chances* with the new notation; he used it only in his *Annuities upon Lives* (1725 and later editions). We have followed the advice of de Moivre and rewritten the proofs in the new notation, feeling confident that the reader will keep in mind that most probabilities were defined as proper rational fractions, a fact which is nearly always obvious from the context.

The second great simplification of the proofs is obtained by the introduction of subscripts. In analyzing some complicated games of chance, for example, Waldegrave's problem, Nicholas Bernoulli and de Moivre had to use the whole alphabet divided into several sections to denote probabilities and expectations of the players corresponding to various states of the game. De Moivre achieved some simplification by using superscripts in a few cases. In many problems they gave the solution for two, three, and four players only and concluded that "the continuation of this rule is manifest," in this way avoiding a general proof which would have been rather unintelligible. Using modern notation with subscripts, it is easy to rewrite such proofs in much shorter form without invalidating the idea of the proof; in fact, we believe that our readers will get a clear idea of the proof because they are accustomed to this symbolism, just as readers in the past understood the original form of the proof because they were educated in that notational tradition.

Comparison of proofs and results in a uniform notation makes evaluating the contributions of various authors easier and minimizes the danger of attributing too much to an individual author. Furthermore, the importance of the results to the following period and to today becomes evident.

The same principle of exposition cannot be used for statistics, because statistics before 1750 was nonmathematical. We shall therefore illustrate the development of statistical methods by typical examples, giving both the original data and their analysis at the time and adding some comments from a modern point of view.

The book is written in textbook style, since our main purpose is to give an account of the most important results in the classical literature. Like most histories of mathematics and science, our exposition concentrates on results which have proved to be of lasting importance.

The persons who laid the foundation of probability theory and statistics were natural philosophers having a broader background and outlook than scientists today. The word "scientist" was coined about the middle of the 19th century, reflecting an ongoing specialization and professionalization. Nevertheless, we shall often use the words "mathematician" and "scientist" to stress certain characteristics of the persons involved.

To convey the flavor of classical works, we shall present quotations of programs from the prefaces of books, the formulation of important problems, and some heated disputes of priority.

We shall point out priorities, but the reader should be aware of the uncertainty involved by taking note of Stigler's Law of Eponymy, (Stigler, 1980), which in its simplest form states that, "No scientific discovery is named after its original inventor."

The driving force behind the development of probability theory and statistics was pressure from society to obtain solutions to important problems for practical use, as well as competition among mathematicians. When a problem is first formulated and its solution indicated, perhaps only by a numerical example, the problem begins a life of its own within the mathematical community; this leads to improved proofs and generalizations of the problem, and we shall see many examples of this phenomenon.

Finally, it should be noted that any history is necessarily subjective, since the weight and interpretation of the events selected depend on the author's interests.

For the serious student of the history of probability theory and statistics, we can only recommend that he or she follow the advice given by de Moivre (1738, p. 235), discussing the works of James and Nicholas Bernoulli on the binomial distribution: "Now the Method which they have followed has been briefly described in my *Miscellanea Analytica*, which the Reader may consult if he pleases, unless they rather chuse, which perhaps would be the best, to consult what they themselves have writ upon that Subject."

1.2 PLAN OF THE BOOK

A fuller title of the book would be *A history of probability theory and statistics and their applications to games of chance, astronomy, demography, and life insurance before 1750, with some comments on later developments.* The topics treated may be grouped into five categories:

Background in mathematics, natural philosophy, and social conditions
Biographies
Probability theory and games of chance
Statistics in astronomy and demography
Life insurance mathematics

Probability theory before 1750 was inspired mainly by games of chance. Dicing, card games, and lotteries, public and private, were important social and economic activities then as today. It is no wonder that intellectual curiosity and economic interests led to mathematical investigations of these activities at a time when the mathematization of science was going on. We shall distinguish three periods.

The period of the foundation of probability theory from 1654 to 1665 begins with the correspondence of Pascal and Fermat on the problem of points, continues with Huygens' treatise on *Reckoning at Games of Chance*, and ends with Pascal's treatise on the *Arithmetical Triangle* and its applications. The correspondence was not published until much later. In his treatise, Pascal solves the problem of points by recursion and finds a division rule, depending on the tail probability of the symmetric binomial. In their correspondence, he and Fermat had solved the same problem also by combinatorial methods. Huygens uses recursion to solve the problem numerically. He also considers an example with a possibly infinite number of games, which he solves by means of two linear equations between the conditional expectations of the two players. All three of them solved the problem of the Gambler's Ruin without publishing their method of solution.

After a period of stagnation of nearly 50 years, there followed a decade with astounding activity and progress from 1708 to 1718 in which the elementary and fragmentary results of Pascal, Fermat, and Huygens were developed into a coherent theory of probability. The period begins with Montmort's *Essay d'Analyse sur les Jeux de Hazard*, continues with de Moivre's *De Mensura Sortis*, Nicholas Bernoulli's letters to Montmort, James Bernoulli's *Ars Conjectandi*, Nicolaas Struyck's *Reckoning of Chances in Games*, and ends with de Moivre's *Doctrine of Chances*. Hence, by 1718 four comprehensive textbooks were available. We shall mention the most important results obtained. They discussed elementary rules of probability calculus, conditional probabilities and expectations, combinatorics, algorithms and recursion formulae, the method of inclusion and exclusion, and examples of using infinite series and limiting processes. They derived the binomial and negative binomial distributions, the hypergeometric distribution, the multivariate version of these distributions, the occupancy distribution, the distribution of the sum of any number of uniformly distributed variables, the Poisson approximation to the binomial, the law of large numbers for the binomial, and an approximation to the tail of the binomial. They solved the problem of points for a game of bowls and for the game of tennis, Waldegrave's problem, the problem of coincidences, and the problem of duration of play, and found the minimax solution for the strategic game Her.

The third period, from 1718 to 1738, was a period of consolidation and steady progress in which de Moivre derived the normal approximation to the binomial distribution, developed a theory of recurring series, improved his solution of the problem of the duration of play, and wrote the second edition of the *Doctrine of Chances*, which became the most important textbook before the publication of Laplace's *Théorie Analytique des Probabilités* in 1812.

We shall discuss these books in detail. We have, however, singled out the most important problems for separate treatment to show how they were solved by joint effort, often in competition among several authors.

Many problems were taken up by the following generation of mathematicians and given solutions that have survived until today. We shall comment on these later developments, usually ending with Laplace's solutions.

The successful development of probability theory did not immediately lead to a theory of statistics. A history of statistical methods before 1750 must therefore build on typical examples of data analysis; we have concentrated here on examples from astronomy and demography.

Astronomers had been aware of the importance of both systematic and random errors since antiquity and tried to minimize the influence of such errors in their planning of observations and data analysis. We shall discuss some data by Tycho Brahe from the end of the 16th century as an example. The mathematization of science in the beginning of the 17th century naturally led many scientists to determine not only the mathematical form of natural laws but also the values of the parameters by fitting equations to data. They inserted the best sets of observations in the equations, as many as the number of parameters, solved for the parameters, calculated the expected values, and studied the deviations between observed and calculated values. Prominent examples are Kepler's three laws on planetary motion derived from his physical theories and data collected by Copernicus and Tycho Brahe. Kepler's data were used by Newton to check his axiomatic theory. Galileo used several sets of observations on the new star of 1572 to compare two hypotheses on the position of the star. We shall also see how Newton used an interpolation polynomial to find the tangent to the orbit of a comet.

A paragon for descriptive statistical analysis of demographic data was provided by Graunt's *Natural and Political Observations made upon the Bills of Mortality* in 1662. Graunt's critical appraisal of the rather unreliable data, his study of mortality by cause of death, his estimation of the same quantity by several different methods, his demonstration of the stability of statistical ratios, and his life table set up new standards for statistical reasoning. Graunt's work led to three different types of investigations: political arithmetic; testing the stability of statistical ratios; and calculation of expectations of life and survivorship probabilities.

Petty also employed Graunt's method of analysis, although less critical, to economic data and coined the term "political arithmetic" for analyses of data of political importance. Similar methods were used by natural philosophers and theologians to analyze masses of data on human and animal populations. The many regular patterns observed were taken as proof of the existence of a supreme being and His "original design." We shall remark only slightly on this line of thought.

It is surprising that probabilists at the time recognized the importance of Graunt's work and without hesitation used their theory on games of chance to describe demographic phenomena. They wrote about the chance of a male birth and the chance of dying at a certain age.

Graunt gave a detailed description and analysis of the yearly variation of the sex ratio at birth in London and Romsey and suggested that similar investigations should be carried out in other places. Arbuthnott used some of Graunt's data extended to his own time to give a statistical proof, based on the symmetric binomial, for the existence of divine providence, a proof that was further strengthened by 'sGravesande. Nicholas Bernoulli compared the observed distribution of the yearly number of male births with a skew binomial distribution, the parameter being estimated from the data, and discussed the probability of the observed number of outliers. His investigation is the first attempt to fit a binomial to data and to test the goodness of fit. Some years later, Daniel Bernoulli used the normal approximation to the binomial in his analysis of deviations between observed and expected values of the number of male births to decide between two hypothetical values of the sex ratio.

Huygens used Graunt's life table to calculate the median and the average remaining lifetime for a person of any given age. He also showed how to calculate survivorship probabilities and joint-life expectations. His results were, however, not published, but similar results were published without proof by James Bernoulli and later proved by Nicholas Bernoulli.

The usefulness of probability theory was convincingly demonstrated by application to problems of life insurance. In the 16th and 17th centuries, states and cities sold life annuities to their citizens to raise money for public purposes. The yearly benefit of an annuity was fixed as a percentage of the capital invested, often as twice the prevailing rate of interest and independent of the nominee's age. In a report from 1671, de Witt showed how to calculate the value of an annuity by means of a piecewise linear life table combined with the age of the nominee and the rate of interest. De Witt's life table was hypothetical, although he referred to some investigations of the mortality of annuitants. In 1693 Halley constructed a life table from observations of the yearly number of deaths in Breslau, calculated the first table of values of annuities as a function of the nominee's age, and explained how to calculate joint-life annuities.

After these ingenious beginnings one would have expected rapid development of both mathematical and practical results in view of the fact that many economic contracts in everyday life depended on life contingencies, but nothing happened for about 30 years. The breakthrough came in 1725 with de Moivre's *Annuities upon Lives*, greatly simplifying both the mathematics and the calculations involved; however, as shown by Simpson, de Moivre went too far in his simplifications. Simpson therefore constructed his own life table for the population of London, and by recursion he calculated tables of values of single- and joint-life annuities for various rates of interest. In the strong competition between de Moivre and Simpson, a comprehensive theory of life annuities was created, and the necessary tables for practical applications provided.

In some chapters in this book we have supplemented the text with problems for the reader, mostly taken from the classical literature.

Although we have not included every classical paper, or every paper commenting on the classical literature, we believe that we have covered the most important ones. However, for various reasons two important results before 1750 have been omitted. The first is Cotes's rule (1722) for estimating a true value by a weighted mean, when observations are of unequal accuracy (see Stigler, 1986, p. 16); the second is Daniel Bernoulli's results (1738) on the theory of moral expectation, the utility of money, and the Petersburg problem (see Todhunter, 1865, Jorland, 1987, and Dutka, 1988).

We shall discuss our reasons for stopping our history at 1750.

By 1750 probability theory had been recognized as a mathematical discipline with a firm foundation and its own problems and methods as described by de Moivre in the *Doctrine of Chances*. A new development began with the introduction of inverse probability by Bayes (1764) and Laplace (1774b).

By 1750 statistics had still not become a mathematical discipline; a mathematical theory of errors and of estimation emerged in the 1750s, as described by Stigler (1986).

Also about 1750, the first phase of the development of a theory of life insurance had been completed. In the 1760s life insurance offices arose so that new and more accurate mortality observations became available. A theory of life assurances was developed, and new ways of calculating and tabulating the fundamental functions were invented.

The reader should note that formulae are numbered with a single number *within sections*. When referring to a formula in *another chapter* the decimal notation is used, (20.5.25) say, denoting formula (25) in §5 of Chap. 20. *Within a chapter* the chapter number is omitted so that only section and formula numbers are given.

1.3 A COMPARISON WITH TODHUNTER'S BOOK

The unquestioned authority on the early history of probability theory is Isaac Todhunter (1820–1884) whose masterpiece, *A History of the Mathematical Theory of Probability from the Time of Pascal to that of Laplace*, was published in 1865. Kendall (1963) has written a short biography of Todhunter in which he gives a precise characterization of his work: "The *History of the Mathematical Theory of Probability* is distinguished by three things. It is a work of scrupulous scholarship; Todhunter himself contributed nothing to the theory of probability except this account of it; and it is just about as dull as any book on probability could be."

We consider Todhunter's *History* an invaluable handbook giving a chronological review of the classical literature grouped according to authors. For the period before 1750, however, we shall argue that Todhunter's account of important topics is incomplete, that he has overlooked the significance of important contributions, and that the trend in the historical development is lost by his organization of the material.

In the many references to Todhunter's *History* in the following we shall omit the year of publication (1865) and give page references only.

As a mathematician Todhunter concentrates on the mathematical theory of probability and disregards the general background, the lives of the persons involved, and the application of their theories to statistics and life insurance.

Todhunter's book is ordered chronologically according to authors; each important author is allotted a separate chapter in which his works are reviewed page by page and commented upon. This method makes it easy for the reader to locate the contributions of each author but difficult to follow the advances made by various authors to the solution of a given problem. We have avoided this dilemma by reviewing the works of each author and referring the detailed treatment of the most important topics to separate chapters that show the historical development for each topic.

More important, however, are the different weights given to many topics by Todhunter and by us. It is not surprising that the significance of a theorem or method differs when viewed from the perspectives of 1865 and today. Todhunter meticulously reports proofs of many results which are without interest today; conversely, he omits proofs of results of great importance. We shall give some examples.

Today one of the most important and interesting topics is the development from James Bernoulli's law of large numbers for the binomial distribution through Nicholas Bernoulli's improved version of James's theorem and his approximation to the binomial tail probability to de Moivre's normal approximation. These three results are treated by Todhunter in less than two pages (pp. 72, 131, 192). He states Bernoulli's theorem without giving his proof; he has overlooked the significance of Nicholas' contribution and gives neither theorem nor proof; he states de Moivre's result for $p = 1/2$ only and indicates the proof by the remark, "Thus by the aid of Stirling's Theorem the value of Bernoulli's Theorem is largely increased." Todhunter has completely overlooked de Moivre's long struggle with this problem, the importance of de Moivre's proof as a model for Laplace's proof, and de Moivre's statement of the theorem for any value of p. Instead of giving the historical development of the method of proof, he gives Laplace's proof (pp. 548–552) because, as he says, previous demonstrations are now superseded by that. This is of course a very peculiar argument for a historian.

It is a common misunderstanding, perhaps due to Todhunter's incomplete account, that de Moivre gave the normal approximation only for the symmetric binomial.

The deficiency of Todhunter's method is most conspicuous in his analysis of the correspondence between Montmort and Nicholas Bernoulli, published in the second edition of Montmort's *Essay* (1713). These closely intertwined letters contain formulations of new problems, usually as a challenge to the recipient; theorems without proofs, sometimes with hints for solution; replies to previous questions; a running commentary on progress with the solution of various problems; and remarks on the contributions of other authors. A single letter often treats five to ten different topics. It is of course impossible to get the gist of these letters in a page-by-page review; rather, it is necessary to give an overview of the contents grouped by subject matter. Todhunter therefore does not realize the importance of Nicholas Bernoulli's work; perhaps he was also under the influence of de Moivre who in the later editions of the *Doctrine* tried to conceal the importance of Bernoulli's results to his own work.

The most difficult topic in probability theory before 1750 was the problem of the duration of play. It was formulated by Montmort in 1708; the first explicit solution was given by Nicholas Bernoulli in 1713. Two solutions were given by de Moivre in 1718, and these were worked out in more detail in 1730 and 1738. Todhunter gives up analyzing this important development, instead he uses Laplace's solution from 1812 to prove de Moivre's theorems. Furthermore, he does not comment on Laplace's solution from 1776 by solving a partial difference equation because this method "since [has] been superseded by that of Generating Functions" (Todhunter, p. 475).

The same procedure is used by Todhunter in his discussion of Waldegrave's problem, the probability of winning a circular tournament, which was solved incompletely by Montmort and de Moivre. A general solution was given by Nicholas Bernoulli, but Todhunter gives only Laplace's proof without noting that Bernoulli's is just as simple.

Todhunter's discussion of the strategic game Her is rather incomplete. He has overlooked the fact that Montmort gives the general form of the player's expectation under randomized strategies and that Waldegrave solves the problem numerically arriving at what today is called the minimax solution. Misled by Todhunter's account, Fisher (1934) solved the "old enigma of card play" by randomization and reached the same solution as Waldegrave did 221 years before.

Todhunter gives unsatisfactory accounts of James Bernoulli's and Montmort's probabilistic discussion of the game of tennis, of the problem of points in a game of bowls, of Montmort's discussion of the occupancy problem, of Simpson's solution of the theory of runs, and of several other problems mentioned in the following chapters.

Kendall's characterization that "it is just as dull as any book on probability could be" applies equally well to several sections of the present book. Detailed proofs of elementary theorems illustrating the historical development are necessarily dull for us, even if they were exciting for them. Pascal, Fermat, Huygens, Hudde, James Bernoulli, Montmort, Nicholas Bernoulli, de Moivre, and Struyck were all intensely interested in solving the problem of the Gambler's Ruin, which today is considered elementary. For statisticians who find examples of games of chance rather dull, it must be a consolation to know that dicing and card playing have their equivalents in sampling from infinite and finite populations, respectively.

1.4 WORKS OF REFERENCE

Gouraud's *Histoire* (1848, 148pp.) gives a nonmathematical and rather uncritical exposition of probability theory and insurance mathematics beginning with Pascal and Fermat and ending with Poisson and Quetelet. It contains many references and was therefore useful for Todhunter when he wrote his *History* (1865, 624pp.).

The first two chapters of Czuber's *Entwicklung der Wahrscheinlichkeitstheorie* (1899, 279pp.) covers nearly the same period as the present book but in less detail. Czuber indicates some methods of proof without giving complete proofs.

The books by Edwards (1987, 174pp.), *Pascal's Arithmetical Triangle*, and Hacking (1975, 209pp.), *The Emergence of Probability*, may be read as an introduction to the present one; they give a more detailed treatment of certain aspects of the history up to the time of Newton and Leibniz.

David (1962, 275pp.) gives a popular history of probability and statistics from antiquity through the time of de Moivre, stressing basic ideas and providing background material for the lives of the great probabilists.

Jordan's book (1972, 619pp.) contains a mathematical account of classical probability theory organized according to topics, with some references to the historical development.

The first 81 pages of Maistrov's book (1974, 281pp.) gives a sketch of the history of probability theory before 1750.

Daston's *Classical Probability in the Enlightenment* (1988, 423pp.) gives a comprehensive, nonmathematical study of the basic ideas in classical probability theory in their relation to games of chance, insurance, jurisprudence, economics, associationist psychology, religion, induction, and the moral sciences, with references to a wealth of background material. Daston's discussion of the history of probabilistic ideas is an excellent complement to our discussion of mathematical techniques and results.

Turning to books on the history of statistics, we mention first Karl Pearson's *The History of Statistics in the 17th and 18th Centuries*, Lectures given at University College, London, 1921–1933, edited by E. S. Pearson (1978, 744pp.). This is a fascinating book with an unusual freshness that conveys Pearson's enthusiasm and last-minute endeavors in preparing his lectures. It describes "the changing background of intellectual, scientific and religious thought," and gives lively biographies with digressions into the fields of mathematics and history of science. Pearson does not discuss statistics in the natural sciences but is mainly concerned with political arithmetic, demography, and the use of statistics for theological purposes. Pearson does not conceal his strong opinions on the subjects treated and the persons involved, which occasionally lead to biased evaluations.

Stigler's *The History of Statistics* (1986, 410pp.) is the first comprehensive history of statistics from 1750 to 1900; it also contains a discussion of Bernoulli's law of large numbers and de Moivre's normal approximation to the binomial.

Westergaard's *Contributions to the History of Statistics* (1932, 280pp.) gives the history of political arithmetic, population statistics, economic statistics, and official statistics before 1900, as well as a short survey of statistical theory. It is a nonmathematical, well-balanced, and scholarly work, with valuable references to the vast literature on descriptive and official statistics.

John's *Geschichte der Statistik* (1884, 376pp.) contains a description of the development of German political science, at that time called statistics, and of political arithmetic and population statistics before 1835.

Meitzen's *Geschichte, Theorie und Technik der Statistik* (1886, 240pp.) discusses the history of official statistics with the main emphasis on its development in Germany.

Following the pioneering work by M. G. Kendall and F. N. David in the 1950s and 1960s, there has been growing interest in the history of probability and statistics, and a great number of papers have been published; the most important, relating to the period before 1750, are listed in the References at the end of this book. Several important papers have been reprinted in *Studies in the History of Statistics and Probability*, Vol. 1 edited by E. S. Pearson and M. G. Kendall (1970) and Vol. 2 edited by M. G. Kendall and R. L. Plackett (1977). A *Bibliography of Statistical Literature Pre-1940* has been compiled by Kendall and Doig (1968).

A comprehensive account of the development of life insurance and its social, economic, and political background before 1914, with some remarks on mathematical results has been given by Braun in *Geschichte der Lebensversicherung und der Lebensversicherungstechnik* (1925, 433pp.).

For the biographies we have of course used the *Dictionary of Scientific Biography*, edited by C. C. Gillispie (1970–1980) and the individual biographies available.

As reference books for the history of mathematics we have used Cantor (1880–1908) and Kline (1972).

For long periods of time there existed a considerable backlog of publications of the Academies at London, Paris, Turin, etc., so that papers were read some years before they were published. Referring to such papers we have used the *date of publication*; in the list of references, however, we have usually added the date of communication to the Academy.