Formulas and Calculations for Drilling Operations

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...An elegant Euler's formula wrapped with imaginary and real numbers resulting in nothing depicts the relationship between the Creator and human intellect...

$$e^{i\pi} + 1 = 0$$

To

Cynthia, Nishanth and Sharon

Contents

Preface

This book is an introductory exposition for drilling engineers, students, lecturers, teachers, software programmers, testers, and researchers. The intent is to provide basic equations and formulas with the calculations for downhole drilling. This book may be a tutorial guide for students, to lecturers and teachers it may be a solution manual, and drilling engineers may find that it is a source for solving problems. Software programmers and testers may use it as a guide as they code, unit test, and validate their implementation, and researchers may use it as a source for further development. Of course, it is very difficult to cover all the aspects and areas of drilling, but this book aims to provide an introduction to exploring the vastness and complexity of drilling engineering. The readers are advised to refer to the books in the bibliography for more details regarding underlying theory. This book is a companion to my other books, *Drilling Engineering, Downhole Drilling Tools, Advanced Drilling Engineering,* and the upcoming *Applied Drilling Engineering Optimization.*

I am grateful to the contributors, the publisher, Phil Carmical, and copyeditor Brittyne Jackson and Mohana Sundaram from Exeter Premedia Services. Also, I thank Dr.João Carlos Plácido and Dr.Dali Gao for helping in formulating some problems. I thank them for their invaluable help. A work of this magnitude with many equations and numbers is bound to have errors even though painstaking efforts have been taken. Needless to say, I request that the readers send errors and comments in effort towards the improvement of this book.

Houston, Texas

1

Basic Calculations

This chapter focuses on different basic calculations such as buoyancy, weight, tension, etc.

1.1 Capacities

Capacities of the pipe, annular capacity, and annular volume can be calculated using the following equations.

The linear capacity of the pipe is

$$C_i = \frac{A_i}{808.5}\,\text{bbl/ft}, \tag{1.1}$$

where A_i is a cross-sectional area of the inside pipe in square inches and equals $0.7854 \times D_i^2$, and D_i is the inside diameter of the pipe in inches.

Volume capacity is

$$V = C_i \times L\ \text{bbl}, \tag{1.2}$$

where L = the length of the pipe, ft.

Annular linear capacity against the pipe is

$$C_o = \frac{A_o}{808.5} \text{ bbl/ft,} \tag{1.3}$$

where A_o, a cross-sectional area of the annulus in square inches, is

$$0.7854 \times \left(D_h^2 - D_o^2\right), \tag{1.4}$$

D_o = the outside side diameter of the pipe, in., and D_h = the diameter of the hole or the inside diameter of the casing against the pipe, in.

Annular volume capacity is

$$V = C_o \times L \text{ bbl.} \tag{1.5}$$

1.2 Displacement

1.2.1 Displacement of the Pipe Based on the Thickness of the Pipe

Open-ended displacement volume of the pipe is

$$V_o = \frac{0.7854\left(D_o^2 - D_i^2\right)}{808.5} \text{ bbl/ft.} \tag{1.6}$$

$$\text{Displacement volume} = V_o \times L \text{ bbl.} \tag{1.7}$$

Close-ended displacement volume of the pipe is

$$V_c = \frac{0.7854\left(D_o^2\right)}{808.5} \text{ bbl/ft.} \tag{1.8}$$

$$\text{Displacement volume} = V_c \times L \text{ bbl.} \tag{1.9}$$

Problem 1.1

Calculate the drill pipe capacity, open-end displacement, closed end displacement, annular volume, and total volume for the following condition: 5,000 feet of 5″ drill pipe with an inside diameter of 4.276″ inside a hole of 8½″.

Solution:

Linear capacity of pipe, using equation 1.1, is

$$C_i = \frac{A_i}{808.5} = \frac{0.7854 \times D_i^2}{808.5} = \frac{0.7854 \times 4.276^2}{808.5} = 0.017762 \text{ bbl/ft.}$$

Pipe volume capacity = $0.017762 \times 5000 = 88.81$ bbl.
Open-end displacement of pipe, using equation 1.6, is

$$V_o = \frac{0.7854\left(D_o^2 - D_i^2\right)}{808.5} = \frac{0.7854\left(5^2 - 4.276^2\right)}{808.5} = 0.006524 \text{ bbl/ft.}$$

Close-end displacement of pipe, using equation 1.8 is

$$V_c = \frac{0.7854\left(D_o^2\right)}{808.5} = \frac{0.7854\left(5^2\right)}{808.5} = 0.024286 \text{ bbl/ft.}$$

Annular volume, using equation 1.5 is

$$\begin{aligned} V &= C_o \times L = \frac{A_o}{808.5} \times L = \frac{0.7854}{808.5} \times \left(D_h^2 - D_o^2\right) \times L \\ &= \frac{0.7854}{808.5} \times \left(8.5^2 - 5^2\right) \times 5000 = 229.5 \text{ bbl.} \end{aligned}$$

Total volume = Pipe volume + Annular volume = 88.81 + 229.50 = 318.31 bbl.

1.3 Buoyancy, Buoyed Weight, and Buoyancy Factor (BF)

The calculations are based on one fluid.

$$\text{Buoyancy} = \frac{\text{Weight of material in air}}{\text{Density of material}} \times \text{Fluid density}. \tag{1.10}$$

$$\text{Buoyed weight} = \left(\frac{\text{Density of material} - \text{Fluid density}}{\text{Density of material}}\right) \times \text{Weight of material in air}. \tag{1.11}$$

$$\text{Buoyancy factor} = \left(\frac{\text{Density of material} - \text{Fluid density}}{\text{Density of material}}\right). \tag{1.12}$$

$$\text{Buoyancy factor} = \left(\frac{\rho_{\mathrm{s}} - \rho_{\mathrm{m}}}{\rho_{\mathrm{s}}}\right) = \left(1 - \frac{\rho_{\mathrm{m}}}{\rho_{\mathrm{s}}}\right), \tag{1.13}$$

where ρ_s is the density of the steel/material, and ρ_m is the density of the fluid/mud.

When the inside and outside fluid densities are different, the buoyancy factor can be given as

$$\text{Buoyancy factor (BF)} = \frac{A_o\left(1 - \frac{\rho_{\mathrm{o}}}{\rho_{\mathrm{s}}}\right) - A_i\left(1 - \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{s}}}\right)}{A_o - A_i}, \tag{1.14}$$

where A_o is the external area of the component, and A_i is the internal area of the component.

1.4 Effective Weight

Effective weight per unit length can be calculated using the following relation. Weight per foot in drilling mud is the weight per foot in air minus the weight per foot of the displaced drilling mud:

$$w_B = w_s + \rho_i A_i - \rho_o A_o, \tag{1.15}$$

$$A_o = \frac{\pi}{4}\left(0.95 \times D_o^2 + 0.05 \times D_{oj}^2\right), \tag{1.16}$$

$$A_i = \frac{\pi}{4}\left(0.95 \times D_i^2 + 0.05 \times D_{ij}^2\right). \tag{1.17}$$

Without tool joints, $A_i = 0.7854 \times D_i^2$, and $A_o = 0.7854 \times D_o^2$.
Using equation 1.15, $w_B = w_s + \rho_i A_i - \rho_o A_o$.
In the above equation, unit weight of the steel can be given as

$$w_s = \rho_s A_s. \tag{1.18}$$

When the inside and outside fluid densities are the same,

$$w_B = A_s(\rho_s - \rho_o) = A_s\rho_s\left(1 - \frac{\rho_o}{\rho_s}\right) = w_s\left(1 - \frac{\rho_o}{\rho_s}\right), \tag{1.19}$$

where $1 - (\rho_o/\rho_s)$ is the buoyancy factor, and where the following values are as follows:

- D_o = outside diameter of component body
- D_{oj} = outside diameter of tool joint
- D_i = inside diameter of component body
- D_{ij} = inside diameter of tool joint
- A_s = cross-sectional area of the steel/material
- ρ_o = annular mud weight at component depth in the wellbore
- ρ_i = internal mud weight at component depth inside the component
- ρ_s = density of the steel/material

Problem 1.2

Calculate the buoyancy factor and buoyed weight of 6,000 ft of 6 5/8″ 27.7 ppf E grade drill pipe in mud of density 10 ppg.

Solution:

Using equation 1.13 and a steel density of 65.4 ppg,

$$\text{Buoyancy factor} = \left(1 - \frac{\rho_m}{\rho_s}\right) = \left(1 - \frac{10}{65.4}\right) = 0.847.$$

Buoyed weight can be calculated using equation 1.11:

$$\begin{aligned}\text{Bouyed weight} &= 0.847 \times 27.7 \times 6000\\ &= 140771.4 \text{ lbf} = 140.8 \text{ kips.}\end{aligned}$$

Problem 1.3

Calculate the buoyed weight of 5,000 ft of 20″ 106.5 ppf casing with drilling mud of density 9 ppg inside and 11 ppg cement outside the casing. Also, estimate the buoyed weight of the casing with the same drilling fluid inside and outside before pumping cement. Neglect the tool joint effects.

Solution:

After pumping cement with full cement behind the casing, the inside diameter of the casing is 18.98 in.

Using equation 1.14 and a steel density of 65.4 ppg,

$$\text{BF} = \frac{A_o\left(1 - \frac{\rho_o}{\rho_s}\right) - A_i\left(1 - \frac{\rho_i}{\rho_s}\right)}{A_o - A_i}$$

$$= \frac{0.7854 \times 20^2\left(1 - \frac{11}{65.4}\right) - 0.7854 \times 19^2\left(1 - \frac{9}{65.4}\right)}{0.7854 \times 20^2 - 0.7854 \times 18.98^2} = 0.5382.$$

Buoyed weight can be calculated using equation 1.11:

$$\begin{aligned}\text{Buoyed weight} &= 0.5382\times 106.5\times 5000\\ &= 286618\ \text{lbf} = 287\ \text{kips.}\end{aligned}$$

Before pumping cement, the buoyed weight can be estimated using equation 1.13 and a steel density of 65.4 ppg:

$$\text{Buoyancy factor} = \left(1-\frac{\rho_\text{m}}{\rho_\text{s}}\right) = \left(1-\frac{9}{65.4}\right) = 0.8623.$$

Buoyed weight can be calculated using equation 1.11:

$$\begin{aligned}\text{Buoyed weight} &= 0.8623\times 106.5\times 5000\\ &= 459220.2\ \text{lbf} = 459.2\ \text{kips.}\end{aligned}$$

Problem 1.4

Calculate the air weight, buoyed weight in drilling fluid, buoyed weight when cement is inside and drilling fluid is in the annulus, buoyed weight when cement is outside and drilling fluid is inside. Casing outside diameter is 9 5/8″, casing inside diameter is 8.681″, drilling fluid density is 10 ppg, cement slurry density is 12 ppg, and the depth of the well is 5,000 ft.

Solution:

$$\text{Air weight} = 47\times 5000 = 235000\ \text{lbf} = 235\ \text{kips.}$$

$$\begin{aligned}\text{Buoyed weight with drilling fluid} &= \left(1-\frac{10}{65.4}\right)\times 5000\times 47\\ &= 199067\ \text{lbf} = 199\ \text{kips.}\end{aligned}$$

Buoyed weight with cement inside and drilling fluid outside is

$$\begin{aligned}\mathrm{BF} &= \frac{A_o\left(1-\dfrac{\rho_\mathrm{o}}{\rho_\mathrm{s}}\right)-A_i\left(1-\dfrac{\rho_\mathrm{i}}{\rho_\mathrm{s}}\right)}{A_o-A_i}\\ &= \frac{0.7854\times 9.625^2\left(1-\dfrac{10}{65.4}\right)-0.7854\times 8.681^2\left(1-\dfrac{12}{65.4}\right)}{0.7854\times 9.625^2-0.7854\times 8.681^2}\\ &= 0.98,\\ &= 0.98\times 5000\times 47 = 230406\ \mathrm{lbf} = 230\ \mathrm{kips}.\end{aligned}$$

Buoyed weight with cement outside and drilling fluid inside is

$$\begin{aligned}\mathrm{BF} &= \frac{A_o\left(1-\dfrac{\rho_\mathrm{o}}{\rho_\mathrm{s}}\right)-A_i\left(1-\dfrac{\rho_\mathrm{i}}{\rho_\mathrm{s}}\right)}{A_o-A_i}\\ &= \frac{0.7854\times 9.625^2\left(1-\dfrac{12}{65.4}\right)-0.7854\times 8.681^2\left(1-\dfrac{10}{65.4}\right)}{0.7854\times 9.625^2-0.7854\times 8.681^2}\\ &= 0.6831,\\ &= 0.6831\times 5000\times 47 = 160541\ \mathrm{lbf} = 160\ \mathrm{kips}.\end{aligned}$$

1.5 Modulus of Elasticity

Modulus of elasticity is

$$E=\frac{\sigma}{\varepsilon}=\frac{F/A}{\Delta L/L}\,\mathrm{psi}, \tag{1.20}$$

where σ = unit stress, psi, ε = unit strain in inch per inch, F = axial force, lbf, A = cross sectional area, in^2, ΔL = total strain or elongation, in., and L = original length, in.

1.6 Poisson's Ratio

$$v = \frac{\varepsilon_{lat}}{\varepsilon_{long}}, \tag{1.21}$$

where ε_{lat} = lateral strain in inches, and ε_{long} = longitudinal or axial strain in inches.

For most metals, Poisson's ratio varies from $\frac{1}{4} - \frac{1}{3}$.

Modulus of elasticity and shear modulus are related to Poisson's ratio as follows:

$$E = 2G(1 + v). \tag{1.22}$$

Modulus of elasticity, shear modulus, and Poisson's ratio for common materials are given in Table 1.1.

1.7 Minimum Yield Strength

Yield strength is defined as the stress that will result in specific permanent deformation in the material. The yield strength can be conveniently determined from the stress strain diagram. Based on the test results, minimum and maximum yield strengths for the tubulars are specified.

Table 1.1 Modulus of elasticity, shear modulus and Poisson's ratio at room temperature.

Metal Alloy	**Modulus of Elasticity**		**Shear Modulus**		**Poisson's Ratio**
	Psi × 10^6	**MPa × 10^6**	**Psi × 10^6**	**MPa × 10^6**	
Aluminum	10	6.9	3.8	2.6	0.33
Copper	16	11	6.7	4.6	0.35
Steel	30	20.7	12	8.3	0.27
Titanium	15.5	10.7	6.5	4.5	0.36
Tungsten	59	40.7	23.2	16	0.28

1.8 Ultimate Tensile Strength

The ultimate tensile strength (UTS) of a material in tension, compression, or shear, respectively, is the maximum tensile, compressive, or shear stress resistance to fracture or rupture. It is equivalent to the maximum load that can be applied over the cross-sectional area on which the load is applied. The term can be modified as the ultimate tensile, compressive, or shearing strength. Ultimate tensile strength of few API pipes are shown in Table 1.2.

1.9 Fatigue Endurance Limit

The endurance limit pertains to the property of a material and is defined as the highest stress or range of cyclic stress that a material can be subjected to indefinitely without causing failure or fracture. In other words, the endurance limit is the maximum stress reversal that can be indefinitely subjected a large number of times without producing fracture. The magnitude of the endurance limit of a material is usually determined from a fatigue test that uses a sample piece of the material.

Table 1.2 API pipe properties.

API	**Yield Stress, psi**		**Minimum Ultimate**	**Minimum**
Grade	**Minimum**	**Maximum**	**Tensile, psi**	**Elongation (%)**
H-40	40,000	80,000	60,000	29.5
J-55	55,000	80,000	75,000	24
K-55	55,000	80,000	95,000	19.5
N-80	80,000	110,000	100,000	18.5
L-80	80,000	95,000	95,000	19.5
C-90	90,000	105,000	100,000	18.5
C-95	95,000	110,000	105,000	18.5
T-95	95,000	110,000	105,000	18
P-110	110,000	140,000	125,000	15
Q-125	125,000	150,000	135,000	18

1.10 Twist

When a rod is subjected to torque it undergoes twist, which is given as

$$\theta = \frac{TL}{GJ} \text{ radians,} \tag{1.23}$$

where θ = angle of twist (radians) (can be $>2\pi$), L = length of section, ft, T = torque, ft-lbf, and G = modulus of rigidity, psi.

$$G = \frac{E}{2(1+\nu)}, \tag{1.24}$$

$$J = \text{Polar moment of inertia (in.}^4) = \frac{\pi}{32}\left(D_o^4 - D_i^4\right), \tag{1.25}$$

and E = modulus of elasticity, psi, and ν = Poisson's ratio.

Problem 1.5

Consider a pipe with the following dimensions carrying an applied tensile load of 5,000 lbs at the bottom. Calculate the maximum stress in the string. The pipe outside diameter = 5 in, the pipe inside diameter = 4 in, the pipe density = 490 lb/ft^3, and the pipe length = 30 ft.

Solution:

The cross sectional area of the pipe is

$$A = \frac{\pi}{4}\left(5^2 - 4^2\right) = 7.08 \text{ in}^2.$$

$$\text{Weight of the pipe} = \frac{\pi}{4}\left(5^2 - 4^2\right) \times 490 \times 30 \times 12 = 721.6 \text{ lbf.}$$

Total force acting at the top of the pipe:

F = Weight of the pipe + Load applied,
$F = 721.6 + 5000 = 5721.6$ lbf.

$$\text{Maximum stress at the top of the pipe} = \sigma = \frac{F}{A} = \frac{5721.6}{7.08}$$
$$= 809 \text{ psi.}$$

Problem 1.6

Calculate the elongation of a cylindrical pipe of 5″ in outside diameter, 4.0 in inside diameter and 10,000 ft long when a tensile load of 20,000 lbf is applied. Assume that the deformation is totally elastic and modulus of elasticity = 30×10^6 psi.

Solution:

From equation $E = \dfrac{F/A}{\Delta L/L}$, the elongation can be written as follows:

$$\Delta L = \frac{F/A}{E/L} = \frac{L \times F}{E \times A} = \frac{L \times F}{E \times \frac{\pi}{4}\left(D_o^2 - D_i^2\right)} = \frac{4L \times F}{E \times \pi \times \left(D_o^2 - D_i^2\right)}.$$

Substituting the values,

$$= \frac{4 \times 10000 \times 12 \times 20000}{30 \times 10^6 \times \pi \times (5^2 - 4^2)} = 11.32 \text{ in.}$$

Problem 1.7

A downhole tool with a length of 30 ft, an outside diameter of 5.5 in., and an inside diameter of 4.75 in. is compressed by an axial force of 30 kips. The material has a modulus of elasticity 30,000 ksi and Poisson's ratio 0.3. Assume the tool is in the elastic range.

Calculate the following:

A. Shortening of tool
B. Lateral strain