STEPHEN P. SCHWARTZ

A BRIEF HISTORY OF

ANALYTIC PHILOSOPHY

FROM RUSSELL TO RAWLS

A Brief History of Analytic Philosophy

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From Russell to Rawls

By Stephen P. Schwartz

To my wife, Diane

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Preface

Analytic philosophy was the dominant Anglo-American philosophical movement of the twentieth century, and remains dominant today. Enough time has now passed that we are able to have historical perspective on this vital philosophical tradition. My aim with this book is to provide a general overview of the leading philosophers, theories, movements, and controversies of analytic philosophy, as well as some idea of its cultural, political, and social setting.

The Anglo-American analytic tradition starts with Bertrand Russell and G. E. Moore at the beginning of the twentieth century. The most recent works that I focus on are from the 1970s. The Epilogue is a brief discussion of the development of analytic philosophy from 1980 to the present day, and a look to the future.

I have not assumed that the reader has any formal background in philosophy. Analytic philosophy is technical, however. It grew out of developments in logic and the foundations of mathematics. Leaving out all technicalities would mean leaving out many interesting and central aspects of analytic philosophy. Rather than clutter the text with explanations of terms and issues that would be familiar to those readers who have studied philosophy, I have provided background snippets at the end of each chapter. These are indicated in the text as [*Background n.m* – Subject]. Those who would find the background helpful can flip to the end of the chapter. Others can skip them. The use of symbolic logic at places in the text is unavoidable. At the end of Chapter 1 I have provided a background on basic symbolism for those unfamiliar with symbolic logic. At the end of each chapter I have appended an annotated list of further readings for those interested in pursuing topics in more depth or following out other tentacles.

For each philosopher that I discuss I have given relevant and representative quotes to illustrate my expositions. I want the philosophers to have a chance to speak for themselves, and to give a sense of how

they sound in their own voices. I hope to inspire readers to seek out the original sources and pursue them on their own. But for many, I realize, this book will be their only chance to engage with many of these texts. For each quote I give the source including page number. Often I give two publication dates. Many of the works I cite are classics that have been reprinted. Thus I may quote from a 1978 reprint of an article that was originally published in 1912. I indicate this by (Author's name 1978/1912, p. *nn*). Refer to the bibliography. I transcribe these quotes exactly as they appear in the original texts. I have not edited them for consistency of spelling and punctuation. I have usually not included the works cited in the text in the suggested further readings, since they are obvious choices for further reading and study.

I have given standard, accepted interpretations when they square with my understanding of the original texts. I have not engaged in historical disputes or attempted to adjudicate among various interpretations of the philosophers' ideas. This history is meant to be a start, not the final word. It would be an appropriate text in courses such as Analytic Philosophy, Twentieth-Century Philosophy, and Contemporary Philosophy.

The structure of the book is basically chronological beginning about 1905 and extending into the 1970s. The history of analytic philosophy is not, however, a direct chronological development. Some strands that differ markedly are simultaneous. Within the tradition of analytic philosophy, criticizing, rethinking, and reworking former notions is crucial. Major figures such as Bertrand Russell and Ludwig Wittgenstein changed their views, attacking their earlier selves. The story of analytic philosophy is the story of analytic philosophers struggling with and against themselves, and each other; struggling with and against its origins, and former movements and doctrines. Later parts of the story can only be understood as reactions to earlier parts. The reader will not find discussion of major thinkers limited to single chapters. Rather, the philosophical stories of Russell, Wittgenstein, Carnap, Quine, and others weave through several chapters. I have provided a useable index for those who want to trace a particular thinker or issue through the history. I have included a list of the leading analytic philosophers at the end of the introduction. No doubt others would have made somewhat different lists, but anybody familiar with the history of analytic philosophy will recognize that the list I give suffers, if at all, only by omission.

I devote a single chapter to ethics in the analytic tradition (Chapter 8). After G. E. Moore's *Principia Ethica* of 1903, ethics was out of the mainstream of analytic philosophy until the 1960s. None of the major

figures of analytic philosophy before 1950 (with the exception of early Moore) did much work on ethics, because they thought that it was outside the province of philosophy. Ethical issues and questions about the analysis of ethical language come up only briefly in earlier chapters. Starting in the 1950s and continuing today a great expansion of interest and work in ethics and value theory has occurred for reasons that I explain in the text. My main focus has been on logic, philosophy of language, metaphysics, and epistemology as was the main focus of analytic philosophy from 1905 to the 1950s and 1960s. I provide only a brief discussion of analytic philosophy since the 1970s for several reasons (Epilogue). For one thing, it is current events, not history. We are too close to have a historical view on the last 30 years of philosophy. For another, work in analytic philosophy has become so specialized and technical that the text would be little more than a collection of technical explanations. Lastly, the volume of publications due to the computer, the Internet, and the consequent increased communications has expanded beyond the point where any overview is possible.

Vincent van Gogh wrote: ''I exaggerate, I sometimes make changes to the subject, but still I don't invent the whole of the painting; on the contrary, I find it readymade – but to be untangled – in the real world.'' This is a good description of history writing, and of philosophy itself. We find the subject in the real world to be untangled. I have tried to untangle the story of analytic philosophy, without too much exaggeration and invention, I hope. But I am an analytic philosopher. I have taught philosophy since the mid-1960s and made my own modest contributions in print. I cannot keep my personal feelings out of the story entirely. I have known, listened to, studied with, talked to, or at least met many of the leading analytic philosophers. I am a participant, not an observer. At places I express my opinions and try to support them briefly. I am personally and passionately involved in the enlightening and edifying enterprise of analytic philosophy.

I would like to thank the following for their help: My colleagues in the Ithaca College Department of Philosophy and Religion, Frederik Kaufman and Craig Duncan, read and made valuable suggestions on parts of the manuscript. John Rosenthal of the Ithaca College Department of Mathematics patiently and kindly helped me with difficult parts of the text. Gerald Hull read much of the manuscript and made many helpful comments, as did Steven Lee on an earlier version. I would also like to thank Eric Lerner for helpful suggestions. My wife, Diane Schwartz, read the entire manuscript as I was writing and gave me valuable suggestions from a non-philosophical perspective. I would also like to thank John Heil and Peter Singer for reading parts of the manuscript and

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Stephen P. Schwartz
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Introduction: What is Analytic Philosophy?

Modern analytical empiricism, of which I have been giving an outline, differs from that of Locke, Berkeley, and Hume by its incorporation of mathematics and its development of a powerful logical technique. (Russell 1945, p. 834)

I have before me on my desk a famous book entitled *What is Mathematics?* In it you will find basic descriptions of number theory, algebra, geometry, topology, and so on. What you will not find is a definition or explanation of what mathematics is, even though the first section of the book is also titled ''What is Mathematics?'' The final answer: ''For scholars and layman alike it is not philosophy but active experience in mathematics itself that alone can answer the question: What is mathematics?'' (Courant and Robbins, 1941, no page number) In the same spirit, my answer to the question ''What is analytic philosophy?'' is – the rest of this book starting with Chapter 1. Analytic philosophy is what the philosophers on the list ''Leading Analytic Philosophers'' (located at the end of this chapter) did philosophically.

If a historian had to give an exact definition in terms of necessary and sufficient conditions of his subject, no histories would get written. I have another book before me on my desk. It is also a classic: *A Brief History of Science* by A. Rupert Hall and Marie Boas Hall (1964). This is a wonderful overview of the development of science from the ancient world to today, but the book contains no definition of ''science, '' no discussion of what science is, or what distinguishes science from other endeavors. If the Halls had to answer these questions first, they would still be working on them and their history would never have been

written. Philosophers of science, even now, do not have settled answers to these questions. The Halls prudently begin their book with the history of the origins of science in the ancient world. They do not worry about the demarcation problem: the problem of how to distinguish science, the subject of their brief history, from other pursuits. It would also be prudent of me to avoid the vexed question of the demarcation of analytic philosophy from other philosophical traditions.

Yet, we have a sense that an overview of mathematics or a history of science does deal with a delimited subject matter. A history of science is not the same as a history of the English novel or a history of modern art. I, along with my philosophical colleagues, recognize that analytic philosophy is a distinct tradition even if we are not quite prepared to draw the exact borders of it. A history of analytic philosophy is not the same as a history of twentieth-century Catholic philosophy, or Marxist philosophy, or twentieth-century Continental philosophy, or twentieth-century American philosophy.

So, so much for prudence. I'll try to say something useful in answer to the question ''What is analytic philosophy?'' (Without having any pretense to being able to give necessary and sufficient conditions for demarcating the tradition.)

Analytic philosophy begins in the first years of the twentieth century and is the dominant tradition in philosophy today. Of course, analytic philosophy is not the same as it was in its early days. Like any movement or tradition, it evolved due to catastrophic political events, advances in technology, the influence of other subjects and disciplines, and its own searching self-scrutiny and criticism.

Analytic philosophy developed out of many sources. Among them were the British tradition of empiricism, mentioned in the leading quote by Bertrand Russell, and developments in the natural sciences in the late nineteenth and early twentieth centuries, especially physics. But the chief impetus was the revolutionary advances in logic, set theory, and the foundations of mathematics in the late nineteenth and very early twentieth centuries. For philosophy the most important of these innovations was the development of symbolic logic, which became the indispensable tool and source of ideas for analytic philosophers. Analytic philosophers got their inspiration, ideas, problems, and methods from British empiricism, formal logic, mathematics, and natural science. (For a definition of empiricism see *Background 1.1* – Epistemology: empiricism versus rationalism, p. 39. For symbolic logic see *Background 1.3* – Mathematical logic of *PM* versus traditional Aristotelian logic and note on symbolism, p. 40.)

How did analytic philosophy evolve and mature? Toward the middle of the twentieth century many analytic philosophers were able to adopt a more distanced and critical attitude to empiricism, science, and mathematics than its earlier practitioners had embraced. This allowed an expansion of analytic ethics, renewed interest in metaphysics, and increased attention to and appreciation of philosophers of the past, starting with the ancient Greeks. Ethicists, metaphysicians, and historians of philosophy brought to their subject the methods of analytic philosophy: clarity of expression, logical argumentation, direct and extensive dialectical interchange among philosophers, and a piecemeal scientific approach to problems, but they left behind the more doctrinaire aspects of the early movement. (All of this is described in detail in the following chapters.)

The name ''analytic philosophy'' refers more to the methods of analytic philosophy than to any particular doctrine that analytic philosophers have all shared.[1] An analytic philosopher analyzes problems, concepts, issues, and arguments. She breaks them down into their parts, dissects them, to find their important features. Insight comes from seeing how things are put together and how they can be prized apart; how they are constructed and how they can be reconstructed. Symbolic logic was and remains the most distinctive tool of analytic philosophers.

Analytic philosophers have always struggled with themselves and each other, their tradition, its origins and ideas. No feature of analytic philosophy has gone unchallenged by other analytic philosophers. After World War II many analytic philosophers in Great Britain reacted against the overreliance, as they saw it, on symbolic logic, natural science, and formal analysis. American philosophers around the same time attacked what they claimed were the unjustified dogmas of analytic philosophy. One of the perennial issues that analytic philosophers focus on is method, including such questions as: How much should we rely on formal logic? Is natural science the only source of reliable knowledge? Should philosophy attempt to be scientific? Analytic philosophy, its methods and doctrines, is one of analytic philosophers' favorite subjects to ponder.

Analytic philosophy is a dialectical enterprise that is always struggling with itself. This is why defining analytic philosophy is so difficult. It is not a unified movement or school. It is loosely organized around a set of problems, methods, and issues but no party line on these has

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[1] The word ''analytic'' in ''analytic philosophy'' does not mean the same as the term used in ''the analytic/synthetic distinction.'' That is a different, but distantly related, concept. (See *Background 1.2* – A priori, analytic, necessary, p. 40.)

ever defined analytic philosophy. Analytic philosophers look to Frege, Russell, Wittgenstein, and Moore as the patriarchs of our large extended and argumentative family.

Geographically, analytic philosophy began almost simultaneously in the very early twentieth century in England and in German-speaking countries. With the rise of the Nazis in the 1930s, most analytic philosophers in Austria and Germany emigrated to Britain or North America. Despite the stresses of World War II, this influx of brilliant minds produced tremendous renewed energy and optimism for analytic philosophy in England and the United States. Today almost all philosophy departments in English-speaking countries would self-describe their orientation as analytic. Interest and participation in analytic philosophy has increased dramatically in Continental Europe and Scandinavia in the past few decades.

As a cultural phenomenon analytic philosophy was an expression of modernism in philosophy.[2] As a loose movement in visual art, literature, and music, modernism began in the nineteenth century but gained cultural dominance after the catastrophic Great War of 1914-18. Like analytic philosophy, modernism cannot be precisely defined. Modernism is characterized by rejection of past traditions; experimentation with new forms that can be shocking and disturbing, reflecting the cultural disillusionment of the post-World War I era; attention to language and method, or surface in the case of painting; anxiety about technology and science, but also utilization of new developments.

In literature the best representative of modernism is James Joyce's *Ulysses*. We also think of such authors as William Faulkner and Virginia Woolf as being paradigmatically modernist. Modernist authors reject the traditional expectation that a novel will comprise a sequential storyline, with plot, character development, and so on. Instead the modernist novel is characterized by fractured time, no identifiable story or plot, stream of consciousness, experimentation with language. In music modernist composers moved away from tunes, harmony, and key structure. Such composers as Stravinsky intentionally made their music dissonant and discordant. Twelve tone composers such as Schoenberg used a new formal system that abandoned traditions of classical Western music. Modernism in painting begins with the impressionists and characterizes the entire development of modern art. Especially beginning with Cezanne, artists more and more got away

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[2] Modernism is not to be confused with modern. ''Modern'' means different things in different contexts. Modern philosophy is usually considered to start with Descartes about 1640. Modern art starts with the impressionists around 1870.

from depicting recognizable objects. They rejected the ''old master'' traditions of pictorial art. Picasso and then others eventually moved toward abstract art. Modernist painters were more interested in the surfaces of their paintings as objects than in other objects which their painting would depict as a mirror or through a window.

The basic aspects of modernism – rejection of past traditions, experimentation with new methods and forms; fascination with and anxiety about technology and use of new technical methods; focusing on method, surface, expression, and language – all characterize analytic philosophy. Analytic philosophy was born from new technical developments in logic and the foundations of mathematics. Analytic philosophers saw themselves, initially, as revolutionary, breaking with the past traditions of Western philosophy. They saw their work as freeing philosophy and even society, from its past forms and obsessions. Analytic philosophers, especially Wittgenstein and those influenced by him, experimented with new ways of expressing their views. Wittgenstein, as you will see in Chapter 2, did not express his philosophy in sequential arguments as did traditional philosophers, nor did several other analytic philosophers. Early analytic philosophers rejected virtually all of past philosophy and would only rely on science to provide knowledge. Analytic philosophers with a formalist bent filled their pages with arcane symbols. The evolution of analytic philosophy exhibits the conflict between formalism and expressionism that we see in modern art. I am thinking of the contrast between the cubists and some abstract painters such as Mondrian on the one hand and van Gogh and the German expressionists on the other. Modernism is reflected in both an extreme formalism and an ardent expressionism. Analytic philosophy went through this internal struggle most stridently just after World War II, but it is present throughout its history. See especially Chapter 4.

Modernism is a point of contact between analytic philosophy and Continental philosophy. Among Continental philosophers, Nietzsche and Kierkegaard can be seen as anticipating or inspiring modernist forms of philosophical activity. Among later Continental philosophers, Heidegger, the French existentialists Camus and Sartre, and later deconstructionist and post-modernist philosophers (despite the title) embody modernism. Viewed as cultural phenomena of the twentieth century, Continental philosophy and analytic philosophy are brothers in arms.

Analytic philosophy, as a cultural phenomenon, was never as obsessively modernist as painting and music in the twentieth century (nor as obsessively modernist as some Continental philosophers such as Heidegger), and always retained a commitment to the essence

of philosophy – reasoned argumentation – although with Wittgenstein and some ordinary language philosophers this is difficult to see through the mists. Analytic philosophers rejected the pretensions of the Enlightenment philosophers but not their commitment to reason. Analytic philosophers aimed to replace what they considered to be the outmoded ways of traditional philosophy with their new techniques based on symbolic logic, the analysis of language, and scientific methods.

This is the story that will unfold in the chapters that follow.

Leading Analytic Philosophers

*Gottlob Frege 1848–1925 German
Bertrand Russell 1872–1970 British
George Edward Moore 1873–1958 British (known as G. E. Moore)
Otto Neurath 1882–1945 Austrian
Moritz Schlick 1882–1936 German
Ludwig Wittgenstein 1889–1951 Austrian/British
Rudolf Carnap 1891–1970 German/American
Hans Reichenbach 1891–1953 German/American
Gilbert Ryle 1900–1976 British
Karl Popper 1902–1994 Austrian/British
*Alfred Tarski 1902–1983 Polish/American
Carl Hempel 1905–1997 German/American
*Kurt Gödel 1906–1978 Austrian/American
Nelson Goodman 1906–1998 American
Willard Van Orman Quine 1908–2000 American (W. V. Quine or
    W. V. O. Quine)
Charles Leslie Stevenson 1908–1979 American (Charles L. Stevenson
    or C. L. Stevenson)
Max Black 1909–1988 Russian/British/American
Alfred Jules Ayer 1910–1989 British (A. J. Ayer)
John Austin 1911–1960 British (J. L. Austin)
Norman Malcolm 1911–1990 American
Wilfrid Sellars 1912–1989 American
*Alan Turing 1912–1954 British
Herbert Paul Grice 1913–1988 British (Paul Grice)
Roderick Chisholm 1916–1999 American
Donald Davidson 1917–2003 American
Gertrude Elizabeth Margaret Anscombe 1919–2001 British (Elizabeth
    Anscombe or G. E. M. Anscombe)
Richard Mervyn Hare 1919–2002 British (R. M. Hare)
Peter Frederick Strawson 1919–2006 British (Peter Strawson or
    P.F. Strawson)

John Jamieson Carswell Smart 1920 Australian (J. J. C. Smart)
Philippa Foot 1920–2010 British
Ruth Barcan Marcus 1921 American
John Rawls 1921–2002 American
Thomas Kuhn 1922–1996 American
Michael Dummett 1925 British
David Malet Armstrong 1926 Australian (D. M. Armstrong)
Stanley Cavell 1926 American
Hilary Putnam 1926 American
*Noam Chomsky 1928 American
Keith Donnellan 1931 American
Richard Rorty 1931–2007 American
Alvin Plantinga 1932 American
John Searle 1932 American
Jaegwon Kim 1934 Korean/American
Thomas Nagel 1937 American
Robert Nozick 1938–2002 American
Saul Kripke 1940 American
Robert Stalnaker 1940 American
David Lewis 1941–2001 American
Peter Singer 1946 Australian

*See *Background 5.1* – Are Frege, Gödel, Tarski, Turing, and Chomsky analytic philosophers? (p. 196). Although I include Frege, Gödel, Tarski, Turing, and Chomsky on this list because of their influence, I do not consider them to be analytic philosophers.

Further Reading

Philosophical Analysis in the Twentieth Century by Scott Soames (Princeton University Press 2003) is a tendentious and controversial two volume history of analytic philosophy. His approach is selective and technical, but useful on the topics he discusses.

Two works on the history and nature of analytic philosophy are *Twentieth-Century Analytic Philosophy* by Avrum Stroll (Columbia University Press 2000) and *What is Analytic Philosophy* Hans-Johann Glock (Cambridge University Press 2008). Glock is generally reliable, but like Soames selective. Stroll is uneven. His extensive discussion of the material I cover in Chapter 7 (the new theory of reference) is unreliable.

A good place to go for individual essays on many of the leading analytic philosophers is *A Companion to Analytic Philosophy* edited by A. P. Martinich and E. David Sosa (Wiley-Blackwell 2005). In this impressive collection each of the 40 articles is written by a different leading scholar.

1

Russell and Moore

*The question which Kant put at the beginning of his philosophy, namely
''How is pure mathematics possible?'' is an interesting and difficult one,
to which every philosophy which is not purely sceptical must find some
answer. (Russell 1959a/1912, p. 84)*

Empiricism, Mathematics, and Symbolic Logic

Bertrand Russell – aristocrat (3rd Earl Russell), anti-war activist, pro-
lific writer, and brilliant philosopher and mathematician – is the father
of Anglo-American analytic philosophy. Russell did the hard work
of expounding and promulgating the new symbolic logic that was to
revolutionize the method of philosophy. Equally important for analytic
philosophy, he introduced others to the works of Gottlob Frege and
Ludwig Wittgenstein, who might otherwise have languished unappre-
ciated. Russell proposed and energetically pursued philosophical issues
that were keenly examined by philosophers throughout the twentieth
century. Without Bertrand Russell's work, especially the work he pro-
duced early in his career in logic and the philosophy of language, there
would have been no Anglo-American analytic philosophy.

Russell says that Frege was the pioneer and no doubt this is true.
''Many matters which, when I was young, baffled me by the vagueness
of all that had been said about them, are now amenable to an exact
technique, which makes possible the kind of progress that is custom-
ary in science. . . . [T]he pioneer was Frege, but he remained solitary
until his old age''(Russell 1963/1944, p. 20).[1] Russell's optimism about

[1]   Citations here and throughout are indicated as (Name date/original publication date
if significantly different, page)

philosophical progress may seem overstated, but not his judgment of Frege. Frege did revolutionary work on the foundations of mathematics and was the first to clarify and investigate issues in the philosophy of language that were central to twentieth-century philosophy and are still central today. Indeed Gottlob Frege *was* the pioneer of the techniques that gave life to analytic philosophy, but he would not have had an impact without Russell's influence. Frege would have remained solitary. Russell brought Frege to the attention of other philosophers and mathematicians, especially in the English-speaking world, and developed and improved Frege's pioneering ideas.

Russell's greatest contribution to logic, philosophy, and mathematics was his publication of *Principia Mathematica* with Alfred North Whitehead (published in three volumes, 1910–13). Based on ideas originally articulated by Frege in the late nineteenth century, Russell developed and founded the field of symbolic logic. Symbolic logic today is central not only to philosophy but to many other areas including mathematics and computer science.[2] In addition to *Principia Mathematica* (often referred to simply as *PM*), Russell expounded the ideas and methods of the new symbolic logic energetically in his *Principles of Mathematics* and many other influential publications early in the twentieth century. The influence, importance, and central role of *PM* cannot be overemphasized. For example, Kurt Gödel titled his historic paper ''On formally undecidable propositions of *Principia Mathematica* and related systems.'' (More on this influential mathematical paper below (see pp. 162–4).)

The methodology that gives analytic philosophy its strength and structure is the logic and philosophy of language generated by the original work of Frege, Russell, and Whitehead.

Their results in logic and the philosophy of language have also had major impacts in other areas of philosophy. The revolution in logic in the early years of the twentieth century gave analytic philosophers the tools to articulate and defend a sophisticated form of empiricism. [*Background 1.1* – Epistemology: empiricism versus rationalism (The background snippets are found at the end of the chapter.)] With the new tools in logic and philosophy of language, philosophers were able to repair the flaws and gaps in thinking of the classical British empiricists. The major gap was the lack of an explanation of how pure mathematics is possible. Modern logic as developed by Frege, Russell, and Whitehead yielded definite results in the foundations of mathematics and the philosophy of language that, though technical

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[2] *Principia Mathematica* was voted number 23 of the 100 most important nonfiction books of the twentieth century – the highest rated philosophy book. (http://www.infoplease.com/ipea/A0777310.html)

and expounded in daunting detail, went to the heart of epistemological issues. Empiricists could claim to have solved the outstanding problems plaguing their theory – namely our knowledge of mathematics – by using the techniques of mathematical logic. (This is explained in the next section.)

Although Russell was uneasy with empiricism, his sympathy was with the classical British empiricists. Virtually all analytic philosophers have shared this sympathy while at the same time becoming increasingly uneasy with the details and presuppositions of classical empiricism. Russell could not accept ''pure empiricism'' – the view that all knowledge is derived from immediate sensory experience – but sought to move only as far from it as was absolutely necessary. Speaking of his very early views Russell says: ''it seemed to me that pure empiricism (which I was disposed to accept) must lead to skepticism . . . ''(Russell 1959b/1924, p. 31). Even worse than skepticism, Russell came to believe that pure empiricism led to solipsism and could not account for our knowledge of scientific laws or our beliefs about the future. Still, Russell always seemed to feel that these were problems for empiricism, not reasons to discard it outright.

Despite his sympathy with empiricism, in places Russell sounds like an unabashed rationalist: ''It is, then, possible to make assertions, not only about cases which we have been able to observe, but about all actual or possible cases. The existence of assertions of this kind and their necessity for almost all pieces of knowledge which are said to be founded on experience shows that traditional empiricism is in error and that there is *a priori* and universal knowledge'' (Russell 1973, p. 292. From a lecture given in 1911). [*Background 1.2* – A priori, analytic, necessary]

Despite his wavering philosophical sympathies, Russell's mathematical logic gave later empiricists the tools to respond to the troubling difficulties with their position that Russell was pointing out. Mathematics is *a priori* and universal, so how can it be empirical? Twentieth-century analytic philosophy got its first shot of energy from a plausible answer to this question – an answer offered by the logical investigations of Frege and Russell.[3]

Frege and Russell were able to use symbolic logic to reconceptualize the very nature of mathematics and our mathematical knowledge (Figure 1.1). I must emphasize that symbolic logic as developed in

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[3] Whitehead, also a brilliant philosopher, logician, and mathematician did much of the technical work of developing symbolic logic but did not play the kind of role that Russell did in publishing it, publicizing it, and making it accessible to fellow philosophers, and showing how fruitful and valuable a tool it was.
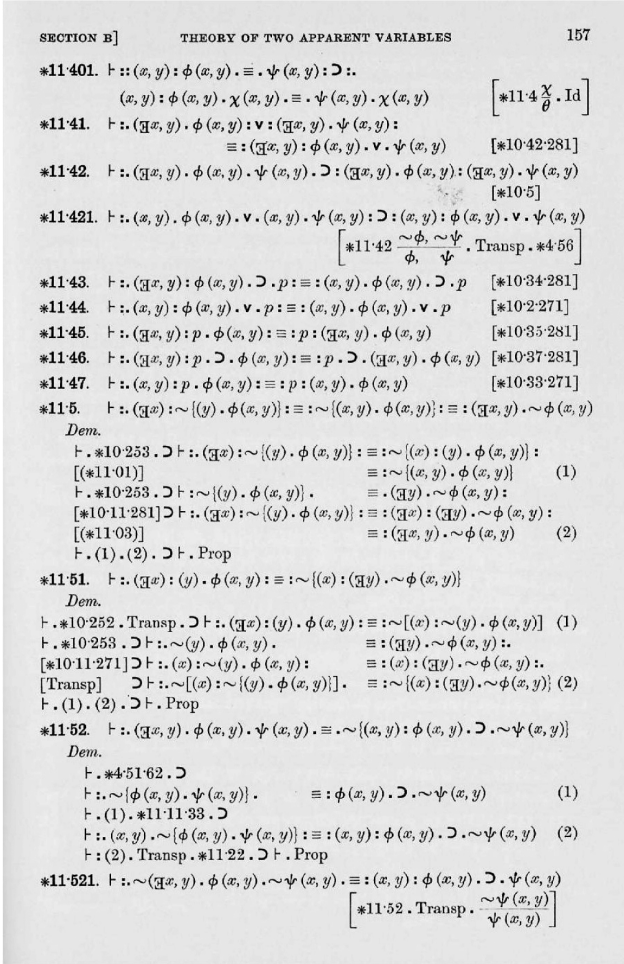
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*11·401. ⊢ :: (x, y) : φ(x, y) . ≡ . ψ(x, y) : ⊃ :.
    (x, y) : φ(x, y) . χ(x, y) . ≡ . ψ(x, y) . χ(x, y)   [*11·4 χ/θ . Id]

*11·41. ⊢ :. (∃x, y) . φ(x, y) : v : (∃x, y) . ψ(x, y) :
        ≡ : (∃x, y) : φ(x, y) . v . ψ(x, y)   [*10·42·281]

*11·42. ⊢ :. (∃x, y) . φ(x, y) . ψ(x, y) . ⊃ : (∃x, y) . φ(x, y) : (∃x, y) . ψ(x, y)   [*10·5]

*11·421. ⊢ :. (x, y) . φ(x, y) . v . (x, y) . ψ(x, y) . ⊃ : (x, y) : φ(x, y) . v . ψ(x, y)   [*11·42 ~φ,~ψ/φ,ψ . Transp . *4·56]

*11·43. ⊢ :. (∃x, y) : φ(x, y) . ⊃ . p : ≡ : (x, y) . φ(x, y) . ⊃ . p   [*10·34·281]

*11·44. ⊢ :. (x, y) : φ(x, y) . v . p : ≡ : (x, y) . φ(x, y) . v . p   [*10·2·271]

*11·45. ⊢ :. (∃x, y) : p . φ(x, y) . ≡ : p . (∃x, y) . φ(x, y)   [*10·35·281]

*11·46. ⊢ :. (∃x, y) : p . ⊃ . φ(x, y) . ≡ : p . ⊃ . (∃x, y) . φ(x, y)   [*10·37·281]

*11·47. ⊢ :. (x, y) : p . φ(x, y) . ≡ : p . (x, y) . φ(x, y)   [*10·33·271]

*11·5. ⊢ :. (∃x) : ~{(y) . φ(x, y)} : ≡ : ~{(x, y) . φ(x, y)} : ≡ : (∃x, y) . ~φ(x, y)
Dem.
⊢ . *10·253 . ⊃ ⊢ :. (∃x) : ~{(y) . φ(x, y)} : ≡ : ~{(x) : (y) . φ(x, y)} :
[(*11·01)]   ≡ : ~{(x, y) . φ(x, y)}   (1)
⊢ . *10·253 . ⊃ ⊢ : ~{(y) . φ(x, y)} .   ≡ . (∃y) . ~φ(x, y) :
[*10·11·281] ⊃ ⊢ :. (∃x) : ~{(y) . φ(x, y)} : ≡ : (∃x) : (∃y) . ~φ(x, y) :
[(*11·03)]   ≡ : (∃x, y) . ~φ(x, y)   (2)
⊢ . (1) . (2) . ⊃ ⊢ . Prop

*11·51. ⊢ :. (∃x) : (y) . φ(x, y) . ≡ : ~{(x) : (∃y) . ~φ(x, y)}
Dem.
⊢ . *10·252 . Transp . ⊃ ⊢ :. (∃x) : (y) . φ(x, y) . ≡ : ~[(x) : ~(y) . φ(x, y)]   (1)
⊢ . *10·253 . ⊃ ⊢ : ~(y) . φ(x, y) .   ≡ : (∃y) . ~φ(x, y) :.
[*10·11·271] ⊢ : . (x) : ~(y) . φ(x, y) :   ≡ : (x) : (∃y) . ~φ(x, y) :.
[Transp]   ⊃ ⊢ :. ~[(x) : ~{(y) . φ(x, y)}] .   ≡ : ~{(x) : (∃y) . ~φ(x, y)}   (2)
⊢ . (1) . (2) . ⊃ ⊢ . Prop

*11·52. ⊢ :. (∃x, y) . φ(x, y) . ψ(x, y) . ≡ . ~{(x, y) : φ(x, y) . ⊃ . ~ψ(x, y)}
Dem.
⊢ . *4·51·62 . ⊃
⊢ :. ~{φ(x, y) . ψ(x, y)} .   ≡ : φ(x, y) . ⊃ . ~ψ(x, y)   (1)
⊢ . (1) . *11·11·33 . ⊃
⊢ :. (x, y) . ~{φ(x, y) . ψ(x, y)} . ≡ : (x, y) : φ(x, y) . ⊃ . ~ψ(x, y)   (2)
⊢ : (2) . Transp . *11·22 . ⊃ ⊢ . Prop

*11·521. ⊢ :. ~(∃x, y) . φ(x, y) . ~ψ(x, y) . ≡ : (x, y) : φ(x, y) . ⊃ . ψ(x, y)
    [*11·52 . Transp . ~ψ(x, y)/ψ(x, y)]

Figure 1.1   This is a page chosen at random from *Principia Mathematica. PM* is extremely daunting and is today studied only by specialists, although it has an accessible introduction that is a brief overview of symbolic logic.

PM was not just the use of symbols – so that for example we use "∨" instead of the word "or" and "(∃x)" for "some." That would be impressive perhaps and simplifying in some ways, but not revolutionary. The revolution in logic, pioneered by Frege, and expounded by *PM* was based on the concept of treating logic mathematically, and then treating mathematics as a form of logic. This is Frege's and Russell's

logicism.[4] [*Background 1.3* – Mathematical logic of *PM* versus traditional Aristotelian logic and a note on symbolism]

Symbolic logic is not only of technical interest for those concerned about the foundations of mathematics. Virtually every philosophy major in every college and university in the United States and elsewhere is required to pass a course in symbolic logic. Not only philosophy majors, but other students as well – computer science majors, mathematics majors, not to mention English majors – take symbolic logic courses. Symbolic logic has also been central to the development of computers, and it is now a branch of mathematics, and is an indispensable tool for theoretical linguistics and virtually anyone working in technical areas of the study of language.

Symbolic logic has been the central motivating force for much of analytic philosophy. Besides giving philosophers the tools to solve problems that have concerned thinkers since the Greeks, the notion that mathematics is logic points to an answer to the question posed by Kant in the quote that opens this chapter. ''How is pure mathematics possible?'' This is an answer that removes mathematics as an obstacle to empiricism. Mathematics is possible because it is analytic.

Logicism

Whitehead and Russell's *PM* was an elaborate argument for logicism, which in turn was based on earlier work by Frege. The logicist program is succinctly stated by Russell and attributed to Frege: ''Frege showed in detail how arithmetic can be deduced from pure logic, without the need of any fresh ideas or axioms, thus disproving Kant's assertion that '$7 + 5 = 12$' is synthetic'' (Russell 1959b, p. 32).

Logicism was one of several responses to difficulties that emerged in the foundations of mathematics toward the end of the nineteenth century. These difficulties perplexed Russell and many others. We can skip over the technicalities for now, and keep in mind that none of the difficulties that troubled Russell would matter for any practical applications of mathematics or arithmetic. You could still balance your checkbook even if the foundations of mathematics had not been put on a firm footing. Nevertheless, to a philosopher of Russell's uncompromising character these difficulties were intellectually troubling. The results of his investigations have thrilled and baffled philosophers ever

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[4]   As opposed, e.g., to psychologism, the view that mathematics is derived from human psychology. Frege was deeply opposed to psychologism.

since and tormented and fascinated (at least a few) students taking Symbolic Logic.

The nature of mathematics and arithmetic is a central problem for philosophers, especially in the area of epistemology. In the argument between the empiricists and rationalists, the question of our knowledge of mathematical facts plays a key role. Even an impure (i.e., moderate) empiricist must answer the question how we know that $7 + 5 = 12$, that the interior angles of a triangle equal $180°$, that there are infinitely many prime numbers, and so on. ''Of course, we know them because we were told them in school and read them in the textbook.'' This answer, while having an appealing simplicity, would disappoint both the rationalist and the empiricist and is abjectly unphilosophical. We know those mathematical facts because we can figure them out, ''see the truth of them,'' especially when we've been shown the proofs or done the calculations. [*Background 1.4* – Proofs that the sum of the interior angles of a triangle is $180°$ and that there are infinitely many prime numbers] And the marvelous thing is, not only that we ''see'' the truths, but also understand that they must be so, could not be otherwise, and are necessary and absolute. No experience could impart such certainty. Mathematical knowledge dooms the empiricist claim that all knowledge is based on experience.

Russell's assertions about geometry in the following quote apply to all of mathematics. (When he uses the term ''idealists'' his description applies to rationalists.)

> Geometry, throughout the 17th and 18th centuries, remained, in the war against empiricism, an impregnable fortress of the idealists. Those who held – as was generally held on the Continent – that certain knowledge, independent of experience, was possible about the real world, had only to point to Geometry: none but a madman, they said, would throw doubt on its validity, and none but a fool would deny its objective reference. The English Empiricists, in this matter, had, therefore, a somewhat difficult task; either they had to ignore the problem, or if, like Hume and Mill, they ventured on the assault, they were driven into the apparently paradoxical assertion that Geometry, at bottom, had no certainty of a different kind from that of Mechanics . . . (Russell 1897, p. 1)[5]

The problem that empiricism has with mathematics is worth pondering. Even if ''$7 + 5 = 12$'' and ''the interior angles of a triangle equal

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[5] Infamously, John Stuart Mill claimed that mathematical truths were based on experience. Few empiricists have agreed with Mill's view. Russell could not accept it and surely Russell is right.

180°'' are derived in some way from experiences of counting and mea-suring angles, it is impossible that, e.g., our knowledge of the infinitude of primes comes from experience. Although perhaps the idea could be led back by many steps to experiences with counting and dividing and so on, I do not see how any experience or observation (other than ''seeing'' the proof) could get one to know with certainty that there are infinitely many prime numbers. Using a computer to generate prime numbers wouldn't help. It would just keep calculating primes, but how could we know it would never get to the last one? There is no possible empirical test that would establish that there are infinitely many primes. Yet the proof is so simple and obvious that there can be no doubt. If you are troubled by the indirect nature of the proof, be assured there are direct proofs. In any case, Euclid's proof assures us there is a larger prime given any series of primes.

Empirical evidence and observations even if pervasive and universal cannot explain the certainty and necessity of mathematical proposi-tions. In the case of a mathematical proposition such as ''$7 + 5 = 12$,'' empirical observations are not evidence or support. If a proposition is based on observational evidence, then there must be possible obser-vations that one could describe that would refute the proposition. No possible observations would refute ''$7 + 5 = 12$.'' If every possible observation, test, and experiment is compatible with the truth of the proposition, then observation, test, and experiment is irrelevant to the proposition. This is the case with the true mathematical propositions that I cited. A simple example should suffice: If I put 7 sheep in the pen, and then 5 more and counted all the sheep and kept getting 11, I would assume that one of the sheep was stolen, escaped, or had been kidnapped by aliens. The last thing I would ever judge is that $7 + 5$ does not equal 12. Indeed, I would never judge that unless I had lost all sense of reason. To repeat, if no possible experience or observation would lead us to give up a proposition, then it is not based on experience or observation. In mathematics we have decisive counterexamples to empiricism: propositions that are true, that we know to be true and in fact are absolutely certain but are not based on observation, test, experiment, or experience.

This much was accepted by Russell and empiricists (other than Mill) and has been accepted by most philosophers since. Our mathematical statements and ones like them are necessarily true and are not based on sensory experience in that they are not empirical scientific results established in the lab or field by the scientific method and observation. The only alternative source seems to be pure reason. The victory cheers of the rationalists are ringing through the ages. Here are clear examples