MODEL-BASED SIGNAL PROCESSING
MODEL-BASED SIGNAL PROCESSING

James V. Candy
Lawrence Livermore National Laboratory
University of California
Santa Barbara, CA

IEEE PRESS

A JOHN WILEY & SONS, INC., PUBLICATION
Praise the Lord, Jesus Christ, Our Savior! In times of great need and distress—He comforts us!
CONTENTS

Preface xv

Acknowledgments xxi

1. Introduction 1
   1.1 Background / 1
   1.2 Signal Estimation / 5
   1.3 Model-Based Processing Example / 7
   1.4 Model-Based Signal Processing Concepts / 11
   1.5 Notation and Terminology / 16
   1.6 Summary / 16
       MATLAB® Notes / 16
       References / 17
       Problems / 17

2. Discrete Random Signals and Systems 21
   2.1 Introduction / 21
   2.2 Deterministic Signals and Systems / 21
   2.3 Spectral Representation of Discrete Signals / 24
       2.3.1 Discrete Systems / 26
       2.3.2 Frequency Response of Discrete Systems / 29
   2.4 Discrete Random Signals / 32
       2.4.1 Motivation / 32
       2.4.2 Random Signals / 36
   2.5 Spectral Representation of Random Signals / 44
2.6 Discrete Systems with Random Inputs / 57
2.7 ARMAX (AR, ARX, MA, ARMA) Models / 60
2.8 Lattice Models / 71
2.9 Exponential (Harmonic) Models / 79
2.10 Spatiotemporal Wave Models / 83
   2.10.1 Plane Waves / 83
   2.10.2 Spherical Waves / 87
   2.10.3 Spatiotemporal Wave Model / 89
2.11 State-Space Models / 92
   2.11.1 Continuous State-Space Models / 92
   2.11.2 Discrete State-Space Models / 98
   2.11.3 Discrete Systems Theory / 102
   2.11.4 Gauss-Markov (State-Space) Models / 105
   2.11.5 Innovations (State-Space) Models / 111
2.12 State-Space, ARMAX (AR, MA, ARMA, Lattice) Equivalence Models / 112
2.13 State-Space and Wave Model Equivalence / 120
2.14 Summary / 124
   MATLAB Notes / 124
   References / 125
   Problems / 127

3. Estimation Theory 135

3.1 Introduction / 135
   3.1.1 Estimator Properties / 136
   3.1.2 Estimator Performance / 137
3.2 Minimum Variance (MV) Estimation / 139
   3.2.1 Maximum a Posteriori (MAP) Estimation / 142
   3.2.2 Maximum Likelihood (ML) Estimation / 143
3.3 Least-Squares (LS) Estimation / 147
   3.3.1 Batch Least Squares / 147
   3.3.2 LS: A Geometric Perspective / 150
   3.3.3 Recursive Least Squares / 156
3.4 Optimal Signal Estimation / 160
3.5 Summary / 167
   MATLAB Notes / 167
   References / 167
   Problems / 168
4. AR, MA, ARMAX, Lattice, Exponential, Wave Model-Based Processors 175

4.1 Introduction / 175
4.2 AR (All-Pole) MBP / 176
   4.2.1 Levinson-Durbin Recursion / 179
   4.2.2 Toeplitz Matrices for AR Model-Based Processors / 185
   4.2.3 Model-Based AR Spectral Estimation / 187
4.3 MA (All-Zero) MBP / 191
   4.3.1 Levinson-Wiggins-Robinson (LWR) Recursion / 193
   4.3.2 Optimal Deconvolution / 198
   4.3.3 Optimal Time Delay Estimation / 201
4.4 Lattice MBP / 207
4.5 ARMAX (Pole-Zero) MBP / 213
4.6 Order Estimation for MBP / 220
4.7 Case Study: Electromagnetic Signal Processing / 227
4.8 Exponential (Harmonic) MBP / 238
   4.8.1 Exponential MBP / 240
   4.8.2 SVD Exponential MBP / 247
   4.8.3 Harmonic MBP / 250
4.9 Wave MBP / 262
4.10 Summary / 271
   MATLAB Notes / 272
   References / 272
   Problems / 275

5. Linear State-Space Model-Based Processors 281

5.1 State-Space MBP (Kalman Filter) / 281
5.2 Innovations Approach to the MBP / 284
5.3 Innovations Sequence of the MBP / 291
5.4 Bayesian Approach to the MBP / 295
5.5 Tuned MBP / 299
5.6 Tuning and Model Mismatch in the MBP / 308
   5.6.1 Tuning with State-Space MBP Parameters / 308
   5.6.2 Model Mismatch Performance in the State-Space MBP / 312
5.7 MBP Design Methodology / 318
5.8 MBP Extensions / 327
   5.8.1 Model-Based Processor: Prediction-Form / 327
   5.8.2 Model-Based Processor: Colored Noise / 329
   5.8.3 Model-Based Processor: Bias Correction / 335
5.9 MBP Identifier / 338
5.10 MBP Deconvolver / 342
5.11 Steady-State MBP Design / 345
  5.11.1 Steady-State MBP / 345
  5.11.2 Steady-State MBP and the Wiener Filter / 349
5.12 Case Study: MBP Design for a Storage Tank / 351
5.13 Summary / 358
MATLAB Notes / 358
References / 359
Problems / 361

6. Nonlinear State-Space Model-Based Processors 367

6.1 Linearized MBP (Kalman Filter) / 367
6.2 Extended MBP (Extended Kalman Filter) / 377
6.3 Iterated-Extended MBP (Iterated-Extended Kalman Filter) / 385
6.4 Unscented MBP (Kalman Filter) / 392
  6.4.1 Unscented Transformations / 393
  6.4.2 Unscented Processor / 397
6.5 Case Study: 2D-Tracking Problem / 404
6.6 Summary / 411
MATLAB Notes / 411
References / 412
Problems / 413

7. Adaptive AR, MA, ARMAX, Exponential Model-Based Processors 419

7.1 Introduction / 419
7.2 Adaption Algorithms / 420
7.3 All-Zero Adaptive MBP / 423
  7.3.1 Stochastic Gradient Adaptive Processor / 424
  7.3.2 Instantaneous Gradient LMS Adaptive Processor / 430
  7.3.3 Normalized LMS Adaptive Processor / 433
  7.3.4 Recursive Least-Squares (RLS) Adaptive Processor / 436
7.4 Pole-Zero Adaptive MBP / 443
  7.4.1 IIR Adaptive MBP / 443
  7.4.2 All-Pole Adaptive Predictor / 445
7.5 Lattice Adaptive MBP / 451
  7.5.1 All-Pole Adaptive Lattice MBP / 451
  7.5.2 Joint Adaptive Lattice Processor / 458
7.6 Adaptive MBP Applications / 460
   7.6.1 Adaptive Noise Canceler MBP / 460
   7.6.2 Adaptive D-Step Predictor MBP / 465
   7.6.3 Adaptive Harmonic MBP / 469
   7.6.4 Adaptive Time-Frequency MBP / 473
7.7 Case Study: Plasma Pulse Estimation Using MBP / 475
7.8 Summary / 481
MATLAB Notes / 481
References / 481
Problems / 483

8. Adaptive State-Space Model-Based Processors 489
8.1 State-Space Adaption Algorithms / 489
8.2 Adaptive Linear State-Space MBP / 491
8.3 Adaptive Innovations State-Space MBP / 495
   8.3.1 Innovations Model / 495
   8.3.2 RPE Approach Using the Innovations Model / 500
8.4 Adaptive Covariance State-Space MBP / 507
8.5 Adaptive Nonlinear State-Space MBP / 512
8.6 Case Study: AMBP for Ocean Acoustic Sound Speed Inversion / 522
   8.6.1 State-Space Forward Propagator / 522
   8.6.2 Sound-Speed Estimation: AMBP Development / 526
   8.6.3 Experimental Data Results / 528
8.7 Summary / 531
MATLAB Notes / 531
References / 532
Problems / 533

9. Applied Physics-Based Processors 539
9.1 MBP for Reentry Vehicle Tracking / 539
   9.1.1 RV Simplified Dynamics / 540
   9.1.2 Signal Processing Model / 542
   9.1.3 Processing of RV Signatures / 546
   9.1.4 Flight Data Processing / 556
   9.1.5 Summary / 559
9.2 MBP for Laser Ultrasonic Inspections / 561
   9.2.1 Laser Ultrasonic Propagation Modeling / 562
   9.2.2 Model-Based Laser Ultrasonic Processing / 563
   9.2.3 Laser Ultrasonics Experiment / 567
## Contents

9.2.4 Summary / 570

9.3 *MBP* for Structural Failure Detection / 571

9.3.1 Structural Dynamics Model / 572
9.3.2 Model-Based Condition Monitor / 574
9.3.3 Model-Based Monitor Design / 577
9.3.4 *MBP* Vibrations Application / 577
9.3.5 Summary / 583

9.4 *MBP* for Passive Sonar Direction-of-Arrival and Range Estimation / 583

9.4.1 Model-Based Adaptive Array Processing for Passive Sonar Applications / 584
9.4.2 Model-Based Adaptive Processing Application to Synthesized Sonar Data / 587
9.4.3 Model-Based Ranging / 590
9.4.4 Summary / 594

9.5 *MBP* for Passive Localization in a Shallow Ocean / 594

9.5.1 Ocean Acoustic Forward Propagator / 595
9.5.2 *AMBP* for Localization / 599
9.5.3 *AMBP* Application to Experimental Data / 603
9.5.4 Summary / 607

9.6 *MBP* for Dispersive Waves / 607

9.6.1 Background / 608
9.6.2 Dispersive State-Space Propagator / 609
9.6.3 Dispersive Model-Based Processor / 612
9.6.4 Internal Wave Processor / 614
9.6.5 Summary / 621

9.7 *MBP* for Groundwater Flow / 621

9.7.1 Groundwater Flow Model / 621
9.7.2 *AMBP* Design / 625
9.7.3 Summary / 627

9.8 Summary / 627

References / 628

### Appendix A Probability and Statistics Overview

A.1 Probability Theory / 631
A.2 Gaussian Random Vectors / 637
A.3 Uncorrelated Transformation: Gaussian Random Vectors / 638

References / 639
Appendix B  SEQUENTIAL MBP and UD-FACTORIZATION  641

B.1 Sequential MBP  / 641
B.2 UD-Factorization Algorithm for MBP  / 644
   References  / 646

Appendix C  SSPACK_PC: AN INTERACTIVE MODEL-BASED PROCESSING SOFTWARE PACKAGE  647

C.1 Introduction  / 647
C.2 Supervisor  / 648
C.3 Preprocessor  / 649
C.4 Postprocessor  / 650
C.5 Algorithms  / 650
C.6 Availability  / 653
   References  / 653

Index  655
This text develops the “model-based approach” to signal processing for a variety of useful model-sets, including what has become popularly termed “physics-based” models. It presents a unique viewpoint of signal processing from the model-based perspective. Although designed primarily as a graduate text, it will prove useful to practicing signal processing professionals and scientists, since a wide variety of case studies are included to demonstrate the applicability of the model-based approach to real-world problems. The prerequisite for such a text is a melding of undergraduate work in linear algebra, random processes, linear systems, and digital signal processing. It is somewhat unique in the sense that many texts cover some of its topics in piecemeal fashion. The underlying model-based approach of this text is uniformly developed and followed throughout in the algorithms, examples, applications, and case studies. It is the model-based theme, together with the developed hierarchy of physics-based models, that contributes to its uniqueness. This text has evolved from two previous texts, Candy ([1], [2]) and has been broadened by a wealth of practical applications to real-world model-based problems.

The place of such a text in the signal processing textbook community can best be explained by tracing the technical ingredients that form its contents. It can be argued that it evolves from the digital signal processing area, primarily from those texts that deal with random or statistical signal processing or possibly more succinctly “signals contaminated with noise.” The texts by Kay ([3], [4], [5]), Therrien [6], and Brown [7] provide the basic background information in much more detail than this text, so there is little overlap with them.

This text additionally prepares the advanced senior or graduate student with enough theory to develop a fundamental basis and go onto more rigorous texts like Jazwinski [8], Sage [9], Gelb [10], Anderson [11], Maybeck [12], Bozic [13],
Kailath [14], and more recently, Mendel [15], Grewel [16], and Bar-Shalom [17]. These texts are rigorous and tend to focus on Kalman filtering techniques, ranging from continuous to discrete with a wealth of detail in all of their variations. The model-based approach discussed in this text certainly includes the state-space models as one of its model classes (probably the most versatile), but the emphasis is on various classes of models and how they may be used to solve a wide variety of signal processing problems. Some more recent texts of about the same technical level, but again, with a different focus, are Widrow [18], Orfanidis [19], Sharf [20], Haykin [21], Hayes [22], Brown [7], and Stoica [23]. Again, the focus of these texts is not the model-based approach but rather a narrow set of specific models and the development of a variety of algorithms to estimate them. The system identification literature and texts therein also provide some overlap with this text, but the approach is again focused on estimating a model from noisy data sets and is not really aimed at developing a model-based solution to a particular signal processing problem. The texts in this area are Ljung ([24], [25]), Goodwin [26], Norton [27] and Soderstrom [28].

The approach we take is to introduce the basic idea of model-based signal processing (MBSP) and show where it fits in terms of signal processing. It is argued that MBSP is a natural way to solve basic processing problems. The more a priori information we know about data and its evolution, the more information we can incorporate into the processor in the form of mathematical models to improve its overall performance. This is the theme and structure that echoes throughout the text. Current applications (e.g., structures, tracking, equalization, and biomedical) and simple examples to motivate the organization of the text are discussed. Next, in Chapter 2, the “basics” of stochastic signals and systems are discussed, and a suite of models to be investigated in the text, going from simple time series models to state-space and wave-type models, is introduced. The state-space models are discussed in more detail because of their general connection to “physical models” and their availability limited to control and estimation texts rather than the usual signal processing texts. Examples are discussed to motivate all the models and prepare the reader for further developments in subsequent chapters. In Chapter 3, the basic estimation theory required to comprehend the model-based schemes that follow are developed establishing the groundwork for performance analysis (bias, error variance, Cramer-Rao bound, etc.). The remaining chapters then develop the model-based processors for various model sets with real-world-type problems discussed in the individual case studies and examples. Chapter 4 develops the model-based scheme for the popular model sets (AR, MA, ARMA, etc.) abundant in the signal processing literature and texts today, following the model-based approach outlined in the first chapter and presenting the unified framework for the algorithms and solutions. Highlights of this chapter include the real-world case studies as well as the “minimum variance” approach to processor design along with accompanying performance analysis. Next we begin to lay the foundation of the “physics-based” processing approach by developing the linear state-space, Gauss-Markov model-set, leading to the well-known Kalman filter solution in Chapter 5. The Kalman filter is developed from the innovations viewpoint with its optimality properties
analyzed within. The solution to the minimum variance design is discussed (tuned
filter) along with a “practical” cookbook approach (validated by theory). Next
critical special extensions of the linear filter are discussed along with a suite of
solutions to various popular signal processing problems (identification, deconvolu-
tion/bias estimation, etc.). Here the true power of the model-based approach using
state-space models is revealed and developed for difficult problems that are easily
handled within this framework. A highlight of the chapter is a detailed processor
design for a storage tank, unveiling all of the steps required to achieve a minimum
variance design. Chapter 6 extends these results even further to the case of nonlinear
state-space models. Theoretically each processor is developed in a logical fashion
leading to some of the more popular structures with example problems throughout.
This chapter ends with a case study of tracking the motion of a super tanker during
docking. Next the adaptive version of the previous algorithms is developed, again,
within the model-based framework. Here many interesting and exciting examples
and applications are presented along with some detailed case studies demonstrat-
ing their capability when applied to real-world problems. Here, in Chapter 7, we
continue with the basic signal processing models and apply them to a suite of
applications. Next, in Chapter 8, we extend the state-space model sets (linear and
nonlinear) to the adaptive regime. We develop the adaptive Kalman-type filters
and apply them to a real-world ocean acoustic application (case study). Finally,
in Chapter 9, we develop a suite of physics-based models ranging from reentry
vehicle dynamics (ARMAX), to nondestructive evaluation using laser ultrasound
(FIR), to a suite of state-space models for vibrating structures, ocean acoustics,
dispersive waves, and distributed groundwater flow. In each case the processor
along with accompanying simulations is discussed and applied to various data sets,
demonstrating the applicability and power of the model-based approach.

In closing, we must mention some of the new and exciting work currently being
performing in nonlinear estimation. Specifically, these are the unscented Kalman
filter [29] (Chapter 6), which essentially transforms the nonlinear problem into
an alternate space without linearization (and its detrimental effects) to enhance
performance, and the particle filter, which uses probabilistic sampling-resampling
theory (Markov chain/Monte Carlo methods) (MCMC) to handle the non-gaussian
type problems. Both approaches are opening new avenues of thought in estimation
that has been stagnant since the 1970s. These approaches have evolved because of
the computer power (especially the MCMC techniques) now becoming available
([29], [30]).

JAMES V. CANDY
Danville, CA
REFERENCES


ACKNOWLEDGMENTS

My beautiful wife, Patricia, and children, Kirstin and Christopher, have shown extraordinary support and patience during the writing of this text. I would like to thank Dr. Simon Haykin whose kindness and gentle nudging helped me achieve the most difficult “first step” in putting together this concept. My colleagues, especially Drs. Edmund Sullivan and David Chambers, have always been inspirational sources of motivation and support in discussing many ideas and concepts that have gone into the creation of this text. The deep questions by UCSB student, Andrew Brown, led to many useful insights during this writing. God bless you all.

JVC
INTRODUCTION

1.1 BACKGROUND

Perhaps the best way to start a text such as this is through an example that will provide the basis for this discussion and motivate the subsequent presentation. The processing of noisy measurements is performed with one goal in mind—to extract the desired information and reject the extraneous [1]. In many cases this is easier said than done. The first step, of course, is to determine what, in fact, is the desired information, and typically this is not the task of the signal processor but that of the phenomenologist performing the study. In our case we assume that the investigation is to extract information stemming from measured data. Many applications can be very complex, for instance, in the case of waves propagating through various media such as below the surface of the earth [2] or through tissue in biomedical [3] or through heterogeneous materials of critical parts in nondestructive evaluation (NDE) investigations [4]. In any case, the processing typically involves manipulating the measured data to extract the desired information, such as location of an epicenter, or the detection of a tumor or flaw in both biomedical and NDE applications.

Another view of the underlying processing problem is to decompose it into a set of steps that capture the strategic essence of the processing scheme. Inherently, we believe that the more “a priori” knowledge about the measurement and its underlying phenomenology we can incorporate into the processor, the better we can expect the processor to perform—as long as the information that is included is correct! One strategy called the model-based approach provides the essence of model-based
signal processing [1]. Some believe that all signal processing schemes can be cast into this generic framework. Simply, the model-based approach is “incorporating mathematical models of both physical phenomenology and the measurement process (including noise) into the processor to extract the desired information.” This approach provides a mechanism to incorporate knowledge of the underlying physics or dynamics in the form of mathematical process models along with measurement system models and accompanying noise as well as model uncertainties directly into the resulting processor. In this way the model-based processor enables the interpretation of results directly in terms of the problem physics. The model-based processor is actually a modeler’s tool enabling the incorporation of any a priori information about the problem to extract the desired information. As depicted in Figure 1.1, the fidelity of the model incorporated into the processor determines the complexity of the model-based processor with the ultimate goal of increasing the inherent signal-to-noise ratio (SNR). These models can range from simple, implicit, nonphysical representations of the measurement data such as the Fourier or wavelet transforms to parametric black-box models used for data prediction, to lumped mathematical representations characterized by ordinary differential equations, to distributed representations characterized by partial differential equation models to capture the underlying physics of the process under investigation. The dominating factor of which model is the most appropriate is usually determined by how severe the measurements are contaminated with noise and the underlying uncertainties. If the SNR of the measurements is high, then simple nonphysical techniques can be used to extract the desired information; however, for low SNR measurements more and more of the physics and instrumentation must be incorporated for the extraction.

This approach of selecting the appropriate processing technique is pictorially shown in Figure 1.1. Here we note that as we progress up the modeling steps...
to increase SNR, the model and algorithm complexity increases proportionally to achieve the desired results. Examining each of the steps individually leads us to realize that the lowest step involving no explicit model (simple) essentially incorporates little a priori information; it is used to analyze the information content (spectrum, time-frequency, etc.) of the raw measurement data to attempt to draw some rough conclusions about the nature of the signals under investigation. Examples of these simple techniques include Fourier transforms and wavelet transforms, among others. Progressing up to the next step, we have black-box models that are basically used as data prediction mechanisms. They have a parametric form (polynomial, transfer function, etc.), but again there is little physical information that can be gleaned from their outputs. At the next step, the gray-box models evolve that can use the underlying black-box structures; however, now the parameters can be used extract limited physical information from the data. For instance, a black-box transfer function model fit to the data yields coefficient polynomials that can be factored to extract resonance frequencies and damping coefficients characterizing the overall system response being measured. Progressing farther up the steps, we finally reach the true model-based techniques that explicitly incorporate the process physics using a lumped physical model structure usually characterized by ordinary differential or difference equations. The top step leads us to processes that are captured by distributed physical model structures in the form of partial differential equations. This level is clearly the most complex, since much computer power is devoted to solving the physical propagation problem. So we see that model-based signal processing offers the ability to operate directly in the physical space of the phenomenologist with the additional convenience of providing a one-to-one correspondence between the underlying phenomenology and the model embedded in the processor. This text is concerned with the various types of model-based processors that can be developed based on the model set selected to capture the physics of the measured data and the inherent computational algorithms that evolve. Before we proceed any further, let us consider a simple example to understand these concepts and their relationship to the techniques that evolve. In the subsequent section we will go even further to demonstrate how an explicit model-based solution can be applied to solve a wave-type processing problem.

However, before we begin our journey, let us look at a relatively simple example to motivate the idea of incorporating a model into a signal processing scheme. Suppose that we have a measurement of a sinusoid at 10 Hz in random noise and we would like to extract this sinusoid (the information) as shown in Figure 1.2a. The data are noisy as characterized by the dotted line with the true deterministic sinusoidal signal (solid line) overlayed in the figure. Our first attempt to analyze the raw measurement data is to take its Fourier transform (implicit sinusoidal model) and examine the resulting frequency bands for spectral content. The result is shown in Figure 1.2b, where we basically observe a noisy spectrum and a set of potential resonances—but nothing absolutely conclusive. After recognizing that the data are noisy, we apply a classical power spectral estimator using an inherent black-box model (implicit all-zero transfer function or polynomial model) with the resulting spectrum shown in Figure 1.2c. Here we note that the resonances have clearly
been enhanced (smoothed) and the noise spectra attenuated by the processor, but their still remains a significant amount of uncertainty in the spectrum. However, observing the resonances in the power spectrum, we proceed next to a gray-box model explicitly incorporating a polynomial model that can be solved to extract the resonances (roots) even further, as shown in Figure 1.2d. Next, we use this extracted model to develop an explicit model-based processor (MBP). At this point we know from the analysis that the problem is a sinusoid in noise, and we design a processor incorporating this a priori information. Noting the sharpness of the peak, we may incorporate a harmonic model that captures the sinusoidal nature of the resonance but does not explicitly capture the uncertainties created by the measurement instrumentation and noise into the processing scheme. The results are shown in Figure 1.2e. Finally, we develop a set of harmonic equations for a sinusoid (ordinary differential equations) in noise as well as the measurement instrumentation model and noise statistics to construct a MBP. The results are shown in Figure 1.2f, clearly demonstrating the superiority of the model-based approach,

Figure 1.2. Sinusoid in noise example. (a) Noisy data and 10Hz sinusoid. (b) Fourier spectrum. (c) Black-box power spectrum. (d) Gray-box polynomial power spectrum. (e) Gray-box harmonic power spectrum. (f) Model-based power spectrum.
when the embedded models are correct. The point of this example is to demonstrate that progressing up the steps incorporating more and more sophisticated models, we can enhance the SNR and extract the desired information.

1.2 SIGNAL ESTIMATION

If a measured signal is free from extraneous variations and is repeatable from measurement to measurement, then it is defined as a deterministic signal. However, if it varies extraneously and is no longer repeatable, then it is a random signal. This section is concerned with the development of processing techniques to extract pertinent information from random signals utilizing any a priori information available. We call these techniques signal estimation or signal enhancement, and we call a particular algorithm a signal estimator or just estimator. Symbolically, we use the caret (\(^\hat{\cdot}\)) notation throughout this text to annotate an estimate (e.g., \(s \rightarrow \hat{s}\)). Sometimes estimators are called filters (e.g., Wiener filter) because they perform the same function as a deterministic (signal) filter except for the fact that the signals are random; that is, they remove unwanted disturbances. Noisy measurements are processed by the estimator to produce “filtered” data.

Estimation can be thought of as a procedure made up of three primary parts:

- Criterion function
- Models
- Algorithm

The criterion function can take many forms and can also be classified as deterministic or stochastic. Models represent a broad class of information, formalizing the a priori knowledge about the process generating the signal, measurement instrumentation, noise characterization, underlying probabilistic structure, and so forth as discussed in the previous section. Finally, the algorithm or technique chosen to minimize (or maximize) the criterion can take many different forms depending on (1) the models, (2) the criterion, and (3) the choice of solution. For example, one may choose to solve the well-known least-squares problem recursively or with a numerical-optimization algorithm. Another important aspect of most estimation algorithms is that they provide a “measure of quality” of the estimator. Usually what this means is that the estimator also predicts vital statistical information about how well it is performing.

Intuitively, we can think of the estimation procedure as the

- Specification of a criterion
- Selection of models from a priori knowledge
- Development and implementation of an algorithm

Criterion functions are usually selected on the basis of information that is meaningful about the process or the ease with which an estimator can be developed.
Criterion functions that are useful in estimation can be classified as deterministic and probabilistic. Some typical functions are as follows:

- **Deterministic**
  - Squared error
  - Absolute error
  - Integral absolute error
  - Integral squared error

- **Probabilistic**
  - Maximum likelihood
  - Maximum a posteriori (Bayesian)
  - Maximum entropy
  - Minimum (error) variance

Models can also be deterministic as well as probabilistic; however, here we prefer to limit their basis to knowledge of the process phenomenology (physics) and the underlying probability density functions as well as the necessary statistics to describe the functions. Phenomenological models fall into the usual classes defined by the type of underlying mathematical equations and their structure, namely linear or nonlinear, differential or difference, ordinary or partial, time invariant or varying. Usually these models evolve to a stochastic model by the inclusion of uncertainty or noise processes.

Finally, the estimation algorithm can evolve from various influences. A preconceived notion of the structure of the estimator heavily influences the resulting algorithm. We may choose, based on computational considerations, to calculate an estimate recursively rather than as a result of a batch process because we require an online, pseudo-real-time estimate. Also each algorithm must provide a measure of estimation quality, usually in terms of the expected estimation error. This measure provides a means for comparing estimators. Thus the estimation procedure is a combination of these three major ingredients: criterion, models, and algorithm. The development of a particular algorithm is an interaction of selecting the appropriate criterion and models, as depicted in Figure 1.3.

Conceptually, this completes the discussion of the general estimation procedure. Many estimation techniques have been developed independently from various viewpoints (optimization, probabilistic) and have been shown to be equivalent. In most cases, it is easy to show that they can be formulated in this framework. Perhaps it is more appropriate to call the processing discussed in this chapter “model based” to differentiate it somewhat from pure statistical techniques. There are many different forms of model-based processors depending on the models used and the manner in which the estimates are calculated. For example, there are process model-based processors (Kalman filters [5], [6], [7]), statistical model-based processors (Box-Jenkins filters [8], Bayesian filters [1], [9]), statistical model-based processors (covariance filters [10]), or even optimization-based processors (gradient filters [11], [12]).