The Power and Beauty of Electromagnetic Fields

Frederic R. Morgenthaler

\[ \nabla \cdot \left[ -\frac{i}{c} (\mathbf{H} \times \mathbf{V})^i - \varepsilon_0 \mathbf{E} \times \mathbf{V} \right] = \mathbf{J}_o + \nabla \cdot [\mathbf{P} \times \mathbf{V}] \]

\[ \nabla \cdot \left[ \frac{i}{c} (\mathbf{E} \times \mathbf{V})^i - \mu_0 \mathbf{H} \times \mathbf{V} \right] = \nabla \cdot [\mu_0 \mathbf{M} \times \mathbf{V}] \]
THE POWER AND BEAUTY OF ELECTROMAGNETIC FIELDS
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F. R. Morgenthaler

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To my loving wife, Barbara,
our splendid daughters,
Ann and Janet,
and their wonderful children, Sarah, Brian,
Douglas, and Tessie
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PREFACE

“The most beautiful thing we can experience is the mysterious. It is the source of all true art and science.”
—Albert Einstein

Anyone, who has taken the trouble to explore the rich legacy that James Clerk Maxwell left to the scientific world, cannot fail to be impressed. Because he made such outstanding and profound contributions in multiple disciplines within a life-span of only 48 years, his achievements are all the more remarkable. They include seminal work in thermodynamics, statistical mechanics, and electromagnetic theory, but it is the latter that we here consider.

Although this text was written to instruct advanced undergraduate and first-year graduate students in the basic concepts of classical macroscopic electromagnetic fields, it was done so with the hope of providing new insights into and appreciation for what is surely one of the supreme achievements in science. Certain topics (described below), that do not appear in traditional texts, are deserving of inclusion both in their own right and because they simplify the development of new material concerning electromagnetic power and energy. The title, The Power and Beauty of Electromagnetic Fields, was chosen because of its multiple meanings.

During a tour of the campus, more than one M.I.T. student has taken a non-scientist friend to the lobby of the Eastman Laboratories (Building 6) to stand before the marble wall on which mathematical symbols that represent Maxwell’s Equations are inscribed in bronze. Usually, the student attempts to convey to his friend just how powerful and useful these compact equations are and how they at first predicted (rather than explained)
the existence of electromagnetic waves propagating at the speed of light, c. How that, in turn, led to their widespread application – to radio, microwaves, television, light, and x-rays; to electrical generators, motors, transducers, control systems, and power-grids; to integrated-circuits, computers, CD-ROMs, and the Internet; to electrocardiograms, pacemakers, magnetic resonance imaging (MRI), computer aided tomography (CAT), biosensors, and other emerging marvels of biomedical technology. And the list goes on and on .... No one can doubt the utility – the power – of electromagnetic theory.

Mathematicians find beauty and elegance in equations – especially if they have general applicability and can be expressed with brevity; many other scientists and engineers share these feelings as well. Surely, Maxwell’s Equations qualify and may be considered beautiful as well as powerful. After all, Albert Einstein pondered their properties and shattered Newtonian concepts of space and time with his Theory of Special Relativity. That in turn led to the introduction (by Minkowski) of four-dimensional space-time (with \(ict\) the imaginary fourth-dimension). Recasting electrodynamics in four-dimensions made evident new symmetries that led to even higher levels of understanding and beauty. Finally, the pinnacle was reached when connections to the world of quantumphysics produced Quantum Electrodynamics (QED) – but the last is not a subject for this text.

But there are other meanings of power and beauty as well. In circuit theory, electrical power is voltage times current and flows into and out of network nodes; in electromagnetic theory, the power flux is commonly defined as the Poynting vector, \(S = E \times H\) (the vector cross-product of the electric and magnetic fields), which is largely exterior to all highly conducting pathways. Likewise, rather than being localized to the electric charges and currents, the Maxwell field energy density, \(W\), with terms proportional to the scalar products \(E \cdot E\) and \(H \cdot H\), is distributed throughout space. These quantities, related to each other at every point by the Poynting Theorem, invite physical interpretation on a per unit area or per unit volume basis. Even at very low-frequencies, that interpretation is very different from the approximate, highly-localized circuit-theory representation.

Nevertheless, as recounted by Julius Stratton [1, pp. 134], the Poynting interpretation was criticized as early as 1902 by H. M. Macdonald [2], and later by George H. Livens [3], and Max Mason and Warren Weaver [4, pp. 264] among other writers. In their thought provoking analogy, Mason and Weaver, while excepting highly localized regions of space having little or no influence outside of them,

“do not believe that ‘Where?’ is a fair or sensible question to ask concerning energy. Energy is a function of configuration, just as beauty of a certain black-and-white design [such as the ‘Tiled-Photons’ example] is a function of configuration. [They] see no more reason or excuse for speaking of a spatial energy density than they would for saying, in the case of a design, that its beauty was distributed over it with a certain density. Such a view would lead one to assign to a perfectly blank square inch in one portion of the design a certain amount of beauty, and to an equally blank square inch in another portion a certain different amount of beauty.”

Many who have pondered that criticism applaud its cleverness – but consider it a false analogy that does little to advance one’s understanding of the issue. After all, they had offered no alternate interpretation of (or replacement for) \(S\) and \(W\). If not in the fields, where does radiating electromagnetic wave power and energy reside? Because circuit-power is confined to the wires that carry electrical currents, it cannot be used to explain radiation without the addition of a rather mysterious “radiation-resistance” to the circuit path. Even with that artifice, when the source-current is turned-off and a
transmitting circuit stops radiating, one expects that the energy, that has been radiated, must be located somewhere in free-space prior to its eventual detection – possibly very far from the transmitter.

During his junior year as an electrical engineering student at M.I.T., this author recalls an instructor telling the class that “although sophomores believe that electrical power is confined to the wires that carry electrical currents in a circuit, we [who have learned about Poynting] know better.” One skeptical student asked the instructor if his belief in the Poynting vector was strong enough that he would be willing to touch the wires of a high-voltage circuit. Without hesitation, the instructor responded, “of course, provided that you show me how I can do so without putting my hand in the field.” We all laughed and became more than a little smug because of our new found superior knowledge – we really thought that we did know better! Nearly forty years later, I learned that we (or at least I) did not; that realization, which came near the end of a long academic career in electromagnetism, was the principal motivation for the writing of this text. The analysis that led to that realization forms an essential component of its content.

Although the “circuit” representation is very useful, conventional wisdom considers it only a low-frequency or quasistatic approximation. In this text, we show that this is not necessarily true and in the process find alternate representations of electromagnetic power and energy that differ from the familiar Poynting theorem values – yet are fully equivalent. The particular choice focussed on features highly-localized power and energy components and emphasizes the circuit rather than the wave nature of these quantities. Moreover, unlike the Poynting vector, this exact representation merges smoothly with well-known quasistatic approximations that have long been used to calculate power flows in both lumped and distributed circuits operating at low-frequencies. As required, the electromagnetic power-conversion density, \( \mathbf{E} \cdot \mathbf{J} \) (the dot product of electric-field and...
current-density), is the same in both the Poynting and all correct alternate representations of electromagnetic power. Maxwell’s Equations and the fields they describe are, of course, left unchanged. It is also possible to alter the Maxwell stress-tensor, $T$, and the associated electromagnetic momentum-density, $G$, in a similar manner without altering the electromagnetic force-density; when all four elements ($S, W, T, G$) are treated similarly, any of the resulting alternate-representations can replace the Maxwell-Poynting form without approximation. One specific choice is termed the Alternate-representation. For time-harmonic fields, the complex Poynting theorem, energy-theorem, and momentum-theorem all have alternate-counterparts. Electromagnetic angular-momentum also has an alternate counterpart that is shown to connect directly with the spin properties of photons.

For certain electromagnetic problems, the Alternate-representation leads to both conceptual and computational simplicity. It is especially useful when dealing with either antenna radiation or quasistatic fields. For example, the power radiated from a Hertzian electric-dipole and its radiation pattern both can be calculated without first finding the electric and/or magnetic field – not a single curl operation need be performed. For other problems, it is the Maxwell-Poynting representation that is mathematically simpler; having a choice adds both flexibility and insight to the process of problem solving. Some features are surprising; for steady-state single-frequency fields, the free-space Alternate power-flux and energy-density are time-independent. Consequently, Alternate reactive power-flows and energies are banished from free-space and restricted to the locations of the charges and currents.

The text is divided into four-parts: Basic Electromagnetic Theory; Four Dimensional Electromagnetism; Electromagnetic Examples; Backmatter. Part I is devoted to a fairly conventional presentation of the integral and differential forms of Maxwell’s Equations in free-space containing electric charges and currents that are subject to Lorentz-forces. Conservation of charge is assumed. The equivalent representation in terms of the magnetic vector-potential and the electric scalar-potential is also given in both Coulomb and Lorenz gauges. Materials with polarization and magnetization that may be electrically-conducting are considered, as are the boundary conditions at material and source interfaces.

When wave-propagation effects are negligible, fields that are mainly electric or magnetic are classified as either electroquasistatic (EQS) or magnetoquasistatic (MQS). The properties of quasistatic fields and their analysis by approximate methods are developed. Electromagnetic power, energy, stress, and momentum are presented in both the Maxwell-Poynting and Alternate representations. So too, are complex versions of power and energy theorems that apply to sinusoidal steady-state fields. For linear media, both homogeneous (source free) and inhomogeneous wave equations are studied in one, two, and three dimensions; extensive use is made of both symmetry and the principle of superposition The concepts of electromagnetic duality, equivalence, and Babinet’s Principle for complementary structures are other important topics that are included; the uniqueness-theorem, induction-theorem, and reciprocity-theorem are also derived.

All derivations contained in Part I are carried out in conventional three-space coordinates with time as a parameter, but it is actually easier to formulate (and generalize) electrodynamics using four-dimensional representations of both fields and forces. It then follows that power, energy, stress, and momentum are unified in terms of an energy-momentum tensor. In Part II, we introduce these concepts and emphasize the utility of expressing the various tensors in terms of four-vector electric and magnetic fields and the four-vector-potential. However, no prior knowledge of such representations is
assumed nor is the reader expected to be familiar with the electrodynamics of moving media which is also discussed. Although these topics must be considered advanced\(^1\), they depend upon straight-forward extensions of the standard vector-calculus – knowledge of which is a prerequisite for almost all texts (including this one) on electromagnetic-theory. Using this approach, an infinity of energy-momentum tensors are found that are fully equivalent to the usual Maxwell-Poynting form; these change the representation of the electromagnetic power, energy, momentum and stress in free-space without altering the four-vector electromagnetic-force density. As noted above, emphasis is given to the particular choice that directly connects with circuit-theory representations of quasistatic-fields. These results are generalized to include the presence of field interactions with matter; special emphasis is given to dielectric and magnetic materials that are linear. Because multiple representations lead to a deeper understanding, four-dimensional electromagnetics is presented in order to complement, illuminate, and (in many cases) simplify, the topics developed in Part I. In addition, the theory certainly possesses great “mathematical-beauty” and so is deserving of inclusion.

In Part III, classic field problems are considered that illustrate how solutions of Maxwell’s Equations can be combined in order to satisfy boundary conditions for a wide variety of examples that, in the main, depend upon topics covered in Part I. Solving problems is the best (perhaps the only) way for a student of electromagnetics to master both physical concepts and mathematical techniques. Gradually, one develops physical intuition concerning fields; most find the process challenging, but the rewards great. It is reassuring that, once gained, mastery of the subject will not become obsolete. Certainly, its application to new materials and devices will continue to refresh electromagnetics, but, if history is any guide, the basic field-equations will not be supplanted; Maxwell seems to have got it very right. The specific examples were chosen, not only because the fields themselves are of interest, but because detailed comparisons can be made between the Alternate and Maxwell-Poynting representations. In many cases, the time-averaged distributions agree exactly. However, in others, there are significant differences and not only is Alternate power and energy highly-localized on currents and charges, but, astonishingly, also in regions of free-space. When compared with the usual Poynting energy-density, the propagation of such distributed and localized forms of Alternate-energy at first seems to create grave paradoxes. However, on closer inspection and analysis of the measurement process – by means of which power is detected, all of these are resolved. In other cases, Alternate-power exists in regions free of electric and magnetic fields or vice-versa. These strange results are also reconciled. In addition to the examples, which are worked through in detail, a chapter of over 120 practice problems is included so that the reader can test his/her understanding of the basic concepts and sharpen problem-solving skills. Many of the problems were created by the author for use in both M.I.T. undergraduate and graduate courses; however, new ones have been added as well.

Part IV contains the Backmatter including Summary, Appendices, Bibliography, and Index. A photo gallery of many (unfortunately not all) electromagnetic luminaries is included so that the reader can humanize the science of electromagnetics. As might be expected, many of the pioneers lived interesting and multi-faceted lives; fascinating

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\(^1\) If considered too demanding (or when there is insufficient time), Part II may be delayed (or bypassed altogether) until selected chapters of Part III have been studied. These contain topics and illustrative examples that depend mainly on knowledge of Part I. Used in this way, the text can serve both undergraduate and graduate students.
biographical information is readily available both in libraries and on the Internet. Eight appendices complete the text. Appendix-A includes elements of Special-Relativity including space-time coordinates and the Lorentz-transformation. Appendix-B is devoted to three-space vector identities and their four-space vector and tensor equivalents. Appendix-C contains important properties of Laplacian-fields and tabulates solutions of Laplace’s Equation. Appendix-D provides expressions for Alternate-power, energy, stress, and momentum in Cartesian, cylindrical, and spherical coordinate systems. Lorentz-torques and electromagnetic angular momentum formulations in both Maxwell-Poynting and Alternate formulations are also included. In Appendix-E, free-space fields associated with pre-specified electric charge and current distributions are considered. The method of images is included, as are formulas for the characteristic impedances of a variety of uniform TEM transmission-lines. Appendix-F reviews properties of Bessel functions and both Chebyshev and Hermite polynomials and provides useful recursion formulas and normalization integrals. Appendix-G discusses a very useful computer program: 4d-em.mac that has built-in knowledge of four-space electrodynamics; it runs under Macsyma and is included on the DVD that accompanies the text. For readers without access to Macsyma, the largely equivalent and freely available Maxima can be substituted and is included. Finally, Appendix-H contains a list of the avi movie files for several of the electromagnetic transients analyzed in the text. These animations are also included and can be viewed with standard media players.

The DVD also includes three electronic versions of the book. [Advanced Level] is the complete text (described above) that is suitable for graduate courses, reference, and self study. [Introductory Level] is intended for undergraduates; [Intermediate Level] for graduate and well prepared undergraduate students. Both omit Part II and abridge Parts III and IV by including appropriate selections of the [Advanced Level] examples, practice problems, and appendices. All three Levels are in PDF form suitable for on-line viewing using Adobe Reader; bookmarks and hyperlinks from the table of contents allow convenient browsing. The on-line versions of Appendix-H permit both the Macsyma Notebooks and the animations to be launched directly from icon hyperlinks.

The difference between the energy-momentum tensors in the Alternate and Maxwell-Poynting representation is itself a four-tensor, II^b, that produces neither electromagnetic-force nor E·J power-density. It is therefore an ephemeral quantity – yet one with components that can be calculated and presented graphically. In a bit of whimsy, that honors the insights of Mason and Weaver, this author has dubbed II^b the “electromagnetic-beauty;” this tensor joins electromagnetic-power to form still another set of meanings for the book title. The author believes that graphical rendering of the “beauty-power-flux” of an electromagnetic-field makes that final meaning plausible. Because selected examples are included in the Summary, the reader can either confirm or deny that judgement.

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JULIUS ADAMS STRATTON (1901–1994)
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LAN JEN CHU (1913–1973)
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