



Formulas and Calculations for Drilling Operations

G. Robello Samuel



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...An elegant Euler's formula wrapped with imaginary and real numbers resulting in nothing depicts the relationship between the Creator and human intellect...

$$e^{i\pi} + 1 = 0$$

To

Cynthia, Nishanth and Sharon

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Contents

Preface	xv
1 Basic Calculations	1
1.1 Capacities	1
1.2 Displacement	2
1.2.1 Displacement of the Pipe Based on the Thickness of the Pipe	2
1.3 Buoyancy, Buoyed Weight, and Buoyancy Factor (BF)	3
1.4 Effective Weight	4
1.5 Modulus of Elasticity	8
1.6 Poisson's Ratio	9
1.7 Minimum Yield Strength	9
1.8 Ultimate Tensile Strength	10
1.9 Fatigue Endurance Limit	10
1.10 Twist	11
1.11 Composite Materials	14
1.12 Friction	15
1.12.1 Coefficient of Friction	15
1.12.2 Types of Friction	16
1.12.3 Friction and Rotational Speed	18
1.13 Gauge and Absolute Pressures	19
1.13.1 Hydrostatic Pressure	20
1.13.2 Mud Gradient	20
1.13.3 Measurement of Pressure	22
1.14 Temperature	23
1.15 Horsepower	25
1.16 Flow Velocity	27

2	Rig Equipment	29
2.1	Overall Efficiency of Engines	29
2.2	Energy Transfer	30
2.3	Blocks and Drilling Line	33
2.4	Derrick Load	34
2.4.1	Block Efficiency Factor	35
2.4.2	Block Line Strength	35
2.5	Ton-Miles (TM) Calculations	44
2.5.1	Drilling Ton-Miles Calculations	44
2.5.2	Coring Ton-Miles Calculations	44
2.5.3	Casing Ton-Miles Calculations	45
2.6	Crown Block Capacity	48
2.7	Line Pull Efficiency Factor	49
2.8	Rotary Power	50
2.9	Mud Pumps	56
2.9.1	Volumetric Efficiency	56
2.9.2	Pump Factor	57
2.10	Energy Transfer	58
2.11	Offshore Vessels	72
2.11.1	Environmental Forces	73
2.11.2	Riser Angle	76
3	Well Path Design	79
3.1	Average Curvature – Average Dogleg Severity (DLS)	79
3.2	Vertical and Horizontal Curvatures	80
3.3	Borehole Curvature	80
3.3.1	Borehole Radius of Curvature	81
3.4	Bending Angle	82
3.5	Tool Face Angle	82
3.6	Borehole Torsion	86
3.6.1	Borehole Torsion – Cylindrical Helical Method	86
3.7	Wellpath Length Calculations	87
3.7.1	Wellpath Trajectory Calculations from Survey Data	88
3.7.1.1	Minimum Curvature Method	88
3.7.1.2	Radius of Curvature Method	89

3.7.2	Natural Curve Method	92
3.7.3	Constant Tool Face Angle Method	92
3.8	Types Of Designs	98
3.9	Tool Face Angle Change	99
3.10	Horizontal Displacement	103
3.11	Tortuosity	105
3.11.1	Absolute and Relative Tortuosity	105
3.11.2	Sine Wave Method	106
3.11.3	Helical Method	107
3.11.4	Random Inclination Azimuth Method	107
3.11.5	Random Inclination Dependent Azimuth Method	108
3.12	Well Profile Energy	109
3.13	Magnetic Reference and Interference	110
3.14	Wellbore Trajectory Uncertainty	112
4	Fluids	119
4.1	Equivalent Mud Weight	119
4.2	Mud Weighting	121
4.3	Common Weighting Materials	123
4.4	Diluting Mud	127
4.5	Base Fluid – Water-Oil Ratios	142
4.6	Fluid Loss	145
4.7	Acidity–Alkalinity	147
4.8	Marsh Funnel	149
4.9	Mud Rheology	150
4.10	Plastic Viscosity, Yield Point and Zero-Sec-Gel	152
4.10.1	Bingham Plastic Model	152
4.10.2	Shear Stress and Shear Rate	152
4.10.3	Power Law	153
5	Hydraulics	159
5.1	Equivalent Mud Weight	159
5.2	Equivalent Circulating Density	160
5.3	Hydraulics: Basic Calculations	161
5.3.1	Critical Velocity	161
5.3.2	Pump Calculations	162

x CONTENTS

5.4	Bit Hydraulics	165
5.4.1	Basic Calculations	165
5.4.2	Optimization Calculations	167
5.4.2.1	Limitation 1 – Available Pump Horsepower	168
5.4.2.2	Limitation 2 – Surface Operating Pressure	168
5.5	Bingham Plastic Model	177
5.5.1	Reynolds Number	177
5.6	Power Law Model	183
5.7	Gel Breaking Pressure	196
5.8	Hole Cleaning – Cuttings Transport	197
5.9	Transport Velocity	198
6	Tubular Mechanics	205
6.1	Drill Collar Length	205
6.2	Bending Stress Ratio (BSR)	207
6.3	Pipe Wall Thickness	207
6.4	Resonant Frequency	209
6.5	Tensions	209
6.6	Drag Force	210
6.7	Side Force Calculation	211
6.8	Torque and Makeup Torque	213
6.9	Buckling	215
6.9.1	Buckling Criteria	215
6.10	Maximum Permissible Dogleg	217
6.11	Length Change Calculations	218
6.11.1	Stretch Due to Axial Load	218
6.11.2	Stretch Due to the Pressure Effect (Ballooning)	218
6.11.3	Stretch Due to Buckling	219
6.11.4	Stretch Due to Temperature	220
6.12	Stresses	221
6.12.1	Radial Stress	221
6.12.2	Hoop Stress (Tangential or Circumferential Stress)	222
6.12.3	Axial Stress	223
6.12.4	Bending Stress with Hole Curvature	224

6.12.5	Bending Stress with Hole Curvature, Pipe Curvature, and Tensile Force	228
6.12.6	Torsional or Twisting Shear Stress	230
6.12.7	Transverse Shear Stress	230
6.12.8	von Mises Stress	233
6.12.9	Stress Ratio	234
6.13	Fatigue Ratio	238
6.14	Bending Stress Magnification Factor	239
6.14.1	BSMF for Tensile Force	239
6.14.2	BSMF for Compressive Force	241
6.15	Slip Crushing	245
6.16	Cumulative Fatigue Calculation	247
7	Drilling Tools	253
7.1	Stretch Calculations	253
7.2	Backoff Calculations	254
7.3	Overpull/Slack-Off Calculations	257
7.4	Motor Calculations	259
7.4.1	Type I Motor	260
7.4.2	Type II Motor	261
7.4.3	Type III Motor	262
7.4.4	Type IV Motor	263
7.5	Stabilizer Calculations	265
7.5.1	Stabilizer Jamming Angle	265
7.5.2	Alignment Angle of Stabilizers with the Wellbore	266
7.6	Percussion Hammer	270
7.7	Positive Displacement Motor (PDM)	271
7.8	Rotor Nozzle Sizing	274
7.9	Downhole Turbine	276
7.10	Jar Calculations	279
7.10.1	Force Calculations for up Jars	279
7.10.2	Force Calculations For Down Jars	279
7.11	Specific Energy	282
8	Pore Pressure and Fracture Gradient	287
8.1	Formation Pressure	287
8.1.1	The Hubert and Willis Method	287

xii CONTENTS

8.1.2	Matthews and Kelly's Correlation	288
8.1.3	Eaton's Method	290
8.1.4	Christman's Method	291
8.2	Leak-off Pressure	296
9	Well Control	301
9.1	Kill Mud Weight	301
9.2	The Length and Density of the Kick	303
9.2.1	Type of Kick	303
9.2.2	Kick Classification	304
9.2.3	Kick Tolerance	305
9.3	Hydrostatic Pressure due to the Gas Column	307
9.4	Leak-off Pressure	307
9.5	Maximum Allowable Annular Surface Pressure (MAASP)	309
9.6	Accumulators	310
9.7	Driller's Method Operational Procedure	312
9.8	Kill Methods	315
9.9	The Riser Margin	316
10	Drilling Problems	317
10.1	Stuck Point Calculations	317
10.2	Differential Sticking Force	321
10.2.1	Method 1	322
10.2.2	Method 2	322
10.2.3	Method 3	324
10.3	Spotting Fluid Requirements	327
10.4	Loss Circulation	328
10.5	Increased ECD Due to Cuttings	330
10.6	Mud Weight Increase Due to Cuttings	331
10.7	Hole Cleaning – Slip Velocity Calculations	333
10.7.1	The Chien Correlation	333
10.7.2	The Moore Correlation	334
10.7.3	The Walker Mays Correlation	335
10.8	Transport Velocity and Transport Ratio	335
10.9	Keyseating	339

11 Cementing	341
11.1 Cement Slurry Requirements	341
11.2 Yield of Cement	341
11.3 Slurry Density	342
11.4 Hydrostatic Pressure Reduction	342
11.5 Contact Time	342
11.6 Gas Migration Potential	347
11.7 Cement Plug	350
12 Well Cost	355
12.1 Drilling Costs	355
12.1.1 Cost Per Foot	355
12.1.2 Coring Costs	360
12.2 Future Value (FV)	361
12.3 Expected Value (EV)	362
12.4 Price Elasticity	362
12.4.1 Ranges of Elasticity	363
Appendix: Useful Conversion Factors	365
Bibliography	371
Index	377
About the Author	387

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Preface

This book is an introductory exposition for drilling engineers, students, lecturers, teachers, software programmers, testers, and researchers. The intent is to provide basic equations and formulas with the calculations for downhole drilling. This book may be a tutorial guide for students, to lecturers and teachers it may be a solution manual, and drilling engineers may find that it is a source for solving problems. Software programmers and testers may use it as a guide as they code, unit test, and validate their implementation, and researchers may use it as a source for further development. Of course, it is very difficult to cover all the aspects and areas of drilling, but this book aims to provide an introduction to exploring the vastness and complexity of drilling engineering. The readers are advised to refer to the books in the bibliography for more details regarding underlying theory. This book is a companion to my other books, *Drilling Engineering*, *Downhole Drilling Tools*, *Advanced Drilling Engineering*, and the upcoming *Applied Drilling Engineering Optimization*.

I am grateful to the contributors, the publisher, Phil Carmical, and copyeditor Brittyne Jackson and Mohana Sundaram from Exeter Premedia Services. Also, I thank Dr.João Carlos Plácido and Dr.Dali Gao for helping in formulating some problems. I thank them for their invaluable help. A work of this magnitude with many equations and numbers is bound to have errors even though painstaking efforts have been taken. Needless to say, I request that the readers send errors and comments in effort towards the improvement of this book.

A handwritten signature in black ink, appearing to read 'Abello' followed by a stylized flourish.

Houston, Texas

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1

Basic Calculations

This chapter focuses on different basic calculations such as buoyancy, weight, tension, etc.

1.1 Capacities

Capacities of the pipe, annular capacity, and annular volume can be calculated using the following equations.

The linear capacity of the pipe is

$$C_i = \frac{A_i}{808.5} \text{ bbl/ft}, \quad (1.1)$$

where A_i is a cross-sectional area of the inside pipe in square inches and equals $0.7854 \times D_i^2$, and D_i is the inside diameter of the pipe in inches.

Volume capacity is

$$V = C_i \times L \text{ bbl}, \quad (1.2)$$

where L = the length of the pipe, ft.

2 FORMULAS AND CALCULATIONS FOR DRILLING OPERATIONS

Annular linear capacity against the pipe is

$$C_o = \frac{A_o}{808.5} \text{ bbl/ft}, \quad (1.3)$$

where A_o , a cross-sectional area of the annulus in square inches, is

$$0.7854 \times (D_h^2 - D_o^2), \quad (1.4)$$

D_o = the outside side diameter of the pipe, in., and D_h = the diameter of the hole or the inside diameter of the casing against the pipe, in.

Annular volume capacity is

$$V = C_o \times L \text{ bbl}. \quad (1.5)$$

1.2 Displacement

1.2.1 Displacement of the Pipe Based on the Thickness of the Pipe

Open-ended displacement volume of the pipe is

$$V_o = \frac{0.7854(D_o^2 - D_i^2)}{808.5} \text{ bbl/ft}. \quad (1.6)$$

$$\text{Displacement volume} = V_o \times L \text{ bbl}. \quad (1.7)$$

Close-ended displacement volume of the pipe is

$$V_c = \frac{0.7854(D_o^2)}{808.5} \text{ bbl/ft}. \quad (1.8)$$

$$\text{Displacement volume} = V_c \times L \text{ bbl}. \quad (1.9)$$

Problem 1.1

Calculate the drill pipe capacity, open-end displacement, closed end displacement, annular volume, and total volume for the following condition: 5,000 feet of 5" drill pipe with an inside diameter of 4.276" inside a hole of 8½".

Solution:

Linear capacity of pipe, using equation 1.1, is

$$C_i = \frac{A_i}{808.5} = \frac{0.7854 \times D_i^2}{808.5} = \frac{0.7854 \times 4.276^2}{808.5} = 0.017762 \text{ bbl/ft.}$$

Pipe volume capacity = $0.017762 \times 5000 = 88.81$ bbl.

Open-end displacement of pipe, using equation 1.6, is

$$V_o = \frac{0.7854 (D_o^2 - D_i^2)}{808.5} = \frac{0.7854 (5^2 - 4.276^2)}{808.5} = 0.006524 \text{ bbl/ft.}$$

Close-end displacement of pipe, using equation 1.8 is

$$V_c = \frac{0.7854 (D_o^2)}{808.5} = \frac{0.7854 (5^2)}{808.5} = 0.024286 \text{ bbl/ft.}$$

Annular volume, using equation 1.5 is

$$\begin{aligned} V &= C_o \times L = \frac{A_o}{808.5} \times L = \frac{0.7854}{808.5} \times (D_h^2 - D_o^2) \times L \\ &= \frac{0.7854}{808.5} \times (8.5^2 - 5^2) \times 5000 = 229.5 \text{ bbl.} \end{aligned}$$

Total volume = Pipe volume + Annular volume = $88.81 + 229.50 = 318.31$ bbl.

1.3 Buoyancy, Buoyed Weight, and Buoyancy Factor (BF)

The calculations are based on one fluid.

4 FORMULAS AND CALCULATIONS FOR DRILLING OPERATIONS

$$\text{Buoyancy} = \frac{\text{Weight of material in air}}{\text{Density of material}} \times \text{Fluid density.} \quad (1.10)$$

$$\text{Buoyed weight} = \left(\frac{\text{Density of material} - \text{Fluid density}}{\text{Density of material}} \right) \times \text{Weight of material in air.} \quad (1.11)$$

$$\text{Buoyancy factor} = \left(\frac{\text{Density of material} - \text{Fluid density}}{\text{Density of material}} \right). \quad (1.12)$$

$$\text{Buoyancy factor} = \left(\frac{\rho_s - \rho_m}{\rho_s} \right) = \left(1 - \frac{\rho_m}{\rho_s} \right), \quad (1.13)$$

where ρ_s is the density of the steel/material, and ρ_m is the density of the fluid/mud.

When the inside and outside fluid densities are different, the buoyancy factor can be given as

$$\text{Buoyancy factor (BF)} = \frac{A_o \left(1 - \frac{\rho_o}{\rho_s} \right) - A_i \left(1 - \frac{\rho_i}{\rho_s} \right)}{A_o - A_i}, \quad (1.14)$$

where A_o is the external area of the component, and A_i is the internal area of the component.

1.4 Effective Weight

Effective weight per unit length can be calculated using the following relation. Weight per foot in drilling mud is the

weight per foot in air minus the weight per foot of the displaced drilling mud:

$$w_B = w_s + \rho_i A_i - \rho_o A_o, \quad (1.15)$$

$$A_o = \frac{\pi}{4} (0.95 \times D_o^2 + 0.05 \times D_{oj}^2), \quad (1.16)$$

$$A_i = \frac{\pi}{4} (0.95 \times D_i^2 + 0.05 \times D_{ij}^2). \quad (1.17)$$

Without tool joints, $A_i = 0.7854 \times D_i^2$, and $A_o = 0.7854 \times D_o^2$.

Using equation 1.15, $w_B = w_s + \rho_i A_i - \rho_o A_o$.

In the above equation, unit weight of the steel can be given as

$$w_s = \rho_s A_s. \quad (1.18)$$

When the inside and outside fluid densities are the same,

$$w_B = A_s (\rho_s - \rho_o) = A_s \rho_s \left(1 - \frac{\rho_o}{\rho_s} \right) = w_s \left(1 - \frac{\rho_o}{\rho_s} \right), \quad (1.19)$$

where $1 - (\rho_o / \rho_s)$ is the buoyancy factor, and where the following values are as follows:

- D_o = outside diameter of component body
- D_{oj} = outside diameter of tool joint
- D_i = inside diameter of component body
- D_{ij} = inside diameter of tool joint
- A_s = cross-sectional area of the steel/material
- ρ_o = annular mud weight at component depth in the wellbore
- ρ_i = internal mud weight at component depth inside the component
- ρ_s = density of the steel/material

Problem 1.2

Calculate the buoyancy factor and buoyed weight of 6,000 ft of 6 5/8" 27.7 ppf E grade drill pipe in mud of density 10 ppg.

Solution:

Using equation 1.13 and a steel density of 65.4 ppg,

$$\text{Buoyancy factor} = \left(1 - \frac{\rho_m}{\rho_s}\right) = \left(1 - \frac{10}{65.4}\right) = 0.847.$$

Buoyed weight can be calculated using equation 1.11:

$$\begin{aligned} \text{Buoyed weight} &= 0.847 \times 27.7 \times 6000 \\ &= 140771.4 \text{ lbf} = 140.8 \text{ kips.} \end{aligned}$$

Problem 1.3

Calculate the buoyed weight of 5,000 ft of 20" 106.5 ppf casing with drilling mud of density 9 ppg inside and 11 ppg cement outside the casing. Also, estimate the buoyed weight of the casing with the same drilling fluid inside and outside before pumping cement. Neglect the tool joint effects.

Solution:

After pumping cement with full cement behind the casing, the inside diameter of the casing is 18.98 in.

Using equation 1.14 and a steel density of 65.4 ppg,

$$\begin{aligned} \text{BF} &= \frac{A_o \left(1 - \frac{\rho_o}{\rho_s}\right) - A_i \left(1 - \frac{\rho_i}{\rho_s}\right)}{A_o - A_i} \\ &= \frac{0.7854 \times 20^2 \left(1 - \frac{11}{65.4}\right) - 0.7854 \times 19^2 \left(1 - \frac{9}{65.4}\right)}{0.7854 \times 20^2 - 0.7854 \times 18.98^2} = 0.5382. \end{aligned}$$

Buoyed weight can be calculated using equation 1.11:

$$\begin{aligned}\text{Buoyed weight} &= 0.5382 \times 106.5 \times 5000 \\ &= 286618 \text{ lbf} = 287 \text{ kips.}\end{aligned}$$

Before pumping cement, the buoyed weight can be estimated using equation 1.13 and a steel density of 65.4 ppg:

$$\text{Buoyancy factor} = \left(1 - \frac{\rho_m}{\rho_s}\right) = \left(1 - \frac{9}{65.4}\right) = 0.8623.$$

Buoyed weight can be calculated using equation 1.11:

$$\begin{aligned}\text{Buoyed weight} &= 0.8623 \times 106.5 \times 5000 \\ &= 459220.2 \text{ lbf} = 459.2 \text{ kips.}\end{aligned}$$

Problem 1.4

Calculate the air weight, buoyed weight in drilling fluid, buoyed weight when cement is inside and drilling fluid is in the annulus, buoyed weight when cement is outside and drilling fluid is inside. Casing outside diameter is 9 5/8", casing inside diameter is 8.681", drilling fluid density is 10 ppg, cement slurry density is 12 ppg, and the depth of the well is 5,000 ft.

Solution:

$$\text{Air weight} = 47 \times 5000 = 235000 \text{ lbf} = 235 \text{ kips.}$$

$$\begin{aligned}\text{Buoyed weight with drilling fluid} &= \left(1 - \frac{10}{65.4}\right) \times 5000 \times 47 \\ &= 199067 \text{ lbf} = 199 \text{ kips.}\end{aligned}$$

8 FORMULAS AND CALCULATIONS FOR DRILLING OPERATIONS

Buoyed weight with cement inside and drilling fluid outside is

$$\begin{aligned} \text{BF} &= \frac{A_o \left(1 - \frac{\rho_o}{\rho_s}\right) - A_i \left(1 - \frac{\rho_i}{\rho_s}\right)}{A_o - A_i} \\ &= \frac{0.7854 \times 9.625^2 \left(1 - \frac{10}{65.4}\right) - 0.7854 \times 8.681^2 \left(1 - \frac{12}{65.4}\right)}{0.7854 \times 9.625^2 - 0.7854 \times 8.681^2} \\ &= 0.98, \\ &= 0.98 \times 5000 \times 47 = 230406 \text{ lbf} = 230 \text{ kips}. \end{aligned}$$

Buoyed weight with cement outside and drilling fluid inside is

$$\begin{aligned} \text{BF} &= \frac{A_o \left(1 - \frac{\rho_o}{\rho_s}\right) - A_i \left(1 - \frac{\rho_i}{\rho_s}\right)}{A_o - A_i} \\ &= \frac{0.7854 \times 9.625^2 \left(1 - \frac{12}{65.4}\right) - 0.7854 \times 8.681^2 \left(1 - \frac{10}{65.4}\right)}{0.7854 \times 9.625^2 - 0.7854 \times 8.681^2} \\ &= 0.6831, \\ &= 0.6831 \times 5000 \times 47 = 160541 \text{ lbf} = 160 \text{ kips}. \end{aligned}$$

1.5 Modulus of Elasticity

Modulus of elasticity is

$$E = \frac{\sigma}{\epsilon} = \frac{F/A}{\Delta L/L} \text{ psi}, \quad (1.20)$$

where σ = unit stress, psi, ϵ = unit strain in inch per inch, F = axial force, lbf, A = cross sectional area, in², ΔL = total strain or elongation, in., and L = original length, in.

1.6 Poisson's Ratio

$$\nu = \frac{\varepsilon_{lat}}{\varepsilon_{long}}, \quad (1.21)$$

where ε_{lat} = lateral strain in inches, and ε_{long} = longitudinal or axial strain in inches.

For most metals, Poisson's ratio varies from $\frac{1}{4}$ - $\frac{1}{3}$.

Modulus of elasticity and shear modulus are related to Poisson's ratio as follows:

$$E = 2G(1 + \nu). \quad (1.22)$$

Modulus of elasticity, shear modulus, and Poisson's ratio for common materials are given in Table 1.1.

1.7 Minimum Yield Strength

Yield strength is defined as the stress that will result in specific permanent deformation in the material. The yield strength can be conveniently determined from the stress strain diagram. Based on the test results, minimum and maximum yield strengths for the tubulars are specified.

Table 1.1 Modulus of elasticity, shear modulus and Poisson's ratio at room temperature.

Metal Alloy	Modulus of Elasticity		Shear Modulus		Poisson's Ratio
	Psi $\times 10^6$	MPa $\times 10^6$	Psi $\times 10^6$	MPa $\times 10^6$	
Aluminum	10	6.9	3.8	2.6	0.33
Copper	16	11	6.7	4.6	0.35
Steel	30	20.7	12	8.3	0.27
Titanium	15.5	10.7	6.5	4.5	0.36
Tungsten	59	40.7	23.2	16	0.28

1.8 Ultimate Tensile Strength

The ultimate tensile strength (UTS) of a material in tension, compression, or shear, respectively, is the maximum tensile, compressive, or shear stress resistance to fracture or rupture. It is equivalent to the maximum load that can be applied over the cross-sectional area on which the load is applied. The term can be modified as the ultimate tensile, compressive, or shearing strength. Ultimate tensile strength of few API pipes are shown in Table 1.2.

1.9 Fatigue Endurance Limit

The endurance limit pertains to the property of a material and is defined as the highest stress or range of cyclic stress that a material can be subjected to indefinitely without causing failure or fracture. In other words, the endurance limit is the maximum stress reversal that can be indefinitely subjected a large number of times without producing fracture. The magnitude of the endurance limit of a material is usually determined from a fatigue test that uses a sample piece of the material.

Table 1.2 API pipe properties.

API	Yield Stress, psi		Minimum Ultimate	Minimum
Grade	Minimum	Maximum	Tensile, psi	Elongation (%)
H-40	40,000	80,000	60,000	29.5
J-55	55,000	80,000	75,000	24
K-55	55,000	80,000	95,000	19.5
N-80	80,000	110,000	100,000	18.5
L-80	80,000	95,000	95,000	19.5
C-90	90,000	105,000	100,000	18.5
C-95	95,000	110,000	105,000	18.5
T-95	95,000	110,000	105,000	18
P-110	110,000	140,000	125,000	15
Q-125	125,000	150,000	135,000	18

1.10 Twist

When a rod is subjected to torque it undergoes twist, which is given as

$$\theta = \frac{TL}{GJ} \text{ radians,} \quad (1.23)$$

where θ = angle of twist (radians) (can be $>2\pi$), L = length of section, ft, T = torque, ft-lbf, and G = modulus of rigidity, psi.

$$G = \frac{E}{2(1+\nu)}, \quad (1.24)$$

$$J = \text{Polar moment of inertia (in.}^4\text{)} = \frac{\pi}{32} (D_o^4 - D_i^4), \quad (1.25)$$

and E = modulus of elasticity, psi, and ν = Poisson's ratio.

Problem 1.5

Consider a pipe with the following dimensions carrying an applied tensile load of 5,000 lbs at the bottom. Calculate the maximum stress in the string. The pipe outside diameter = 5 in, the pipe inside diameter = 4 in, the pipe density = 490 lb/ft³, and the pipe length = 30 ft.

Solution:

The cross sectional area of the pipe is

$$A = \frac{\pi}{4} (5^2 - 4^2) = 7.08 \text{ in}^2.$$

$$\text{Weight of the pipe} = \frac{\pi}{4} (5^2 - 4^2) \times 490 \times 30 \times 12 = 721.6 \text{ lbf.}$$

12 FORMULAS AND CALCULATIONS FOR DRILLING OPERATIONS

Total force acting at the top of the pipe:

$$F = \text{Weight of the pipe} + \text{Load applied},$$
$$F = 721.6 + 5000 = 5721.6 \text{ lbf.}$$

$$\text{Maximum stress at the top of the pipe} = \sigma = \frac{F}{A} = \frac{5721.6}{7.08}$$
$$= 809 \text{ psi.}$$

Problem 1.6

Calculate the elongation of a cylindrical pipe of 5" in outside diameter, 4.0 in inside diameter and 10,000 ft long when a tensile load of 20,000 lbf is applied. Assume that the deformation is totally elastic and modulus of elasticity = 30×10^6 psi.

Solution:

From equation $E = \frac{F/A}{\Delta L/L}$, the elongation can be written as follows:

$$\Delta L = \frac{F/A}{E/L} = \frac{L \times F}{E \times A} = \frac{L \times F}{E \times \frac{\pi}{4} (D_o^2 - D_i^2)} = \frac{4L \times F}{E \times \pi \times (D_o^2 - D_i^2)}$$

Substituting the values,

$$= \frac{4 \times 10000 \times 12 \times 20000}{30 \times 10^6 \times \pi \times (5^2 - 4^2)} = 11.32 \text{ in.}$$

Problem 1.7

A downhole tool with a length of 30 ft, an outside diameter of 5.5 in., and an inside diameter of 4.75 in. is compressed by an axial force of 30 kips. The material has a modulus of elasticity 30,000 ksi and Poisson's ratio 0.3. Assume the tool is in the elastic range.

Calculate the following:

- A. Shortening of tool
- B. Lateral strain