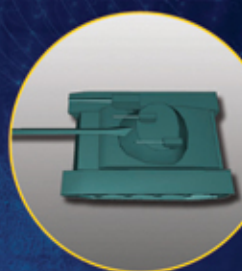
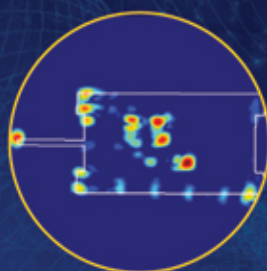
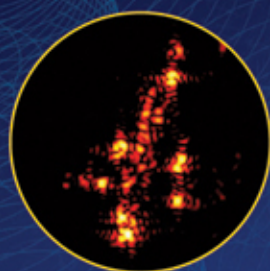


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INVERSE SYNTHETIC APERTURE RADAR IMAGING WITH MATLAB[®] ALGORITHMS



CANER ÖZDEMİR

 WILEY

Inverse Synthetic Aperture Radar Imaging with MATLAB Algorithms

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Inverse Synthetic Aperture Radar Imaging with MATLAB Algorithms

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Mersin University

Mersin, Turkey



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To:

*My wife,
My three daughters,
My brother,
My father,
and the memory of my beloved mother*

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Preface

Inverse synthetic aperture radar (ISAR) has been proven to be a powerful signal processing tool for imaging moving targets usually on the two-dimensional (2D) down-range cross-range plane. ISAR imagery plays an important role especially in military applications such as target identification, recognition, and classification. In these applications, a critical requirement of the ISAR image is to achieve sharp resolution in both down-range and cross-range domains. The usual way of obtaining the 2D ISAR image is by collecting the frequency and aspect diverse backscattered field data from the target. For synthetic aperture radar (SAR) and ISAR scenarios, there is always a trade-off between the down-range resolution and the frequency bandwidth. In contrast to SAR, the radar is usually fixed in the ISAR geometry and the cross-range resolution is attained by target's rotational motion, which is generally unknown to the radar engineer.

In order to successfully form an ISAR image, the target's motion should contain some degree of rotational component with respect to radar line of sight (RLOS) direction during the coherent integration time (or dwell time) of the radar system. But in some instances, especially when the target is moving along the RLOS direction, the target's viewing angle width is insufficient to be able to form an ISAR image. This restriction can be eliminated by utilizing bistatic or multistatic configurations that provide adequate look-angle diversity of the target. Another challenging problem occurs when the target's rotational velocity is sufficiently high such that the target's viewing angle width is not small during the dwell time of the radar. The target's translational movement is another issue that has to be addressed before displaying the final motion-free ISAR image. Therefore, motion effects have to be removed or mitigated with the help of motion compensation algorithms.

This book is devoted to the conceptual description of ISAR imagery and the explanation of basic ISAR research. Although the primary audience will be graduate students and other interested researchers in the fields of electrical

engineering and physics, I hope that colleagues working in radar research and development or in a related industry may also benefit from the book. Numerical or experimental examples in Matlab technical language are provided for the presented algorithms with the aim of improving the understanding of the algorithms by the reader.

The organization of the book is as follows. In the first chapter, an overview of Fourier theory, which plays an important and crucial role in radar imaging, is presented to provide a fair knowledge of Fourier-based signal processing basics. Noting that the ISAR imaging can also be treated as a signal processing tool, an understanding of signal processing and Fourier theory will be required to get the full benefit from the chapters within the book. The next chapter is devoted to radar fundamentals. Since ISAR itself is a radar, the key parameters of the radar concept that is related to ISAR research are revisited. These include electromagnetic scattering, radar cross section, the radar equation, and the radar waveforms. Then, before stepping into inverse problem of ISAR, the forward problem of SAR is reviewed in Chapter 3. SAR and ISAR provide dual problems and share dual algorithms with similar difficulties. Therefore, understanding the ISAR imagery could not be complete without understanding the SAR concepts. In the SAR chapter, therefore, important concepts of SAR such as resolution, pulse compression, and image formation are given together with associated Matlab codes. Furthermore, some advanced concepts and trends in SAR imaging are also presented.

After providing the fundamentals for SAR imaging, we provide the detailed imaging procedure for conventional ISAR imaging and the basic ISAR concepts with associated Matlab codes in Chapter 4. The topics include range profile concept, range/cross-range resolutions, small-angle small-bandwidth ISAR imaging, large-angle wide-bandwidth ISAR imaging, polar reformatting, and three-dimensional ISAR imaging. In Chapter 5, we provide some design aspects that are used to improve the quality of the ISAR image. Down sampling/up sampling, image aliasing, point spread function and smoothing are covered in this chapter. Several imaging tricks and fine-tuning procedures such as zero-padding and windowing that are used for enhancing the image quality are also presented.

In Chapter 6, range-Doppler ISAR image processing is given in detail. ISAR waveforms, ISAR receiver for these waveforms, quadrature detection, Doppler shift phenomena, and range-Doppler ISAR imaging algorithms are presented. The design examples with Matlab codes are also provided. In Chapter 7, scattering center representation, which has proven to be a sparse but an effective model of ISAR imaging, is presented. We provide algorithms to reconstruct both the image and the field data from the scattering centers with good fidelity. In Chapter 8, motion compensation (MOCOMP), one of the most important and challenging problems of ISAR imagery, is taken up in detail. The concepts include Doppler effect due to target motion, translational and motion compensation routines, range tracking, and Doppler tracking subjects. Algorithms and numerical examples with Matlab codes are

provided for the most popular MOCOMP techniques, namely, cross-correlation method, minimum entropy method, and joint-time frequency (JTF)-based motion compensation. In the final chapter, applications of the ISAR imaging concept to different but related engineering problems are presented. The employment of ISAR imagery to the antenna scattering problem (i.e., antenna SAR) and also to the antenna coupling problem (i.e., antenna coupling SAR) are explained. The imaging algorithms together with numerical examples are given. In addition, the application of the SAR/ISAR concept to the ground penetrating radar application is presented.

All MATLAB files may be accessed on the following ftp site: ftp://ftp.wiley.com/public/sci_tech_med/inverse_synthetic.

CANER ÖZDEMİR

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C.Ö.

Basics of Fourier Analysis

1.1 FORWARD AND INVERSE FOURIER TRANSFORM

Fourier transform (FT) is a common and useful mathematical tool that is utilized in numerous applications in science and technology. FT is quite practical, especially for characterizing nonlinear functions in nonlinear systems, analyzing random signals, and solving linear problems. FT is also a very important tool in radar imaging applications as we shall investigate in the forthcoming chapters of this book. Before starting to deal with the FT and inverse Fourier transform (IFT), a brief history of this useful linear operator and its founders is presented.

1.1.1 Brief History of FT

Jean Baptiste Joseph Fourier, a great mathematician, was born in 1768 in Auxerre, France. His special interest in heat conduction led him to describe a mathematical series of sine and cosine terms that can be used to analyze propagation and diffusion of heat in solid bodies. In 1807, he tried to share his innovative ideas with researchers by preparing an essay entitled “On the Propagation of Heat in Solid Bodies.” The work was examined by Lagrange, Laplace, Monge, and Lacroix. Lagrange’s oppositions caused the rejection of Fourier’s paper. This unfortunate decision caused colleagues to wait for 15 more years to read his remarkable contributions on mathematics, physics, and, especially, signal analysis. Finally, his ideas were published in the book *The Analytic Theory of Heat* in 1822 [1].

Discrete Fourier transform (DFT) was developed as an effective tool in calculating this transformation. However, computing FT with this tool in the

19th century was taking a long time. In 1903, Carl Runge studied the minimization of the computational time of the transformation operation [2]. In 1942, Danielson and Lanczos utilized the symmetry properties of FT to reduce the number of operations in DFT [3]. Before the advent of digital computing technologies, James W. Cooley and John W. Tukey developed a fast method to reduce the computation time in DFT. In 1965, they published their technique that later on became famous as the fast Fourier transform (FFT) [4].

1.1.2 Forward FT Operation

The FT can be simply defined as a certain linear operator that maps functions or signals defined in one domain to other functions or signals in another domain. The common use of FT in electrical engineering is to transform signals from time domain to frequency domain or vice versa. More precisely, forward FT decomposes a signal into a continuous spectrum of its frequency components such that the time signal is transformed to a frequency-domain signal. In radar applications, these two opposing domains are usually represented as “spatial frequency” (or wave number) and “range” (distance). Such use of FT will be examined and applied throughout this book.

The forward FT of a continuous signal $g(t)$ where $-\infty < t < \infty$ is described as

$$\begin{aligned} G(f) &= \mathcal{F}\{g(t)\} \\ &= \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi ft} dt. \end{aligned} \tag{1.1}$$

To appreciate the meaning of FT, the multiplying function $e^{-j2\pi ft}$ and operators (multiplication and integration) on the right side of Equation 1.1 should be investigated carefully: The term $e^{-j2\pi ft}$ is a complex phasor representation for a sinusoidal function with the single frequency of f_i . This signal oscillates only at the frequency of f_i and does not contain any other frequency component. Multiplying the signal in interest, $g(t)$, with the term $e^{-j2\pi ft}$ provides the similarity between each signal, that is, how much of $g(t)$ has the frequency content of f_i . Integrating this multiplication over all time instances from $-\infty$ to ∞ will sum the f_i contents of $g(t)$ over all time instants to give $G(f_i)$; that is, the amplitude of the signal at the particular frequency of f_i . Repeating this process for all the frequencies from $-\infty$ to ∞ will provide the frequency spectrum of the signal; that is, $G(f)$. Therefore, the transformed signal represents the continuous spectrum of frequency components; that is, representation of the signal in “frequency domain.”

1.1.3 IFT

This transformation is the inverse operation of the FT. IFT, therefore, synthesizes a frequency-domain signal from its spectrum of frequency components

to its time-domain form. The IFT of a continuous signal $G(f)$ where $-\infty < f < \infty$ is described as

$$\begin{aligned} g(t) &= \mathcal{F}^{-1} \{G(f)\} \\ &= \int_{-\infty}^{\infty} G(f) \cdot e^{j2\pi ft} df. \end{aligned} \quad (1.2)$$

1.2 FT RULES AND PAIRS

There are many useful Fourier transform rules and pairs that can be very helpful when applying the FT or IFT to different real-world applications. We will briefly revisit them to remind the reader of the properties of FT. Provided that FT and IFT are defined as in Equations 1.1 and 1.2, respectively, FT pair is denoted as

$$g(t) \xleftrightarrow{\mathcal{F}} G(f), \quad (1.3)$$

where \mathcal{F} represents the forward FT operation from time domain to frequency domain. The IFT operation is represented by \mathcal{F}^{-1} and the corresponding alternative pair is given by

$$G(f) \xleftrightarrow{\mathcal{F}^{-1}} g(t). \quad (1.4)$$

Here, the transformation is from frequency domain to time domain. Based on these notations, the properties of FT are listed briefly below.

1.2.1 Linearity

If $G(f)$ and $H(f)$ are the FTs of the time signals $g(t)$ and $h(t)$, respectively, the following equation is valid for the scalars a and b :

$$a \cdot g(t) + b \cdot h(t) \xleftrightarrow{\mathcal{F}} a \cdot G(f) + b \cdot H(f). \quad (1.5)$$

Therefore, the FT is a linear operator.

1.2.2 Time Shifting

If the signal is shifted in time with a value of t_o , then its frequency domain signal is multiplied with a phase term as listed below:

$$g(t - t_o) \xleftrightarrow{\mathcal{F}} e^{-j2\pi ft_o} \cdot G(f) \quad (1.6)$$

1.2.3 Frequency Shifting

If the time signal is multiplied by a phase term of $e^{j2\pi f_o t}$, then the FT of this time signal is shifted in frequency by f_o :

$$e^{j2\pi f_o t} \cdot g(t) \xleftrightarrow{\mathcal{F}} G(f - f_o) \quad (1.7)$$

1.2.4 Scaling

If the time signal is scaled by a constant a , then the spectrum is also scaled with the following rule:

$$g(at) \xleftrightarrow{\mathcal{F}} \frac{1}{|a|} G\left(\frac{f}{a}\right), \quad a \in \mathbb{R}, a \neq 0. \quad (1.8)$$

1.2.5 Duality

If the spectrum signal $G(f)$ is taken as a time signal $G(t)$, then the corresponding frequency-domain signal will be the time reversal equivalent of the original time-domain signal, $g(t)$:

$$G(t) \xleftrightarrow{\mathcal{F}} g(-f). \quad (1.9)$$

1.2.6 Time Reversal

If the time is reversed for the time-domain signal, then the frequency is also reversed in the frequency-domain signal:

$$g(-t) \xleftrightarrow{\mathcal{F}} G(-f). \quad (1.10)$$

1.2.7 Conjugation

If the conjugate of the time-domain signal is taken, then the frequency-domain signal is conjugated and frequency-reversed:

$$g^*(t) \xleftrightarrow{\mathcal{F}} G^*(-f). \quad (1.11)$$

1.2.8 Multiplication

If the time-domain signals $g(t)$ and $h(t)$ are multiplied in time, then their spectrum signals $G(f)$ and $H(f)$ are convolved in frequency:

$$g(t) \cdot h(t) \xleftrightarrow{\mathcal{F}} G(f) * H(f). \quad (1.12)$$

1.2.9 Convolution

If the time-domain signals $g(t)$ and $h(t)$ are convolved in time, then their spectrum signals $G(f)$ and $H(f)$ are multiplied in the frequency domain:

$$g(t) * h(t) \xleftrightarrow{\mathcal{F}} G(f) \cdot H(f). \quad (1.13)$$

1.2.10 Modulation

If the time-domain signal is modulated with sinusoidal functions, then the frequency-domain signal is shifted by the amount of the frequency at that particular sinusoidal function:

$$\begin{aligned} g(t) * \cos(2\pi f_o t) &\xleftrightarrow{\mathcal{F}} \frac{1}{2} (G(f + f_o) + G(f - f_o)) \\ g(t) * \sin(2\pi f_o t) &\xleftrightarrow{\mathcal{F}} \frac{j}{2} (G(f + f_o) - G(f - f_o)). \end{aligned} \quad (1.14)$$

1.2.11 Derivation and Integration

If the derivative or integration of a time-domain signal is taken, then the corresponding frequency-domain signal is given as below:

$$\begin{aligned} \frac{d}{dt} g(t) &\xleftrightarrow{\mathcal{F}} 2\pi f \cdot G(f) \\ \int_{-\infty}^t g(\tau) d\tau &\xleftrightarrow{\mathcal{F}} \frac{1}{j2\pi f} G(f) + \pi G(0) \cdot \delta(f). \end{aligned} \quad (1.15)$$

1.2.12 Parseval's Relationship

A useful property that was claimed by Parseval is that since the FT (or IFT) operation maps a signal in one domain to another domain, the signals' energies should be exactly the same as given by the following relationship:

$$\int_{-\infty}^{\infty} |g(t)|^2 dt \xleftrightarrow{\mathcal{F}} \int_{-\infty}^{\infty} |G(f)|^2 df. \quad (1.16)$$

1.3 TIME-FREQUENCY REPRESENTATION OF A SIGNAL

While the FT concept can be successfully utilized for the stationary signals, there are many real-world signals whose frequency contents vary over time. To be able to display these frequency variations over time, joint time-frequency (JTF) transforms/representations are used.

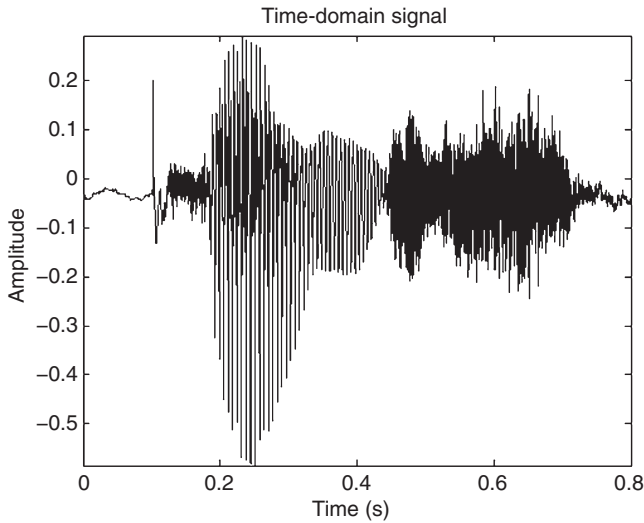


FIGURE 1.1 The time-domain signal of “prince” spoken by a lady.

1.3.1 Signal in the Time Domain

The term “time domain” is used while describing functions or physical signals with respect to time either continuous or discrete. The time-domain signals are usually more comprehensible than the frequency-domain signals since most of the real-world signals are recorded and displayed versus time. A common equipment to analyze time-domain signals is the *oscilloscope*. In Figure 1.1, a time-domain sound signal is shown. This signal is obtained by recording an utterance of the word “prince” by a lady [5]. By looking at the occurrence instants in the x -axis and the signal magnitude in the y -axis, one can analyze the stress of the letters in the word “prince.”

1.3.2 Signal in the Frequency Domain

The term “frequency domain” is used while describing functions or physical signals with respect to frequency either continuous or discrete. Frequency-domain representation has been proven to be very useful in numerous engineering applications while characterizing, interpreting, and identifying signals. Solving differential equations and analyzing circuits and signals in communication systems are a few applications among many others where frequency-domain representation is much more advantageous than time-domain representation. The frequency-domain signal is traditionally obtained by taking the FT of the time-domain signal. As briefly explained in Section 1.1, FT is generated by expressing the signal onto a set of basis functions, each of which is a sinusoid with the unique frequency. Displaying the measure of the similarities of the original time-domain signal to those particular unique

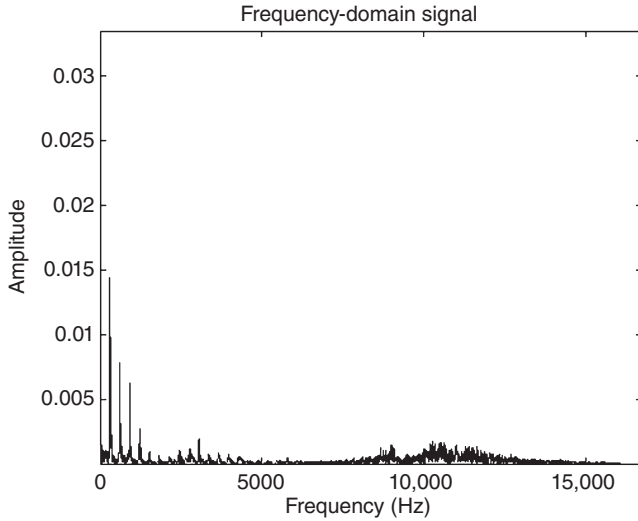


FIGURE 1.2 The frequency-domain signal (or the spectrum) of “prince.”

frequency bases generates the Fourier transformed signal, or the frequency-domain signal. Spectrum analyzers and network analyzers are the common equipment which analyze frequency-domain signals. These signals are not as quite perceivable when compared to time-domain signals. In Figure 1.2, the frequency-domain version of the sound signal in Figure 1.1 is obtained by using the FT operation. The signal intensity value at each frequency component can be read from the y -axis. The frequency content of a signal is also called the spectrum of that signal.

1.3.3 Signal in the (JTF) Plane

Although FT is very effective for demonstrating the frequency content of a signal, it does not give the knowledge of frequency variation over time. However, most of the real-world signals have time-varying frequency content such as speech and music signals. In these cases, the single-frequency sinusoidal bases are not suitable for the detailed analysis of those signals. Therefore, JTF analysis methods were developed to represent these signals both in time and frequency to observe the variation of frequency content as the time progresses.

There are many tools to map a time-domain or frequency-domain signal onto the JTF plane. Some of the most well-known JTF tools are the short-time Fourier transform (STFT) [6], the Wigner–Ville distribution [7], the Choi–Willams distribution [8], the Cohen’s class [9], and the time-frequency distribution series (TFDS) [10]. Among these, the most appreciated and commonly used is the STFT or the spectrogram. The STFT can easily display the

variations in the sinusoidal frequency and phase content of local moments of a signal over time with sufficient resolution in most cases.

The spectrogram transforms the signal onto two-dimensional (2D) time-frequency plane via the following famous equation:

$$\begin{aligned} STFT\{g(t)\} &\triangleq G(t, f) \\ &= \int_{-\infty}^{\infty} g(\tau) \cdot w(\tau - t) e^{-j2\pi f\tau} d\tau. \end{aligned} \quad (1.17)$$

This transformation formula is nothing but the short-time (or short-term) version of the famous FT operation defined in Equation 1.1. The main signal, $g(t)$, is multiplied with a shorter duration window signal, $w(t)$. By sliding this window signal over $g(t)$ and taking the FT of the product, only the frequency content for the windowed version of the original signal is acquired. Therefore, after completing the sliding process over the whole duration of the time-domain signal $g(t)$ and putting corresponding FTs side by side, the final 2D STFT of $g(t)$ is obtained.

It is obvious that STFT will produce different output signals for different duration windows. The duration of the window affects the resolutions in both domains. While a very short-duration time window provides a good resolution in the time domain, the resolution in the frequency domain becomes poor. This is because of the fact that the time duration and the frequency bandwidth of a signal are inversely proportional to each other. Similarly, a long duration time signal will give a good resolution in the frequency domain while the resolution in the time domain will be bad. Therefore, a reasonable compromise has to be attained about the duration of the window in time to be able to view both domains with fairly good enough resolutions.

The shape of the window function has an effect on the resolutions as well. If a window function with sharp ends is chosen, there will be strong sidelobes in the other domain. Therefore, smooth waveform type windows are usually utilized to obtain well-resolved images with less sidelobes with the price of increased main lobe beamwidth; that is, less resolution. Commonly used window types are Hanning, Hamming, Kaiser, Blackman, and Gaussian.

An example of the use of spectrograms is demonstrated in Figure 1.3. The spectrogram of the sound signal in Figure 1.1 is obtained by applying the STFT operation with a Hanning window. This JTF representation obviously demonstrates the frequency content of different syllables when the word “prince” is spoken. Figure 1.3 illustrates that while the frequency content of the part “prin . . .” takes place at low frequencies, that of the part “. . . ce” occurs at much higher frequencies.

JTF transformation tools have been found to be very useful in interpreting the physical mechanisms such as scattering and resonance for radar applications [11–14]. In particular, when JTF transforms are used to form the 2D image of electromagnetic scattering from various structures, many useful physical features can be displayed. Distinct time events (such as scattering from

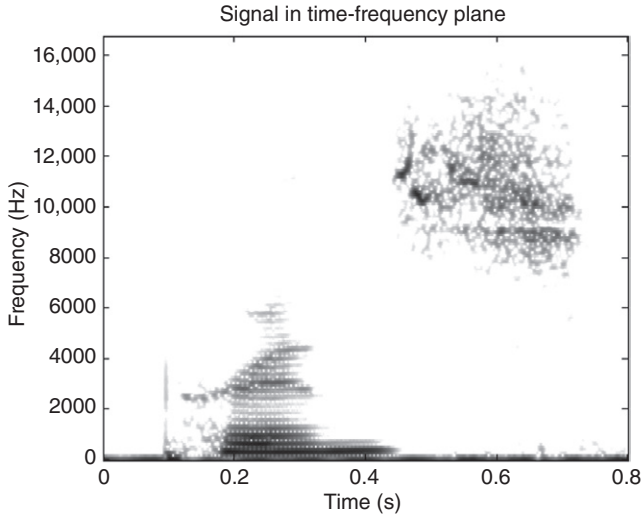


FIGURE 1.3 The time-frequency representation of the word “prince.”

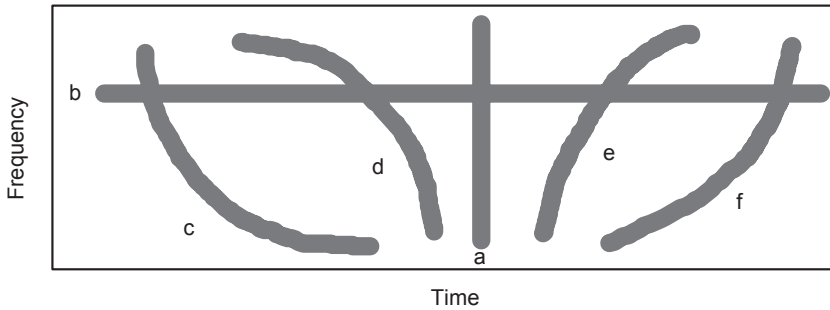


FIGURE 1.4 Images of scattering mechanisms in the joint time-frequency plane: (a) scattering center, (b) resonance, (c and d) dispersion due to material, (e and f) dispersion due to geometry of the structure.

point targets or specular points) show up as vertical line in the JTF plane as depicted in Figure 1.4a. Therefore, these scattering centers appear at only one time instant but for all frequencies. A resonance behavior such as scattering from an open cavity structure shows up as horizontal line on the JTF plane. Such mechanisms occur only at discrete frequencies but over all time instants (see Fig. 1.4b). Dispersive mechanisms, on the other hand, are represented on the JTF plane as slanted curves. If the dispersion is due to the material, then the slope of the image is positive as shown in Figure 1.4c,d. The dielectric coated structures are the good examples of this type of dispersion. The reason

for having a slanted line is because of the modes excited inside such materials. As frequency increases, the wave velocity changes for different modes inside these materials. Consequently, these modes show up as slanted curves in the JTF plane. Finally, if the dispersion is due to the geometry of the structure, this type of mechanism appears as a slanted line with a negative slope. This style of behavior occurs for such structures such as waveguides where there exist different modes with different wave velocities as the frequency changes as seen in Figure 1.4e,f.

An example of the use of JTF processing in radar application is shown in Figure 1.5 where spectrogram of the simulated backscattered data from a dielectric-coated wire antenna is shown [14]. The backscattered field is collected from the Teflon-coated wire ($\epsilon_r = 2.1$) such that the tip of the electric field makes an angle of 60° with the wire axis as illustrated in Figure 1.5. After the incident field hits the wire, infinitely successive scattering mechanisms occur. The first four of them are illustrated on top of Figure 1.5. The first return comes from the near tip of the wire. This event occurs at a discrete time that

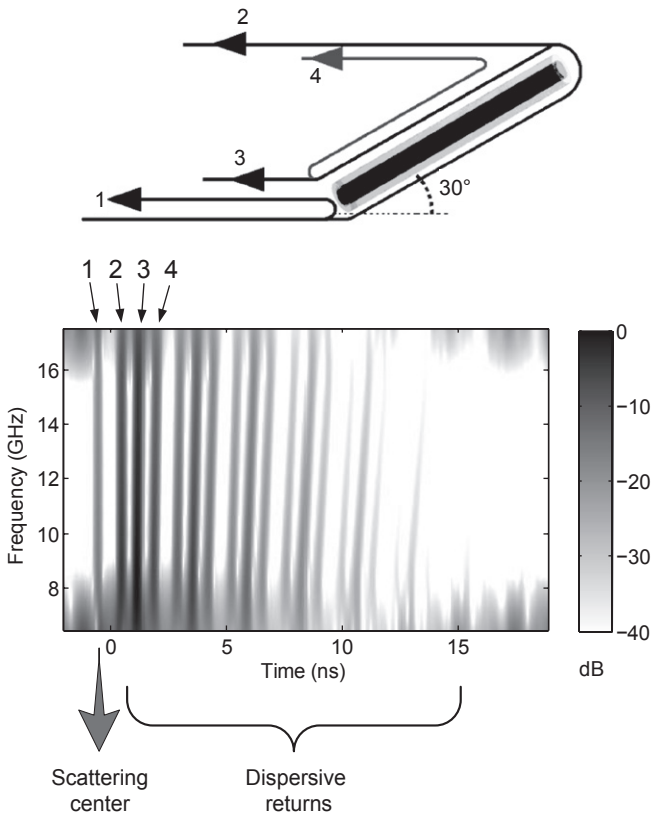


FIGURE 1.5 JTF image of a backscattered measured data from a dielectric-coated wire antenna using spectrogram.