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Understanding and Managing Model Risk

A Practical Guide for Quants, Traders and Validators

MASSIMO MORINI

Understanding and Managing Model Risk

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Massimo Morini

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Preface

One fundamental reason for writing this book is that I do not think that models can ‘kill Wall Street’, as someone was heard to say during the credit crunch. Shortsighted policies and regulations, and bad incentives for market players, are much more likely killers (see Chapter 1 for some precise results regarding the role they can play in fuelling a crisis). And yet I am perplexed when I hear some fellow modellers deny any responsibility, saying ‘Models were not a problem. The problem was in the data and the parameters! The problem was in the application!’. As a researcher and bank quant, I find these disclaimers paradoxical. Models in finance are tools to quantify prices or risks. This includes mathematical relations, a way to use data or judgement to compute the parameters, and indications on how to apply them to practical issues. Only by taking all these things together can we talk of ‘a model’. Modellers should stay away from the temptation to reduce models to a set of mathematical functions that can be thought of separately from the way they are specified and from the way they are applied. If this were the case, models would really be only blank mathematical boxes and people would be right to consider them useless, when not outright dangerous.

This is not the definition of models considered in this book. I think that mathematical models are magnificent tools that can take our understanding of markets, and our capability to act in markets, to levels impossible to reach without quantitative aids. For this to be true, we must understand the interaction between mathematics and the reality of markets, data, regulations and human behaviour, and control for this in our management of model risk.

The fact that thousands of technical papers speak of very advanced models, and just a handful focus on model risk and how to manage it, is one of our problems. Too often models have been used to create a false sense of confidence rather than to improve our understanding. Increasing the complexity of the mathematical details to hide our ignorance of the underlying system is an abuse of the beauty and power of mathematics.

At the same time we have relegated model validation and risk management to become a formal and boring topic for bureaucrats. So I do not find it strange that this book has been written not by a risk manager or a validator, but by a front office quant who has spent the last ten years inventing new models, implementing them, and helping practitioners to use them for buying, selling and hedging derivatives. No one has seen how many unexpected consequences the practical use of models can have more often than a front office quant. This forces us to think of model robustness and of the effect of different calibrations or estimations of parameters. While risk managers and validators can at times afford to take a formal approach to model risk, front office quants must go deeper into the mathematical aspects of models for their implementation, and are also those who then have to deal with the most practical side of model risk.

I have also been helped by the fact that I am a researcher and trainer in the field of quantitative finance, am up-to-date with the variety of models developed by quants and enjoy the benefit of many discussions with my fellow researchers and students about the use and misuse of models. Another important element is the role I have been allowed to play in the study of the foundations of modelling at my bank, and the close collaboration with a wise and far-sighted risk management and validation group team during my last years at Intesa Sanpaolo.

In this book I have tried to avoid the two opposite extremes that I have seen too often. On one hand, training material on risk management often gives a lot of details on formal compliance or simple techniques to produce numbers that are acceptable to put in reports, but lacks the quantitative approach that would be needed to understand models deeply, and the practical examples on how real risks can arise from the use of models and hit the business of your bank or institution. Now a consensus is growing, even among regulators, that we need something different. On the other hand, many papers on financial models are weighed down with mathematics and numerics, but just a few focus on the consequences that different modelling choices can have on enterprise-wide risk and on the analysis of financial conditions and practical misuses that can lead to model losses. It is also rare to find papers that show how many alternative models are possible giving you the same good fit and efficient calibration but leading to completely different pricing and risk assessment for complex products. Before the crisis models did not play the role of allowing as transparent as possible a translation of assumptions into numbers. They have often hidden poor and oversimplified assumptions under a lot of numerical and mathematical details.

In this book you will find the rigorous mathematical foundations and the most recent developments in financial modelling, but they are analyzed taking into account the regulatory and accountancy framework, and they are explained through a wide range of practical market cases on different models and different financial products, to display where model risk hides and how it can be managed. The consequences of model assumptions when applied in the business, including explanation of model errors and misunderstandings, the comparison of different models and the analysis of model uncertainty are a focus of this book, to build up a practical guide for reducing the likelihood of model losses.

Those who like mathematics, will find as much of it as they can desire, especially in the second part of the book. But in the first part of the book there are also hundreds of pages of explanations in plain words, without formulas, that I strongly advise should not be ignored. They are sometimes the only way to think about the real purposes for which formulas are developed, and they are often the only way to explain models to many who will use them. Those who do not really like mathematics will be happy to see that in these pages all concepts are also explained without formulas. But please, do make an effort to engage with the mathematics. Here it is explained, often from the foundations, and always put in relation to practice, you may be surprised to find just how useful it can be. This also makes the book suitable for students that want to see financial models within the context of their application, and for users that have to choose different models and want to explore their hidden consequences.

Some of the mathematical complexities we have seen in models in the past decade are probably useless or even disturbing. But financial problems are seriously complex, and very often a high level of mathematical ability is really needed. I do think, however, that the high level of theoretical complexity reached by models must be balanced by a practical and not-too-complex approach to model risk management. In what follows you will find all the mathematics needed to understand models, but you will not find complex theoretical and mathematical frameworks for how to perform model risk management or validation. We want to reduce model risk, not to compound the risk of complex models with the risk of complex model validation. We keep our distance from fascinating but overcomplex frameworks that are often inapplicable and inhibit fresh thinking.

My aim is to help regulators, senior management, traders, students, and also quants themselves to a deeper understanding and awareness of the financial implications of quantitative models. Even more importantly, I want to provide quants, risk managers and validators with tools for investigating and displaying effectively the reasons for choosing one model and rejecting another, and for understanding and explaining why in many cases model uncertainty is unavoidable and models must not be used to create a false sense of confidence or as a shield for dangerous business decisions. Before the recent crisis, this analysis and this explanation failed too often and the consequences have been harsh.

In any case: if the book fails to fulfil this role, at least it has reached such a size that it can be used by quants and technical traders to stop physically any dangerous model misuse or misunderstanding. The sheer weight of its pages will force the errants to stop and think about what they are doing, without, one hopes, leaving any permanent physical consequences.

A final remark is in order. No book should even try to be a definitive work on model risk. If this were the case, we might feel entitled to stop thinking about and doubting our model tools, and a crisis worse than the one we have just seen would be forthcoming. In spite of the range of models and markets considered, this search for risks, errors and misunderstanding in the development and use of models is necessarily very partial and incomplete. But I am confident that coming with me on this quest will make you a better hunter.

One of the exercises for the reader is to spot the model risks that managed to escape the nets of this book, or survive defiantly among its pages, and propose solutions. I have even set up a website: www.managingmodelrisk.com.

HOW WE PROCEED

The book is divided into two parts. In the first, I want to build solid knowledge of the theory and the empirical evidence underlying the best practice of model risk management, constructing a practical scheme for model choice and model validation. I want the reader to not just accept each step passively, but to gain a thorough understanding of why it is useful and of how it must be applied in different situations. Since the different possible situations in financial markets are a continuous infinity of a high order, the only way to gain confidence is to explore each step deeply through market examples. I have tried in the examples to keep a practical and ‘teaching’ approach, as confirmed by the number of ‘handwritten’ figures that come from my courses for practitioners.

This book covers a wide range of asset classes. The lion’s share is probably played by interest rates and credit, which is not surprising because in almost all banks model risk management has a special focus on these asset classes. The most natural examples in the first part of the book, that deals with errors in model assumptions and model application, come from credit, where these issues have emerged most often, particularly in the recent credit crunch. The second part of this book deals with more technical errors, particularly in computational methods, hedging, and mathematical techniques. Here, most of the examples come from interest rates, because it is here that the most advanced techniques were developed and applied. These two asset classes are also those that are experiencing the most changes in modelling approach now. However, equity modelling is mentioned very often throughout the book, and actually the majority of the issues dealt with in the book can have an application within complex equity models, as I often point out. We also speak of cross-currency, and liquidity and hybrid modelling have sections devoted to them.

Below is an extended summary of the contents

In Chapter 1 we want to understand what Model Risk really means in practice. To achieve this goal:

- We study the foundations of quantitative pricing and their relationship with the actual workings of the markets.
- We see the most relevant analyses of model risk given in the literature, and we test them on the reality of the past crises, from the stock market crash of 1987 to the LTCM collapse, and the Russian default, up to the credit crunch, to see which model errors really led to large losses and how this risk could be managed.
- We investigate the links between the way we use models and the accounting standards, in particular the concepts of fair value, mark-to-market and levels 1, 2 and 3 for pricing.
- We describe the prescriptions of regulators to see which constraints they set on modelling and which indications they give on model risk management.

In Chapter 2 we consider three market examples, so as to apply the scheme for Model Validation and Model Risk Management developed at the end of Chapter 1.

- We consider three asset classes: credit, equity and interest rates.
- For each asset class we consider a few payoffs, and apply to them a range of different models, including the most popular modelling alternatives in the market. One goal of this chapter is to understand how to perform model comparison and model choice.
- We show how to deal with model uncertainty with provisions such as Reserves and Model Lines or Limits to Exposure. We perform market intelligence and show how to interpret the results of it with reverse engineering.
- The first example is introduced here for the first time, for the other two we analyze the existing literature and then go beyond it.

In Chapter 3 we look at stress-testing to understand the core risk of a payoff by using models, an issue already tackled in the previous chapter, and we look at the stress-testing of models to understand their weaknesses, an issue resumed later in Chapter 6.

- We devote particular attention to avoiding the pitfalls that are most likely to occur when performing stress-testing.
- We investigate what cases of stress one should consider (market conditions, payoff features, characteristics of the counterparties...) and we see a few examples of how to use historical and cross-section market information to design stress scenarios.
- As a playground we display here, via stress-testing, the errors in the practice of credit derivatives that were at the center of the crisis, including the still widespread copula and mapping methods, and present alternatives to these.

In Chapter 4 we consider the most painful event in terms of model losses: when a model consensus in the market suddenly breaks down and is replaced by a radically different standard.

- We carry the study on with the purpose of understanding the mechanisms of consensus change, already considered in the first chapter, so as to be not fully unprepared for the changes that will happen in the future.
- The first example of the death of a model, and the birth of a new one, regards the changes that happened recently to the pricing of even the simplest interest rate derivatives: the separation of discounting and forwarding, the multiplication of term-structures and the explosion of basis spreads. In this analysis we investigate the hidden assumptions of a modelling framework, by seeing how the traditional mathematical representation of interest rates taught in books must be replaced by a different approach.
- The second example, related to the first one, deals with the inclusion of liquidity and funding in pricing. Since we are still in the middle of this transformation of pricing foundations, we can now study the risks to which we would be exposed depending on the direction the market takes.

The second part of this book is devoted to those aspects of the practice in the financial markets where model risk management is most crucial.

In Chapter 5 we focus on hedging, an activity based on models but dangerously overlooked by the research in quantitative finance, or addressed in a theoretical way unrelated to practice. We take a different approach.

- We study how models are used in real hedging, and how this differs from their use in pricing. These differences must be studied and the intrinsic risks understood and managed. The principal example is on local and stochastic volatility models for equity options.
- We look at how to perform a P&L-Explain test, where one tests the hedging performance of a model. We want to understand the limitations of this technique but also what it can actually tell us about the appropriateness of a model.

In Chapter 6 we focus on computational methods, in order to understand how they must be assessed, stress-tested, and their efficiency monitored.

- We focus on approximations since these can hide the sneakiest model risk. In fact when market conditions change approximations often break down, but the market may take some time to react.
- The examples we see regard mostly the approximations used in the interest rate market, for example convexity adjustment, BGM-model approximations or the SABR formula. In testing them we also show the problems they are having in the current market conditions.
- We see how an approximation can be tested against an exact method or against a more precise numerical procedure. We also show examples and exercises of the risks in simulation and numerical integration.

In Chapter 7 we analyze the risks associated with two common operations: interpolation and extrapolation. We show two approaches:

- How to use non-trivial market information in order to minimize the need for extrapolation. We see this in particular for the volatility smile.
- How to use the mathematical properties of some quantities in order to make interpolation more consistent and avoid the use of extrapolation. Here we focus on the correlation skew.

In Chapter 8 we tackle the risk involved in correlation modelling from two different perspectives:

- We present useful technical solutions for modelling and parameterizing correlations, with examples from different asset classes where correlations need to have different properties.
- We explore the most common errors made when devising assumptions about correlation, such as assuming rigid relations for factors that have a degree of independence (the 1-correlation risk) and conversely the risk of taking as unrelated those things that have structural links (the 0-correlation risk). Two market cases are observed.

In Chapter 9 we complete the treatment of a topic that is covered in almost all other chapters: calibration. We look at exposing the residual mode uncertainty that remains after a calibration, and minimizing this uncertainty by enrichment of the calibration set.

- Introducing some model risk management tools needed to perform diagnostics of a calibration procedure, such as assessing the stability of the resulting model.

Chapter 10 is devoted to an issue that at times is not included in a narrow definition of model risk, but has high relevance: the risk of errors in the description of the payoff.

- We consider the case when the errors arise from a superficial interpretation of the termsheet or of the legal prescriptions. We see an example that has a strong impact on the pricing of counterparty risk.
- We consider the errors that arise from simplifications introduced to ease the mathematical representation of a payoff. The example is on Index options.

Chapter 11 considers an application of models which is typical of hedge funds or proprietary trading desks: using models for statistical or model arbitrage, exploiting temporary inconsistencies among related products. We see in practice two classic examples:

- Capital-structure arbitrage, based on equity and bonds/CDS, and here addressed with a recent structural model.
  Cap-swaption arbitrage in a Libor market model.
- We show by looking at empirical results how “arbitrage trades" can be easier to risk manage as directional trades on market uncertainty.

WHAT ELSE YOU WILL FIND IN THIS BOOK

In explaining model risk and model validation, we describe in detail practical examples where we cover a number of relevant topics for today’s finance, not mentioned, or only hinted at, in the above summary:

- Correlation modelling for equity with stochastic volatility, interest rates, FX rates, default events.
- The comparison of local vs stochastic volatility models both in terms of hedging and in terms of pricing path-dependent/forward-start derivatives.
- The most dangerous correlation errors in the computation of wrong-way counterparty risk.
- The modern pricing of interest rate derivatives with multiple curves for basis swaps and alternative discounting curves.
- The up-to-date treatment of the impact of funding liquidity in pricing.

- The impact of market illiquidity on the way we compute prices, and its relation to model uncertainty.
- How to set quantitative triggers to detect when a market formula is going to break down.
- Bubbles, arbitrage and market completeness in practice.
- A detailed account of the development of the credit crunch and its relationship with model choices and model errors.
- Diagnostic tools used on the behaviour of a model, such as the way to compute the model-implied evolution of volatilities and smiles.
- What is really explained by P&L-Explain tests.
- Different examples of reverse-engineering to understand which models can have generated observable prices.
- The analysis of the most relevant problems when using copulas for default events, the impossibility to control the timing of related events, and a solution to this.
- The analysis of gap risk using different models that treat information differently.
- The meaning, advantages and risks of taking into account the default of our institution in pricing (DVA).
- Detailed examples from asset classes including credit, interest rates, equity, cross-currency and funding.
- The analysis of the behaviour of the SABR model and the limits of its pricing formulas.
- The large number of changes to modelling standards which are required by the post-crisis market patterns.
- The risks hidden within the pricing procedures for plain vanilla derivatives.
- An alternative way to model correlations that can explain the correlation skew.
- Counterparty risk adjustment and the indetermination associated with an unclear legal definition of default payments.
- The reality of the lack of fundamental information in markets and the role this plays in derivatives marketing and trading.
- Dealing with funding liquidity and credit simultaneously and the risks of double-counting, loss of competitiveness or excessively aggressive behaviour.
- New analysis on the pricing of Bermudan swaptions and CMS derivatives.
- We explore the popular issue of calibrating a model to European options and then applying it to early exercise American/Bermudan derivatives.
- The explanation via liquidity and counterparty risk of the presence of basis swaps, and the hedging consequences of multiple curves.
- The explanation and a non-standard analysis of a range of models that include local and stochastic volatility models, jump models, the Libor market model for interest rate derivatives, structural models, copulas, mapping methods, reduced form credit models.
- Two analyses of correlation risk in hedging, for equity and for rates.
- And much more... but not inflation, nor the variance-gamma model!

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“The wide world is all about you; you can fence yourselves in, but you cannot forever fence it out.”

Gildor in ‘The Lord of the Rings’, by J.R.R Tolkien

“There was no way, without full understanding, that one could have confidence that conditions the next time might not produce erosion three times more severe than the time before. Nevertheless, officials fooled themselves into thinking they had such understanding and confidence, in spite of the peculiar variations from case to case. A mathematical model was made to calculate erosion. This was a model based not on physical understanding but on empirical curve fitting . . . Similar uncertainties surrounded the other constants in the formula. When using a mathematical model careful attention must be given to uncertainties in the model.”

Richard Feynman, from ‘Rogers’ Commission Report into the Challenger Crash Appendix F – Personal Observations on Reliability of Shuttle’

“It does not do to leave a live dragon out of your calculations, if you live near him.”

‘The Hobbit’, by J.R.R Tolkien

“Official management, on the other hand, claims to believe the probability of failure is a thousand times less. One reason for this may be an attempt to assure the government of NASA perfection and success in order to ensure the supply of funds. The other may be that they sincerely believed it to be true, demonstrating an almost incredible lack of communication between themselves and their working engineers.”

Richard Feynman, from ‘Rogers’ Commission Report into the Challenger Crash Appendix F – Personal Observations on Reliability of Shuttle’

“Now, therefore, things shall be openly spoken that have been hidden from all but a few until this day . . . And I will begin that tale, though others shall end it . . . You may tarry, or come back, or turn aside into other paths, as chance allows. The further you go, the less easy will it be to withdraw.”

Elrond in ‘The Lord of the Rings’, by J.R.R Tolkien

Part I

Theory and Practice of Model Risk Management

1
Understanding Model Risk

1.1 WHAT IS MODEL RISK?

In the last years, during and after the credit crunch, we have often read in the financial press that errors on 'models' and lack of management of 'model risk' were among the main causes of the crisis. A fair amount of attacks regarded mathematical or quantitative models, like the notorious Gaussian copula, that were accused to be wrong and give wrong prices for complex derivative, in particular credit and mortgage-related derivatives. These criticisms to valuation models have been shared also by bank executives and people that are not unexperienced on the reality of financial markets. In spite of this it is not very clear when a model must be considered *wrong*, and as a consequence it is not clear what model risk is.

We can probably all agree that *model risk is the possibility that a financial institution suffers losses due to mistakes in the development and application of valuation models*, but we need to understand which mistakes we are talking about.

In the past, model validation and risk management focused mainly on detecting and avoiding errors in the mathematical passages, the computational techniques and the software implementation that we have to perform to move from model assumptions to the quantification of prices. These sources of errors are an important part of model risk, and it is natural that model risk management devotes a large amount of effort to avoid them. We will devote a share of the second part of this book to related issues. However, they regard that part of model risk that partially overlaps with a narrow definition of operational risk: the risk associated to lack of due diligence in tasks for which it is not very difficult to define what should be the right execution. Is this what model validation is all about? In natural science, the attempt to eliminate this kind of error is not even part of model validation. It is called *model verification*, since it corresponds to verifying that model assumptions are turned correctly into numbers. The name model validation is instead reserved to the activity of assessing if *the assumptions of the model are valid*. Model assumptions, not computational errors, were the focus of the most common criticisms against quantitative models in the crisis, such as 'default correlations were too low'.

The errors that we can make in the assumptions underlying our models are the other crucial part of model risk, probably underestimated in the past practice of model risk management. They are the most relevant errors in terms of impact on the reputation of a financial institution that works with models. A clear example is what happened with rating agencies when the subprime crisis burst. When they were under the harshest attacks, rating agencies tried to shield themselves from the worst criticisms by claiming that the now evident underestimation of the risk of credit derivatives was not due to wrong models, but to mistakes made in the software implementation of the models. Many market operators, that knew the models used by rating agencies, did not believe this justification, and it had no other effect than increasing the perception that wrong models were the real problem. What is interesting to notice is that admitting wrong software appeared to them less devastating for their reputation than admitting wrong models.

Unfortunately, errors in mathematics, software or computational methods are easy to define and relatively easy to detect, although this requires experience and skills, as we will see in the second part of the book. Errors in model assumptions, instead, are very difficult to detect. It is even difficult to define them. How can we, as the result of some analysis, conclude that a model, intended as a set of assumptions, has to be considered wrong? We need to understand when a valuation model must be called wrong in order to answer to our first crucial question, *what is model risk?*

In this section we look for the answer. The first sources we use to clarify this issue are the words of a few legendary quants that in the past have tried to say when models are right or wrong in order to give a definition of model risk. You will see that not even among quants there is consensus about what model risk is. But then, when we apply these approaches to past crises to understand how they could have protected us from the worst model losses, we will see that the different approaches can lead to similar practical prescriptions.

1.1.1 The Value Approach

As early as 1996, before both the LTCM collapse and the credit crunch, the two events that put most critical pressure on the risk involved in using mathematical pricing models, one of living legends of quantitative finance, Emanuel Derman, wrote a paper titled *Model Risk*. This is a natural starting point to define our subject, also because it can be seen as the foundation of one of the two main schools of thought about model risk. The views of the author on the subject are further specified by a later paper written in 2001 that addresses model validation prescriptions, under the title *The Principles and Practice of Verifying Derivatives Prices*.

Derman notices first that the previous years had seen the emergence of an *'astonishingly theoretical approach to valuation of risky products. The reliance on models to handle risk'*, he points out, *'carries its own risk'*. Derman does not give a definition of model risk, but he indicates some crucial questions that a model validator should have in mind:

1. Is the payoff accurately described?
2. Is the software reliable?
3. Has the model been appropriately calibrated to the prices of the simpler, liquid constituents that comprise the derivative?
4. *'Does the model provides a realistic (or at least plausible) description of the factors that affect the derivative's value?'*

Can we deduce a definition of model risk from these points? The first two points are not trivial. When speaking of approximations and numerics in Chapter 6 we will talk of errors to avoid in implementation, and we even devote the entire Chapter 10 to the errors that can be made in the description of a payoff. However, these points do not add to our understanding of Derman's ideas about the nature of the errors we can make in model assumptions.

The third point instead underlines a feature that models must have: the capability to price consistently with the market the simpler instruments related to a derivative, namely to perform the so-called *calibration*. This is an important issue, on which we will focus later on. But not even this point clarifies what model risk is. All banks, now, calibrate their models to liquid market prices. For any asset class or financial product there are many models which are different from each other and yet can all be calibrated very well to the market. Once we have satisfied this calibration constraint, are we sure that model risk has been eliminated, or instead the core of model risk is crucially linked to the fact that we have *different* models allowing for good calibration, so that calibration does not solve our model uncertainty?

A better clarification is given in the fourth point. From this we can deduce a definition of model risk. Once we are sure that we have correctly implemented payoff and software, and our model appears calibrated to the liquid underlying products, we have a residual risk that seems to be the core of model risk:

> *Model risk is the risk that the model is not a realistic/plausible representation of the factors affecting the derivative's* <u>*value*</u>

This is confirmed when Derman says that for less liquid or more exotic derivatives one must verify the *'reasonableness of the model itself'*. There is more. Derman (1996) gives an account of the things that can go wrong in model development, and he starts from some examples where lack of realism is surely the crucial problem:

> *'You may have not taken into account all the factors that affect valuation ... You may have incorrectly assumed certain stochastic variables can be approximated as deterministic .... You may have assumed incorrect dynamics ... You may have made incorrect assumptions about relationships'.* E. Derman, Model Risk.

So, is Derman saying that we should try to find out what the *true model* is? No, in fact he never uses those somewhat exoteric concepts like the *true model* or *right model*. He states, and it is hard to disagree, that a model is always an *'attempted simplification of a reality'*, and as such there can be no true or perfectly realistic model. But realism and reasonableness, coupled with simplicity, must remain crucial goals of a modeller, and their lack creates model risk.

Is Derman saying that we must look for realism and reasonableness in all aspects of the model? Not either. We must care for those aspect that have a relevant impact, limiting the analysis to '*the factors that affect the derivative's value*'.

This approach to model risk is probably the one shared by most practitioners of finance and beyond, and does not appear too far away from the views expressed more recently by Derman. For example, in the 'Financial Modeler's Manifesto', written with Paul Wilmott, another legend of quant finance, we read among the principles that a modeler should follow *'I will never sacrifice reality for elegance without explaining why I have done so. Nor I will give the people who use my model false comfort about its accuracy'*. We refer to this, and to Derman's recent book 'Models Behaving Badly – Why Confusing Illusion with Reality Can Lead to Disaster, on Wall Street and in Life', whose title is already very telling, for more about Derman's views.

It is clear to everyone that knows finance and does not confuse it with mathematics and not even with physics, that there is not such a thing as the 'true value' of a derivative that the model should be able to compute. However realism and capability to describe the actual behaviour of the relevant risk-factors are crucial principles to judge a model, and more realistic models should be preferred. Somewhat, we can say that the right model and the right value do not exist in practice, but wrong models and wrong values do exist, they can be detected and we should commit ourselves to find models giving values as 'little wrong' as possible, and then manage the residual unavoidable risk. This is the reason why we talk of 'Value approach'.

There are cases where we can all agree that the price given by some models does not correspond to the value of a derivative. Most of these cases are trivial. If we are selling an out-of-the money option on a liquid volatile underlying, the model we use must incorporate some potential future movement of the underlying. We cannot use a deterministic model, assuming no volatility. Otherwise we would be selling the option for nothing, based on an assumption that can be disproved just waiting a bit and seeing the price of the underlying move in the market.

We will see other examples which are less trivial and yet we can easily spot that some assumptions are not realistic. To give an example regarding the infamous credit models, you will see in Chapter 2 the case of default predicted exactly by spreads going to infinity according to standard structural models or in Chapter 3, speaking of Gaussian copula, again a default predicted exactly, and some years in advance, by the default of another company. These assumptions are unrealistic and yet they are hidden in two very common models. When they do not impact in a relevant way the value of a derivative, we can consider them harmless simplifications. When, like in the examples we will analyze, we can show that they impact strongly the value of a derivative, we should raise a warning. At times it is more difficult to say when a relevant feature of a model is realistic or not; in this case we will have to use our judgement, collect as much information as possible and try to make the best possible choice.

You may at first think that everyone must agree with such a reasonable and no-nonsense approach, and with the definition of model risk it implies. It is not like that. A view on Model Risk that starts from completely different foundations is analyzed in the next section.

1.1.2 The Price Approach

If Derman has been one of the fathers of quantitative modelling between the end of the eighties and the nineties, Riccardo Rebonato marked the development of interest rate models – the field where the most dramatic quantitative developments have been done – between the end of the nineties and the subsequent decade. He has been a master in bridging the gap between complex mathematics and market practice. After the turn of the century Rebonato wrote a paper titled *Theory and Practice of Model Risk Management* that presents a view on the subject strongly different, at first sight, from the classic view explained above.

Rebonato (2003) takes the point of view of a financial institution, which is worried not only of the material losses associated to model risk, but even more of the effect that evidence of model risk mismanagement can have on the reputation of a financial institution and its perceived ability to control its business. Under this point of view, this classic definition of model risk and model validation are misplaced. In fact derivatives need to be marked-to-market, as we will see in Section 1.3, and this means that the balance-sheet value of a derivative must come as much as possible from market prices.

If this is the situation, what should the main concern of a model validation procedure be? Should we worry so much that '*the model provides a realistic (or at least plausible) description of the factors that affect the derivative's value'?* Well . . . at least this is not the first concern we must have, since, to use the words of Rebonato, *'Requiring that a product should be marked to market using a more sophisticated model (ie, a model which makes more realistic assumptions) can be equally misguided if . . . the market has not embraced the "superior" approach.'*

These considerations lead Rebonato to an alternative definition of model risk, that has become so popular that we can consider it the motto of a different approach to model risk, the Price approach:

> *'Model risk is the risk of occurrence of a significant difference between the mark-to-model value of a complex and/or illiquid instrument, and the* price *at which the same instrument is revealed to have traded in the market'.* Rebonato R., *Theory and Practice of Model Risk Management*

Rebonato (2003) justifies this view pointing out that the real losses that hit an institution's balance sheet usually do not appear *'because of a discrepancy between the model value and* *the "true" value of an instrument'*, but through the mark-to-market process, because of a discrepancy between the model value and the market price.

Fair enough. It is hard to disagree with such statements. As long as the market agrees with our model valuation, we do not have large losses due to models. When we evaluate with a model which is the same one used to reach market prices, we do not have model losses arising from mark-to-market thus we have no accounting losses. More interestingly, we can also avoid material losses, because, if the market agrees with our valuation model, we can always sell an asset or extinguish a liability at the price at which we have booked. This is true even if the market model is, to use the words of Rebonato, *'unreasonable, counterintuitive, perhaps even arbitrageable'*.[1]

This has another implication. When the market price can be observed quite frequently, there is little time during which the model price and market price of a derivative can diverge, so that big model risk is unlikely to be generated. If a bank notices a mispricing, this will be controlled by provisions such as stop-losses and will not generate losses so big to worry an institution, although they can worry a single trader. The problem arises with very complex or illiquid products, for which market prices are not observed frequently. Then the model price of a derivative and its market price can diverge a lot, and when eventually the market price gets observed a large and sudden loss needs to be written in the balance-sheet, with effects on a bank which are also reputational.

The different definition of model risk given by Rebonato (2003) requires, at least at first sight, a different approach to model validation. Large losses with reputational damage emerge when a sudden gap opens between market price and model booking. This can happen for three reason:

1. The reason can be that we were using a model different from the market consensus, and when we are forced to compare ourselves with the market – because of a transaction or because the market consensus has become visible – this difference turns into a loss. From this comes the first prescription of the Price approach, given strongly in Rebonato (2003), to gather as much information as possible on the approach currently used by the majority of the market players. This can be done through different channels. We follow Rebonato (2003) and we add some more of our own, which have become more important after Rebonato's paper was written.
   A. Some channels are based on the idea that if we can observe prices from counterparties, then we can perform *reverse-engineering* of these prices, namely we can understand which models were used to generate them. Examples of how this can be performed are in Chapter 2, in Section 4.1 and throughout the book. How can we collect counterparty prices when the market is not liquid?
      - getting as much information as possible about the deals which are struck in the market or other closeout prices such as those for unwindings and novations.
      - analyzing the collateral regulations with counterparties. Collateral is the amount of guarantees (usually cash) exchanged between banks in order to protect the reciprocal

[1] Some could argue that losses may arise, even if we use the same model used by the market, from the fact that we are hedging with an unreasonable model. We discuss similar issues in Chapter 5, where we will see that the above argument has some solid foundations, but also that real hedging strategies do not follow strictly model assumptions, so that it can be difficult to quantify the hedging losses due to unreasonableness of a valuation model. According to Rebonato (2003), in any case, losses incurred because of an 'incorrect' hedging strategy are unlikely to be of such magnitude to have a major impact, and thus should not be the focus of model risk management. More recently, Nawalkha and Rebonato (2011) points out that when a derivative is hedged, losses due to model errors may cancel out, at least in part, between the derivative and the hedge.

exposures from counterparty risk. The amount of collateral must be kept equal to the expected discounted exposure, that corresponds approximately to the price of all deals existing between two counterparties. We can observe this frequent repricing from our counterparties, in some cases also specifically for a single deal, to get information on the models they use.
- monitoring broker quotes (that usually do not have the same relevance as prices of closed deals) and consensus pricing systems such as Mark-it Totem. This is a service that collects quotes from market operators on a range of different over-the-counter derivatives, eliminates the quotes that appear not in line with the majority, and then computes an average of the accepted quotations. The market operators whose quotes were accepted get informed about the average. There are derivatives for which this service provides a very relevant indication of market consensus. Today, this is considered an important source of market information.

B. A few channels suggested by Rebonato (2003) regard gathering *market intelligence* by
- attending conferences and other technical events where practitioners present their methodologies for evaluating derivatives.
- asking the salesforce for any information they have about counterparty valuations. Additionally, salespeople can inform us if the prices computed with our models appear particularly competitive in the market (are we underestimating risk?) or are regularly beaten by competitors' prices (are we being too conservative?).
- Rebonato (2003) says finally that 'contacts with members of the trader community at other institutions are invaluable'. We can rephrase it, less formally, as follows: keep in touch with your college mates that work in other banks and make them speak out about the model they use at the third pint of beer at the pub.

2. If, thanks to any of the above channels, we are confident that we are using the same model prevailing in the market and this model is not changing, the only cause for large gaps between our booking and market prices can be the model/operational errors like software bugs or errors in describing the payoff. Therefore these errors must be avoided.
3. The two points above do not appear to help us in the past examples of big market losses. In 1987 there appeared to be a market consensus on the use of something similar to the Black and Scholes formula to price equity derivatives. After the market crash in October 1987 the pricing approach changed dramatically, with a clear appearance of the smile. The market consensus had moved from a lognormal model to some approximation of a model with fat-tails, may it be a stochastic volatility model or a model admitting jumps, and this was a big source of losses. Those that had sold out-of-the-money puts for nothing had to book a loss not only because of the fall of the underlying, but also because the volatility used by market player to evaluate them became much higher than the one used for at-the-money options. Even following the above points 1) and 2) of the Price approach, we would have been completely exposed to such losses. Similar market shifts in the pricing approach to interest rate derivatives characterized the aftermath of the LTCM crisis in 1998. And we have recently experienced the most dramatic event of this type with the subprime crisis and the fall of the Gaussian copula based pricing framework for CDOs. This gives the third way in which we can have a large gap between the way we were pricing and the market price: even if we are using the market consensus model, the market consensus can suddenly change. This issue is taken into account by Rebonato (2003) that, after presenting knowledge of the market approach as the first task of a model risk manager, adds that *'the next important task of the risk manager is to surmise how today's accepted pricing methodology might change in the future.'*