Bandwidth-Efficient Digital Modulation with Application to Deep-Space Communications
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Bandwidth-Efficient Digital Modulation with Application to Deep-Space Communications

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# Table of Contents

**Foreword** ........................................................................................................... vii

**Preface** ............................................................................................................. ix

**Chapter 1: Introduction** .................................................................................. 1

**Chapter 2: Constant Envelope Modulations** .................................................. 3

- 2.1 The Need for Constant Envelope ................................................................. 3
- 2.2 Quadriphase-Shift-Keying and Offset (Staggered) Quadriphase-Shift-Keying ........................................................................................................... 4
- 2.3 Differentially Encoded QPSK and Offset (Staggered) QPSK ........................ 8
- 2.4 $\pi/4$-QPSK: A Variation of Differentially Encoded QPSK with Instantaneous Amplitude Fluctuation Halfway between That of QPSK and OQPSK .................. 9
- 2.5 Power Spectral Density Considerations ...................................................... 12
- 2.6 Ideal Receiver Performance ...................................................................... 12
- 2.7 Performance in the Presence of Nonideal Transmitters ............................ 12
  - 2.7.1 Modulator Imbalance and Amplifier Nonlinearity ............................ 12
  - 2.7.2 Data Imbalance ................................................................................... 26
- 2.8 Continuous Phase Modulation ................................................................... 26
  - 2.8.1 Full Response—MSK and SFSK ....................................................... 27
  - 2.8.2 Partial Response—Gaussian MSK ................................................... 57
- 2.9 Simulation Performance ............................................................................ 113

**References** ...................................................................................................... 116

**Chapter 3: Quasi-Constant Envelope Modulations** ........................................ 125

- 3.1 Brief Review of IJF-QPSK and SQORC and their Relation to FQPSK .......... 129
- 3.2 A Symbol-by-Symbol Cross-Correlator Mapping for FQPSK ..................... 136
- 3.3 Enhanced FQPSK ....................................................................................... 143
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4 Interpretation of FQPSK as a Trellis-Coded Modulation</td>
<td>146</td>
</tr>
<tr>
<td>3.5 Optimum Detection</td>
<td>147</td>
</tr>
<tr>
<td>3.6 Suboptimum Detection</td>
<td>152</td>
</tr>
<tr>
<td>3.6.1 Symbol-by-Symbol Detection</td>
<td>152</td>
</tr>
<tr>
<td>3.6.2 Average Bit-Error Probability Performance</td>
<td>159</td>
</tr>
<tr>
<td>3.6.3 Further Receiver Simplifications and FQPSK-B Performance</td>
<td>161</td>
</tr>
<tr>
<td>3.7 Cross-Correlated Trellis-Coded Quadrature Modulation</td>
<td>166</td>
</tr>
<tr>
<td>3.7.1 Description of the Transmitter</td>
<td>168</td>
</tr>
<tr>
<td>3.7.2 Specific Embodiments</td>
<td>172</td>
</tr>
<tr>
<td>3.8 Other Techniques</td>
<td>177</td>
</tr>
<tr>
<td>3.8.1 Shaped Offset QPSK</td>
<td>177</td>
</tr>
<tr>
<td>References</td>
<td>184</td>
</tr>
<tr>
<td>Chapter 4: Bandwidth-Efficient Modulations with More Envelope Fluctuation</td>
<td>187</td>
</tr>
<tr>
<td>4.1 Bandwidth-Efficient TCM with Prescribed Decoding Delay—Equal Signal Energies</td>
<td>190</td>
</tr>
<tr>
<td>4.1.1 ISI-Based Transmitter Implementation</td>
<td>190</td>
</tr>
<tr>
<td>4.1.2 Evaluation of the Power Spectral Density</td>
<td>195</td>
</tr>
<tr>
<td>4.1.3 Optimizing the Bandwidth Efficiency</td>
<td>204</td>
</tr>
<tr>
<td>4.2 Bandwidth-Efficient TCM with Prescribed Decoding Delay—Unequal Signal Energies</td>
<td>212</td>
</tr>
<tr>
<td>References</td>
<td>218</td>
</tr>
<tr>
<td>Chapter 5: Strictly Bandlimited Modulations with Large Envelope Fluctuation (Nyquist Signaling)</td>
<td>219</td>
</tr>
<tr>
<td>5.1 Binary Nyquist Signaling</td>
<td>219</td>
</tr>
<tr>
<td>5.2 Multilevel and Quadrature Nyquist Signaling</td>
<td>223</td>
</tr>
<tr>
<td>References</td>
<td>223</td>
</tr>
<tr>
<td>Chapter 6: Summary</td>
<td>225</td>
</tr>
<tr>
<td>6.1 Throughput Performance Comparisons</td>
<td>225</td>
</tr>
<tr>
<td>References</td>
<td>226</td>
</tr>
</tbody>
</table>
Foreword

The Deep Space Communications and Navigation Systems Center of Excellence (DESCANSO) was recently established for the National Aeronautics and Space Administration (NASA) at the California Institute of Technology's Jet Propulsion Laboratory (JPL). DESCANSO is chartered to harness and promote excellence and innovation to meet the communications and navigation needs of future deep-space exploration.

DESCANSO's vision is to achieve continuous communications and precise navigation—any time, anywhere. In support of that vision, DESCANSO aims to seek out and advocate new concepts, systems, and technologies; foster key scientific and technical talents; and sponsor seminars, workshops, and symposia to facilitate interaction and idea exchange.

The Deep Space Communications and Navigation Series, authored by scientists and engineers with many years of experience in their respective fields, lays a foundation for innovation by communicating state-of-the-art knowledge in key technologies. The series also captures fundamental principles and practices developed during decades of deep-space exploration at JPL. In addition, it celebrates successes and imparts lessons learned. Finally, the series will serve to guide a new generation of scientists and engineers.

Joseph H. Yuen
DESCANSO Leader
Preface

Traditional modulation methods adopted by space agencies for transmitting telecommand and telemetry data have incorporated subcarriers as a simple means of separating different data types as well ensuring no overlap between the radio frequency (RF) carrier and the modulated data’s frequency spectra. Unfortunately, subcarrier modulation suffers from a number of disadvantages, namely, greater spacecraft complexity, additional losses in the modulation/demodulation process, and most important, at least from the standpoint of this monograph, a large, occupied bandwidth. One effort to mitigate the latter was to replace the more traditional square-wave subcarriers with sine-wave carriers, but this was not considered to be an acceptable solution for all space-exploration missions.

In the early digital communication years (i.e., 1960s and 1970s), bandwidth occupancy was really not an issue because of low data rates and the requirement for only a few data channels (subcarriers). Consequently, other attempts at limiting bandwidth occupancy were not considered at that time. As missions became more complex, however, the RF spectrum became more congested, and data rates continued to grow, thus requiring an attendant increase in subcarrier frequencies (equivalently, occupied bandwidth) and along with that, an increased susceptibility to interference from different spacecraft. A point came at which it was no longer feasible to use subcarrier-based modulation methods. Fortunately, during this same period, improved bandwidth-efficient modulation methods that directly modulated the carrier were being developed, which, along with improved data formatting methods (e.g., packet transfer frame telemetry) to handle the multiple channel separation problem, eliminated the need for subcarriers. Combining the packet telemetry format with any of the direct modulation methods and applying
additional spectral pulse shaping to the latter now made it possible to transmit messages at a high data rate while using a comparatively small bandwidth.

The purpose of this monograph is to define, describe, and then give the performance (power and bandwidth) of digital communication systems that incorporate a large variety of the bandwidth-efficient modulations referred to above. In addition to considering the ideal behavior of such systems, we shall also cover their performance in the presence of a number of practical (non-ideal) transmitter and receiver characteristics such as modulator and phase imbalance, imperfect carrier synchronization, and transmitter nonlinearity. With regard to the latter, the requirement of operating the transmitter at a high power efficiency, i.e., running the power amplifier in a saturated or near-saturated condition, implies that one employ a constant envelope modulation. This constraint restricts the type of modulations that can be considered, which in turn restricts the amount of spectral occupancy and power efficiency that can be achieved. Relaxing the constant envelope condition (which then allows for a more linear but less efficient transmitter power amplifier operation) potentially eases the restrictions on power and bandwidth efficiency to the extreme limit of Nyquist-type signaling, which, in theory, is strictly bandlimited and capable of achieving the maximum power efficiency. Because of this inherent trade-off between envelope (or more correctly, instantaneous amplitude) fluctuation of the modulation and the degree of power and bandwidth efficiency attainable, we have chosen to structure this monograph in a way that clearly reflects this issue. In particular, we start by discussing strictly constant envelope modulations and then, moving in the direction of more and more envelope fluctuation, end with a review of strictly bandlimited (Nyquist-type) signaling. Along the way, we consider a number of quasi-constant envelope modulations that have gained considerable notoriety in recent years and represent a good balance among the above-mentioned power and bandwidth trade-off considerations.

Finally, it should be mentioned that although the monograph attempts to cover a large body of the published literature in this area, the real focus is on the research and the results obtained at the Jet Propulsion Laboratory (JPL). As such, we do not offer this document to the readership as an all-inclusive treatise on the subject of bandwidth-efficient modulations but rather one that, as the title reflects, highlights the many technical contributions performed under NASA-funded tasks pertaining to the development and design of deep-space communications systems. When taken in this context, we hope that, in addition to being informative, this document will serve as an inspiration to future engineers to continue the fine work that was initiated at JPL and has been reported on herein.

Marvin K. Simon

June 2001
Chapter 1
Introduction

The United States Budget Reconciliation Act of 1993 mandates reallocation of a minimum of 200 MHz of spectrum below 5 GHz for licensing to nonfederal users. One of the objectives is to promote and encourage novel spectrum-inspired technology developments and wireless applications. Many user organizations and communications companies have been developing advanced modulation techniques in order to more efficiently use the spectrum.

In 1998, the international Space Frequency Coordination Group (SFCG) adopted a spectral mask that precludes the use of a number of classical modulation schemes for missions launched after 2002. The SFCG has recommended several advanced modulations that potentially could reduce spectrum congestion. No one technique solves every intended application. Many trade-offs must be made in selecting a particular technique, the trade-offs being defined by the communications environment, data integrity requirements, data latency requirements, user access, traffic loading, and other constraints. These new modulation techniques have been known in theory for many years, but have become feasible only because of recent advances in digital signal processing and microprocessor technologies.

This monograph focuses on the most recent advances in spectrum-efficient modulation techniques considered for government and commercial applications. Starting with basic, well-known digital modulations, the discussion will evolve to more sophisticated techniques that take on the form of constant envelope modulations, quasi-constant envelope modulations, nonconstant envelope modulations, and finally Nyquist-rate modulations. Included in the discussion will be a unified treatment based on recently developed cross-correlated trellis-coded quadrature modulation (XTCQM), which captures a number of state-of-the-art spectrally efficient modulation schemes. Performance analysis, computer simulation results, and their hardware implications will be addressed. Comparisons of
different modulation schemes recommended by the Consultative Committee for Space Data Systems (CCSDS), an international organization for cross support among space agencies, for SFCG will be discussed.
Chapter 2
Constant Envelope Modulations

2.1 The Need for Constant Envelope

Digital communication systems operate in the presence of path loss and atmospheric-induced fading. In order to maintain sufficient received power at the destination, it is required that a device for generating adequate transmitter output power based on fixed- but-limited available power be employed, examples of which are traveling-wave tube amplifiers (TWTAs) and solid-state power amplifiers (SSPAs) operated in full- saturation mode to maximize conversion efficiency. Unfortunately, this requirement introduces amplitude modulation-amplitude modulation (AM-AM) and amplitude modulation-phase modulation (AM-PM) conversions into the transmitted signal. Because of this, modulations that transmit information via their amplitude, e.g., quadrature amplitude modulation (QAM), and therefore need a linear amplifying characteristic, are not suitable for use on channels operated in the above maximum transmitter power efficiency requirement. Another consideration regarding radio frequency (RF) amplifier devices that operate in a nonlinear mode at or near saturation is the spectral spreading that they reintroduce due to the nonlinearity subsequent to bandlimiting the modulation prior to amplification. Because of the need for the transmitted power spectrum to fall under a specified mask imposed by regulating agencies such as the FCC or International Telecommunications Union (ITU), the modulation must be designed to keep this spectral spreading to a minimum. This constraint necessitates limiting the amount of instantaneous amplitude fluctuation in the transmitted waveform in addition to imposing the requirement for constant envelope.

1 An approach whereby it might be possible to generate QAM-type modulations using separate nonlinearly operated high-power amplifiers on the inphase (I) and quadrature (Q) channels is currently under investigation by the author.
Because of the above considerations regarding the need for high transmitter power efficiency, it is clearly desirable to consider modulations that achieve their bandwidth efficiency by means other than resorting to multilevel amplitude modulation. Such constant envelope modulations are the subject of discussion in the first part of this monograph. Because of the large number of possible candidates, to keep within the confines of a reasonable size book, we shall restrict our attention to only those that have some form of inphase-quadrature phase (I-Q) representation and as such an I-Q form of receiver.

### 2.2 Quadriphase-Shift-Keying and Offset (Staggered) Quadriphase-Shift-Keying

$M$-ary phase-shift-keying ($M$-PSK) produces a constant envelope signal that is mathematically modeled in complex form\(^2\) as

$$
\tilde{s}(t) = \sqrt{2P}e^{j(2\pi f_c t + \theta(t) + \theta_c)} = \tilde{S}(t)e^{j(2\pi f_c t + \theta_c)}
$$

where $P$ is the transmitted power, $f_c$ is the carrier frequency in hertz, $\theta_c$ is the carrier phase, and $\theta(t)$ is the data phase that takes on equiprobable values $\beta_i = (2i - 1)\pi/M$, $i = 1, 2, \cdots, M$, in each symbol interval, $T_s$. As such, $\theta(t)$ is modeled as a random pulse stream, that is,

$$
\theta(t) = \sum_{n=-\infty}^{\infty} \theta_n p(t - nT_s)
$$

where $\theta_n$ is the information phase in the $n$th symbol interval, $nT_s < t \leq (n+1)T_s$, ranging over the set of $M$ possible values $\beta_i$ as above, and $p(t)$ is a unit amplitude rectangular pulse of duration $T_s$ seconds. The symbol time, $T_s$, is related to the bit time, $T_b$, by $T_s = T_b \log_2 M$ and, thus, the nominal gain in bandwidth efficiency relative to binary phase-shift-keying (BPSK), i.e., $M = 2$, is a factor of $\log_2 M$. The signal constellation is a unit circle with points uniformly spaced by $2\pi/M$ rad. Thus, the complex signal transmitted in the $n$th symbol interval is

$$
\tilde{s}(t) = \sqrt{2P}e^{j(2\pi f_c t + \theta_n + \theta_c)}, \quad nT_s < t \leq (n+1)T_s, \quad n = -\infty, \cdots, \infty
$$

\(^2\)The actual (real) transmitted signal is $s(t) = \text{Re} \{\tilde{s}(t)\} = \sqrt{2P} \cos \left(2\pi f_c t + \theta(t) + \theta_c\right)$. 

Note that because of the assumed rectangular pulse shape, the complex baseband signal \( \hat{S}(t) = \sqrt{2P}e^{j\theta_n} \) is constant in this same interval and has envelope \( |\hat{S}(t)| = \sqrt{2P} \).

A special case of M-PSK that has an I-Q representation is quadriphase-shift-keying (QPSK), and corresponds to \( M = 4 \). Here it is conventional to assume that the phase set \( \{\beta_i\} \) takes on values \( \pi/4, 3\pi/4, 5\pi/4, 7\pi/4 \). Projecting these information phases on the quadrature amplitude axes, we can equivalently write QPSK in the \( n \)th symbol interval in the complex I-Q form

\[
\tilde{s}(t) = \sqrt{P} (a_{In} + ja_{Qn}) e^{j(2\pi f_c t + \theta_c)}, \quad nT_s < t \leq (n + 1)T_s
\]  

(2.2-4)

where the information amplitudes \( a_{In} \) and \( a_{Qn} \) range independently over the equiprobable values \( \pm 1 \). Here again, because of the assumed rectangular pulse shape, the complex baseband signal \( \hat{S}(t) = \sqrt{P} (a_{In} + ja_{Qn}) \) is constant in this same interval. The real transmitted signal corresponding to (2.2-4) has the form

\[
s(t) = \sqrt{P} m_I(t) \cos(2\pi f_c t + \theta_c) - \sqrt{P} m_Q(t) \sin(2\pi f_c t + \theta_c),
\]

where

\[
m_I(t) = \sum_{n=-\infty}^{\infty} a_{In} p(t - nT_s), \quad m_Q(t) = \sum_{n=-\infty}^{\infty} a_{Qn} p(t - nT_s) \quad (2.2-5)
\]

If one examines the form of (2.2-4) it becomes apparent that a large fluctuation of the instantaneous amplitude between symbols corresponding to a 180-deg phase reversal can occur when both \( a_{In} \) and \( a_{Qn} \) change polarity at the same time. As mentioned in Sec. 2.1, it is desirable to limit the degree of such fluctuation to reduce spectral regrowth brought about by the transmit amplifier nonlinearity, i.e., the smaller the fluctuation, the smaller the sidetone regeneration and vice versa. By offsetting (staggering) the I and Q modulations by \( T_s/2 \), one guarantees the fact that \( a_{In} \) and \( a_{Qn} \) cannot change polarity at the same time. Thus, the maximum fluctuation in instantaneous amplitude is now limited to that corresponding to a 90-deg phase reversal (i.e., either \( a_{In} \) or \( a_{Qn} \), but not both, change polarity). The resulting modulation, called offset (staggered) QPSK (OQPSK), has a signal of the form

\[\text{One can think of the complex carrier as being modulated now by a complex random pulse stream, namely, } \tilde{a}(t) = \sum_{n=-\infty}^{\infty} \left(a_{In} + ja_{Qn}\right) p(t - nT_s).\]
While it is true that for M-PSK with $M = 2^m$ and $m$ an arbitrary integer, the information phases can be projected on the I and Q coordinates and as such obtain, in principle, an I-Q transmitter representation, it should be noted that the number of possible I-Q amplitude pairs obtained from these projections exceeds $M$. Consequently, decisions on the resulting I and Q multilevel amplitude signals at the receiver are not independent in that each pair of amplitude decisions does not necessarily render one of the transmitted phases. Therefore, for $M \geq 8$ it is not practical to view M-PSK in an I-Q form.

The detection of an information phase can be obtained by combining the detections on the I and Q components of this phase. The receiver for QPSK is illustrated in Fig. 2-1(a) while the analogous receiver for OQPSK is illustrated in Fig. 2-1(b). The decision variables that are input to the hard-limiting threshold devices are

$$y_{In} = a_{In} \sqrt{P} T_s + N_{In}$$  \hspace{1cm} (2.2-7)$$
$$y_{Qn} = a_{Qn} \sqrt{P} T_s + N_{Qn}$$

where for QPSK

$$N_{In} = \text{Re} \left\{ \int_{nT_s}^{(n+1)T_s} \tilde{N}(t) \, dt \right\}$$  \hspace{1cm} (2.2-8)$$
$$N_{Qn} = \text{Im} \left\{ \int_{nT_s}^{(n+1)T_s} \tilde{N}(t) \, dt \right\}$$

whereas for OQPSK
In either case, \( N_{I,n} \), \( N_{Q,n} \) are zero mean Gaussian random variables (RVs) with variance \( \sigma_N^2 = N_0 T_s / 2 \) and thus conditioned on the data symbols, \( y_{I,n} \), \( y_{Q,n} \) are also Gaussian RVs with the same variance.

**Fig. 2-1(a).** Complex form of optimum receiver for ideal coherent detection of QPSK over the AWGN.

**Fig. 2-1(b).** Complex form of optimum receiver for ideal coherent detection of OQPSK over the AWGN.
2.3 Differentially Encoded QPSK and Offset (Staggered) QPSK

In an actual coherent communication system transmitting M-PSK modulation, means must be provided at the receiver for establishing the local demodulation carrier reference signal. This means is traditionally accomplished with the aid of a suppressed carrier-tracking loop [1, Chap. 2]. Such a loop for M-PSK modulation exhibits an M-fold phase ambiguity in that it can lock with equal probability at the transmitted carrier phase plus any of the M information phase values. Hence, the carrier phase used for demodulation can take on any of these same M phase values, namely, \( \theta_c + \beta = \theta_c + 2i\pi/M, \ i = 0, 1, 2, \ldots, M - 1 \). Coherent detection cannot be successful unless this M-fold phase ambiguity is resolved.

One means for resolving this ambiguity is to employ differential phase encoding (most often simply called differential encoding) at the transmitter and differential phase decoding (most often simply called differential decoding) at the receiver following coherent detection. That is, the information phase to be communicated is modulated on the carrier as the difference between two adjacent transmitted phases, and the receiver takes the difference of two adjacent phase decisions to arrive at the decision on the information phase.\(^4\) In mathematical terms, if \( \Delta \theta_n \) were the information phase to be communicated in the nth transmission interval, the transmitter would first form \( \theta_n = \theta_{n-1} + \Delta \theta_n \) modulo \( 2\pi \) (the differential encoder) and then modulate \( \theta_n \) on the carrier.\(^5\) At the receiver, successive decisions on \( \theta_{n-1} \) and \( \theta_n \) would be made and then differenced modulo \( 2\pi \) (the differential decoder) to give the decision on \( \Delta \theta_n \). Since the decision on the true information phase is obtained from the difference of two adjacent phase decisions, a performance penalty is associated with the inclusion of differential encoding/decoding in the system.

For QPSK or OQPSK, the differential encoding/decoding process can be performed on each of the I and Q channels independently. A block diagram of a receiver for differentially encoded QPSK (or OQPSK) would be identical to that shown in Fig. 2-1(a) [or Fig. 2-1(b)], with the inclusion of a binary differential decoder in each of the I and Q arms following the hard-decision devices [see

---

\(^4\) Note that this receiver (i.e., the one that makes optimum coherent decisions on two successive symbol phases and then differences these to arrive at the decision on the information phase) is suboptimum when \( M > 2 \) [2]. However, this receiver structure, which is the one classically used for coherent detection of differentially encoded M-PSK, can be arrived at by a suitable approximation of the likelihood function used to derive the true optimum receiver, and at high signal-to-noise ratio (SNR), the difference between the two becomes mute.

\(^5\) Note that we have shifted our notation here insofar as the information phases are concerned so as to keep the same notation for the phases actually transmitted.
Figs. 2-2(a) and 2-2(b).\(^6\) Inclusion of differentially encoded OQPSK in our discussion is important since, as we shall see later on, other forms of modulation, e.g., minimum-shift-keying (MSK), have an I-Q representation in the form of pulse-shaped, differentially encoded OQPSK.

### 2.4 \(\pi/4\)-QPSK: A Variation of Differentially Encoded QPSK with Instantaneous Amplitude Fluctuation Halfway between That of QPSK and OQPSK

Depending on the set of phases, \(\{\Delta\beta_i\}\), used to represent the information phase, \(\Delta\theta_n\), in the \(n\)th transmission interval, the actual transmitted phase, \(\theta_n\), in this same transmission interval can range either over the same set, \(\{\beta_i\} = \{\Delta\beta_i\}\), or over another phase set. If for QPSK, we choose the set \(\Delta\beta_i = 0, \pi/2, \pi, 3\pi/2\) to represent the information phases, then starting with an initial transmitted phase chosen from the set \(\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\), the subsequent transmitted phases, \(\{\theta_n\}\), will also range over the set \(\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\) in every transmission interval. This is the conventional form of differentially encoded QPSK, as discussed in the previous section. Now suppose instead that the set \(\Delta\beta_i = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4\) is used to represent the information phases, \(\{\Delta\theta_n\}\). Then, starting, for example, with an initial phase chosen from the set \(\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\), the transmitted phase in the next interval will range over the set \(0/2, \pi, 3\pi/2\). In the following interval, the transmitted phase will range over the set \(\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\), and in the interval following that one, the transmitted phase will once again range over the set \(0, \pi/2, \pi, 3\pi/2\). Thus, we see that for this choice of phase set corresponding to the information phases, \(\{\Delta\theta_n\}\), the transmitted phases, \(\{\theta_n\}\), will alternatively range over the sets \(0, \pi/2, \pi, 3\pi/2\) and \(\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\). Such a modulation scheme, referred to as \(\pi/4\)-QPSK [3], has an advantage relative to conventional differentially encoded QPSK in that the maximum change in phase from transmission to transmission is 135 deg, which is halfway between the 90-deg maximum phase change of OQPSK and 180-deg maximum phase change of QPSK.

In summary, on a linear additive white Gaussian noise (AWGN) channel with ideal coherent detection, all three types of differentially encoded QPSK, i.e., conventional (nonoffset), offset, and \(\pi/4\) perform identically. The differences among the three types on a linear AWGN channel occur when the carrier demodulation phase reference is not perfect, which corresponds to nonideal coherent detection.

\(^6\)Since the introduction of a 180-deg phase shift to a binary phase sequence is equivalent to a reversal of the polarity of the binary data bits, a binary differential encoder is characterized by \(a_n = a_{n-1}b_n\) and the corresponding binary differential decoder is characterized by \(b_n = a_{n-1}a_n\) where \(\{b_n\}\) are now the information bits and \(\{a_n\}\) are the actual transmitted bits on each channel.
Fig. 2.2(a). Complex form of optimum receiver for ideal coherent detection of differentially encoded QPSK over the AWGN.
Fig. 2-2(b). Complex form of optimum receiver for ideal coherent detection of differentially encoded OQPSK over the AWGN.
2.5 Power Spectral Density Considerations

The power spectral densities (PSD) of QPSK, OQPSK, and the differentially encoded versions of these are all identical and are given by

\[ S(f) = PT_s \left( \frac{\sin \pi f T_s}{\pi f T_s} \right)^2 \]  

(2.5-1)

We see that the asymptotic (large \( f \)) rate of rolloff of the PSD varies as \( f^{-2} \), and a first null (width of the main lobe) occurs at \( f = 1/T_s = 1/2T_b \). Furthermore, when compared with BPSK, QPSK is exactly twice as bandwidth efficient.

2.6 Ideal Receiver Performance

Based upon the decision variables in (2.2-7) the receiver for QPSK or OQPSK makes its I and Q data decisions from

\[ \hat{a}_{I,n} = \text{sgn} \, y_{I,n} \]
\[ \hat{a}_{Q,n} = \text{sgn} \, y_{Q,n} \]

(2.6-1)

which results in an average bit-error probability (BEP) given by

\[ P_b(E) = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right), \quad E_b = PT_b \]  

(2.6-2)

and is identical to that of BPSK. Thus, we conclude that ideally BPSK, QPSK, and OQPSK have the identical BEP performance although the latter two occupy half the bandwidth.

2.7 Performance in the Presence of Nonideal Transmitters

2.7.1 Modulator Imbalance and Amplifier Nonlinearity

The deleterious effect on receiver performance of modulator phase and amplitude imbalance and amplifier nonlinearity has been studied by several researchers [3–10]. With regard to modulator imbalances, the primary source of degradation comes about because of the effect of the imbalance on the steady-state lock point of the carrier tracking loop, which has a direct impact on the determination of
accurate average BEP performance. Here, we summarize some of these results for QPSK and OQPSK, starting with modulator imbalance acting alone and then later on in combination with amplifier nonlinearity. We begin our discussion with a description of an imbalance model associated with a modulator for generating these signals.

### 2.7.1.1 Modulator Imbalance Model

QPSK can be implemented with two balanced modulators, one on each of the I and Q channels, as illustrated in Fig. 2-3. Each of these modulators is composed of two AM modulators with inputs equal to the input nonreturn-to-zero (NRZ) data stream and its inverse (bit polarities inverted). The difference of the outputs of the two AM modulators serves as the BPSK transmitted signal on each channel. A mathematical description of the I and Q channel signals in the presence of amplitude and phase imbalances introduced by the AM modulators is

\[s_I(t) = \frac{\sqrt{P}}{2} m_I(t) \left[ \cos(2\pi f_c t + \theta_{cI}) + \Gamma_I \cos(2\pi f_c t + \theta_{cI} + \Delta\theta_{cI}) \right]
\]

\[+ \frac{\sqrt{P}}{2} \left[ \cos(2\pi f_c t + \theta_{cI}) - \Gamma_I \cos(2\pi f_c t + \theta_{cI} + \Delta\theta_{cI}) \right] \] (2.7-1a)

\[s_Q(t) = \frac{\sqrt{P}}{2} m_Q(t) \left[ \sin(2\pi f_c t + \theta_{cQ}) + \Gamma_Q \sin(2\pi f_c t + \theta_{cQ} + \Delta\theta_{cQ}) \right]
\]

\[+ \frac{\sqrt{P}}{2} \left[ \sin(2\pi f_c t + \theta_{cQ}) - \Gamma_Q \sin(2\pi f_c t + \theta_{cQ} + \Delta\theta_{cQ}) \right] \] (2.7-1b)

\[s(t) = s_I(t) + s_Q(t)\]

where \(\theta_{cI}, \theta_{cQ}\) are the local oscillator carrier phases associated with the I and Q balanced modulators, \(\Gamma_I, \Gamma_Q\) (both assumed to be less than unity) are the relative amplitude imbalances of these same modulators, and \(\Delta\theta_{cI}, \Delta\theta_{cQ}\) are the phase imbalances between the two AM modulators in each of the I and Q

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7 To be consistent with the usage in Ref. 8, we define the transmitted signal as the sum of the I and Q signals, i.e., \(s(t) = s_I(t) + s_Q(t)\) rather than their difference as in the more traditional usage of (2.2-5). This minor switch in notation is of no consequence to the results that follow.