Wolfgang Hofmann | Jürgen Schlabbach | Wolfgang Just

REACTIVE **POONER** COMPENSATION A PRACTICAL GUIDE



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REACTIVE POWER COMPENSATION

REACTIVE POWER COMPENSATION A PRACTICAL GUIDE

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Foreword and Acknowledgements

The book gives a general overview and also specific deep knowledge about the segment "compensation of reactive power". Network quality, power losses, energy saving and reduction of CO_2 are discussed within 22 chapters forming a technical "dictionary". It is written to be accessible for all specialists; including engineers, electricians and students. The purpose of this book is to extend the knowledge in this specified field. This "technical guide" answers a lot of questions arising in controlling reactive power e.g. at parallel operation with generators.

This book is based on a book in German published by the VDE Verlag GmbH (Berlin and Offenbach) in the year 2003. Some chapters were revised and adapted or entirely rewritten to conform to new technological developments and changing standards. The chapters have been primarily revised by:

Jürgen Schlabbach – Chapters 1 incl. 4; 18 and 22 Wolfgang Hofmann – Chapters 5 incl. 16 and 21 Wolfgang Just – Chapters 17, 19 and 20

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Basics of Reactive Power

1.1 Chapter Overview

This chapter deals with the definitions and fundamentals of active, reactive and apparent power in the case of sinusoidal and non-sinusoidal current and voltage. The differences between power factor, taking account of only the fundamental frequency components, and distortion factor, taking account of higher frequency components as well, are explained. Equivalent mechanical models are presented to explain the behaviour of inductance and capacitance and the generation of reactive power.

1.2 Phasors and Vector Diagrams

Motors, discharge lamps, transformers, generators with lagging power factor, as well as cables and overhead lines with high current loading, need reactive power to build up the magnetic field, sometimes called the consumption of reactive or inductive power. Other equipment and consumers, such as rectifiers with capacitive smoothing, compact fluorescent lamps, capacitors, generators with leading power factor and overhead transmission lines and cables in no-load or low-load operation, need reactive power to build up the electric field, an effect called the generation of reactive or capacitive power. In contrast to active power, reactive power is not converted into heat, light or torque, but fluctuates between the source (e.g. capacitor) and the drain (e.g. motor). Compared with pure active power, the current is increased as the active current and the reactive current are added to the apparent current according to their amount and phase angle.

When dealing with AC and three-phase systems, it should be noted that currents and voltages are generally not in phase. The phase position depends on the amount of inductance, capacitance and ohmic resistance at the impedance.

The time course, for example of a current or voltage, varies in accordance with

$$u(t) = \hat{u}\cos\left(\omega t + \varphi_U\right) \tag{1.1a}$$

$$i(t) = i\cos\left(\omega t + \varphi_I\right) \tag{1.1b}$$

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Figure 1.1 Vector diagram and time course of AC voltage [1].

as can be shown in a line diagram, see Figure 1.1. In the case of sinusoidal variables, these can be shown at the complex numerical level by rotating pointers, which rotate in a mathematically positive sense (counter-clockwise) with angular velocity ω as follows:

$$\underline{U} = \sqrt{2}U \cdot e^{(j\omega t + \varphi_U)} \tag{1.2a}$$

$$\underline{I} = \sqrt{2I} \cdot e^{(j\omega t + \varphi_I)} \tag{1.2b}$$

The time course in this case is obtained as a projection onto the real axis, as in Figure 1.1. The terms for the designation of resistances and admittances are stipulated in DIN 40110 [2] and in IEC 60027-7 [3]. These specify the following:

Resistance	R	Active resistance
Reactance	X	Reactance
Conductance	G	Active conductance
Susceptance	В	Susceptance

The generic term for resistances is given as impedance or apparent impedance

$$\underline{Z} = R + jX \tag{1.3a}$$

The generic term for conductance is admittance or apparent admittance

$$\underline{Y} = G + jB \tag{1.3b}$$

The reactance depends on the particular frequency under consideration and can be calculated for capacitances or inductances from

$$X_C = \frac{1}{\omega C} \tag{1.4a}$$

$$X_L = \omega L \tag{1.4b}$$

For sinusoidal variables, the current through a capacitor, or the voltage at an inductance, can be calculated as follows:

$$i(t) = C \cdot \frac{du(t)}{dt} \tag{1.5a}$$

$$u(t) = L \cdot \frac{di(t)}{dt}$$
(1.5b)

The derivation for sinusoidal variables establishes that the current achieves, by an inductance, its maximum value a quarter period after the voltage. When considering the process at the complex level, the pointer for the voltage precedes the pointer for the current by 90°. This corresponds to multiplication by +j.

For capacitance, on the other hand, the voltage does not reach its maximum value until a quarter period after the current, the voltage pointer lagging behind the current by 90°, which corresponds to multiplication by -j. This enables the relationship between current and voltage for inductances and capacitances to be expressed in a complex notation. Thus

$$\underline{U} = j\omega L \cdot \underline{I} \tag{1.6a}$$

$$\underline{I} = \frac{1}{j\omega C} \cdot \underline{U} \tag{1.6b}$$

Vectors are used to describe electrical processes. They are therefore used in DC, AC and three-phase systems. Vector systems can, by definition, be chosen as required, but must not be changed during an analysis or calculation. It should also be noted that the appropriate choice of the vector system is of substantial assistance in describing and calculating special tasks. The need for vector systems is clear if one considers Kirchhoff's law, for which the positive directions of the active and reactive powers are then also stipulated.

For reasons of comparability and transferability, the vector system for the three-phase network (L1,L2,L3 components or RYB) should also be used for other component systems (e.g. symmetrical components), which describe the three-phase network.

If vectors are drawn as shown in Figure 1.2, the active and reactive powers, for instance output by a generator in overexcited operation, are positive. This vector system is designated as a generator vector system (GVS). Accordingly, the active and reactive powers consumed by



Figure 1.2 Definition of vectors for current, voltage and power in three-phase AC systems [1]: (a) power system diagram; (b) electrical diagram for symmetrical conditions (positive sequence system).



Figure 1.3 Vector diagram of current, voltage and power [1]: (a) related to consumer vector system – CVS); (b) related to power generation (generation vector system – GVS).

the load (e.g. motor) are positive when choosing the consumer vector system (CVS). Figure 1.3 shows the phasor diagram of an ohmic–inductive load in the generator and in the consumer vector system.

1.3 Definition of Different Types of Power

The definitions and explanantions are given in accordance with DIN 40110 [2]. The instantaneous value of the power p(t) in an AC system is calculated as follows:

$$p(t) = u(t) \cdot \dot{i}(t) \tag{1.7}$$

with i(t) and u(t) as the instantaneous values of current and voltage. Generally the product of current and voltage is oscillating and shows positive and negative values within one period. The mean value of the oscillating power is called active power \overline{P} :

$$\overline{P} = \frac{1}{T} \int_{0}^{T} u(t) \cdot i(t) dt$$
(1.8)

In the case of sinusoidal current and voltage

$$u(t) = \hat{u}\cos\left(\omega t + \varphi_U\right) \tag{1.9a}$$

$$i(t) = \hat{i}\cos\left(\omega t + \varphi_I\right) \tag{1.9b}$$

The instantaneous value of the power p(t) as the product of the instantaneous values of current and voltage is

$$p(t) = \hat{u}\hat{i}\cos(\omega t + \varphi_U)\cos(\omega t + \varphi_I)$$
(1.10a)

After some numerical operations and with $\varphi = \varphi_U - \varphi_I$, the following equation is obtained:

$$p(t) = \frac{\hat{u}\hat{i}}{2}\cos\varphi + \frac{\hat{u}\hat{i}}{2}\cos(2\omega t + \varphi)$$
(1.10b)

Equation 1.10b indicates that the power p(t) oscillates with twice the frequency of the current and voltage; its mean value is called active power P:

$$P = \frac{\hat{u}\hat{i}}{2}\cos\varphi \tag{1.11a}$$

The term $\hat{ui}/2$ is called apparent power S:

$$S = \frac{\hat{u}\hat{i}}{2} \tag{1.11b}$$

If one eliminates φ_I in the above equations the following is obtained:

$$p(t) = \frac{\hat{u}\hat{i}}{2}\cos\varphi + \frac{\hat{u}\hat{i}}{2}\cos\varphi \cdot \cos\left(2\omega t + 2\varphi_U\right) + \frac{\hat{u}\hat{i}}{2}\sin\varphi \cdot \sin\left(2\omega t + 2\varphi_U\right)$$
(1.12)

The term $(\hat{u}\hat{i}/2)\sin\varphi$ is called reactive power Q. The reactive power oscillates with twice the frequency of the current and voltage; its mean value is zero:

$$Q = \frac{\hat{u}\hat{i}}{2}\sin\varphi \tag{1.11c}$$

The reactive power in the CVS is positive if the phase angle φ is between 0° and +180°; that is, if the voltage pointer leads the current pointer. In this case the reactive power is called the inductive power, which is the power drawn from the system by a reactance. If the voltage pointer lags behind the current pointer, which is when the phase angle φ is between 0° and -180° , the reactive power becomes negative. This is called capacitive power, as it is the power supplied to the system by a capacitance.

In general, the following equation is valid

$$|Q| = \sqrt{S^2 - P^2} \tag{1.13}$$

for the amplitudes of the active power P, reactive power Q and apparent power S are defined as above. If rms values are used instead of peak values, as is common in calculating power systems, the active, reactive and apparent power become

$$P = P = U \cdot I \cos(\varphi_U - \varphi_I) \tag{1.14a}$$

$$Q = U \cdot I \sin(\varphi_U - \varphi_I) \tag{1.14b}$$

$$S = U \cdot I \tag{1.14c}$$

The quotient from active power *P* and reactive power *S* is called the power factor λ . In the case of sinusoidal currents and voltages the power factor is identical to the distortion factor of the fundamental frequency $\cos \varphi_1$.

Figure 1.4 indicates the time course of current and voltage at an ohmic–inductive consumer load and the resulting active, reactive and apparent power.

1.4 Definition of Power for Non-Sinusoidal Currents and Voltages

Active power can only be converted if current and voltage have equal frequency, as the integral for current and voltage of unequal frequency in accordance with

$$\overline{P} = \frac{1}{T} \int_{0}^{T} u(t) \cdot i(t) dt$$
(1.8)

makes no contribution.

If current and voltage both have a non-sinusoidal waveform

$$u(t) = \sum_{k=1}^{N} \hat{u}_{k} \cos(k\omega_{1}t + \varphi_{U,k})$$
(1.15a)

$$i(t) = \sum_{l=1}^{N} \hat{i}_{l} \cos(l\omega_{1}t + \varphi_{l,l})$$
 (1.15b)

the instantaneous value of the power is calculated as

$$p(t) = \sum_{k=l=1}^{N} \frac{\hat{u}_{k}\hat{i}_{l}}{2} \cos(\varphi_{U,k} - \varphi_{I,l}) + \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{\hat{u}_{k}\hat{i}_{l}}{2} \cos((k+l)\omega_{1}t + \varphi_{U,k} + \varphi_{I,l}) + \sum_{\substack{k=1\\k \neq l}}^{N} \sum_{l=1}^{N} \frac{\hat{u}_{k}\hat{i}_{l}}{2} \cos((k-l)\omega_{1}t + \varphi_{U,k} - \varphi_{I,l})$$
(1.16)



Figure 1.4 Current, voltage and powers at an ohmic–inductive consumer load: (a) current and voltage; (b) active, reactive and apparent power.

The first summand describes the active power, whereby the component with k = l = 1 represents the fundamental component active power and the summands where k = l > 1 render the harmonic active powers. The second summand renders the reactive power Q and the third summand the distortion power Q_d . The time course of these powers oscillates non-sinusoidally about the zero-frequency mean value. Note that the higher frequencies of voltage and current generate active power as well, if their frequencies are the same.

The correlation between the powers is as follows (active part of fundamental current I_{w1} ; reactive part of fundamental current I_{b1} ; harmonic part of current I_{v}):

$$S^2 = P_1^2 + Q_1^2 + Q_d^2 (1.17a)$$

$$S^{2} = U^{2} \left(I_{w1}^{2} + I_{b1}^{2} + \sum_{\nu=2}^{H} I_{\nu}^{2} \right)$$
(1.17b)

The active power P_1 and the reactive power Q_1 are related to the fundamental frequency of current and voltage, and the distortion power Q_d is related to the harmonic currents and the fundamental frequency of the voltage:

$$P_1 = U \cdot I_1 \cdot \cos \varphi \tag{1.18a}$$

$$Q_1 = U \cdot I_1 \cdot \sin \varphi \tag{1.18b}$$

$$Q_d = U \cdot \sqrt{\sum_{\nu=2}^H I_\nu^2}$$
(1.18c)

The different terms are represented in a three-dimensional diagram as in Figure 1.5.

The power factor λ , which is defined as the quotient of active power and apparent power, is generally defined as follows:

$$\lambda = \frac{|P|}{\sqrt{(P^2 + Q_1^2 + Q_d^2)}}$$
(1.19)

The displacement factor $\cos \varphi_1$ is defined as the quotient of active power and apparent power with fundamental frequency (in the case of sinusoidal voltage and non-sinusoidal current):

$$\cos\varphi_1 = \frac{P_1}{\sqrt{P_1^2 + Q_1^2}} = \frac{P_1}{S_1}$$
(1.20)

The power factor λ and displacement factor $\cos \varphi_1$ are related to each other by the fundamental content g_i of the current:

$$\lambda = g_i * \cos \varphi_1 \tag{1.21}$$



Figure 1.5 Active, reactive, apparent and distortion power, power factor and displacement factor.

The fundamental content g_i is defined as the quotient of the rms value of fundamental current to the total rms value:

$$g_i = \frac{I_1}{I} \tag{1.22}$$

The total rms value also includes the higher frequency components of the current as well:

$$I = \sqrt{\sum_{\nu=1}^{H} I_{\nu}^{2}}$$
(1.23)

1.5 Equivalent Mechanical Model for Inductance

An equivalent model from mechanics can illustrate, as in Figure 1.6, the phenomena of inductance, capacitance, active and reactive power. A train with mass *m* is accelerated by the locomotive to its final velocity *v*. The pointers of force and velocity are in the same direction, and the power and energy supplied are positive as well. When the force is increased or decreased in a stepwise fashion, the velocity of the train does not change stepwise, but increases or decreases by means of an exponential function. The energy supplied, in the case of increasing force, or not supplied, in the case of decreasing force, is stored in the movement of the train, which is identical to the phenomena of storage and discharge of electrical energy in an inductance. The mechanical energy W_{mec} is given by

$$W_{mec} = m \cdot \frac{v^2}{2} \tag{1.24a}$$

and the electrical energy W_{el} by

$$W_{el} = L \cdot \frac{I^2}{2} \tag{1.25a}$$



Figure 1.6 Force and velocity while accelerating and decelerating a train [4].

Comparing electrical and mechanical phenomena, the equivalents are:

Voltage	=	Mechanical force
Current	=	Velocity
Inductance	=	Physical mass
Capacitance	=	Spring constant
Electrical energy	=	Mechanical energy
Electrical power	=	Mechanical power

If the force to accelerate the train is a sinusoidal function it is obvious that the velocity of the train does not change synchronously (with the same frequency), but with a time delay, see Figure 1.7. The maximal values of velocity and mechanical force have a time delay or phase shift similar to the phase shift between voltage and current at an inductance, which is described by the term 'reactive power'. Reactive power in this case is reactive power by an inductance. It is always present if the phasors of mechanical force (equivalent to the voltage) and velocity (equivalent to the current) have opposite directions and different signs. Reactive power W_{mag} in inductances stored in the magnetic field is

$$W_{mag} = L \cdot \frac{I^2}{2} \tag{1.25b}$$



Figure 1.7 Equivalent electrical and mechanical model (inductance and mass): (a) starting point; (b) accelerating – energy supply (imported); (c) decelerating – energy generation (exported); (d) exported energy (voltage switched off); (e) time course of current, voltage and power.

In the mechanical model the equivalent of the magnetically stored energy is the kinetic energy of the moving mass:

$$W_{kin} = m \cdot \frac{v^2}{2} \tag{1.24b}$$

1.6 Equivalent Mechanical Model for Capacitance

Reactive power can be compensated by capacitors, which store energy in the electric field:

$$W_{cap} = C \cdot \frac{U^2}{2} \tag{1.26}$$

The equivalent of a capacitor in the mechanical model is a spring, which stores energy (potential energy)

$$W_{pot} = k \cdot \frac{F^2}{2} \tag{1.27}$$

with mechanical force F and spring constant k. If a laminated spring (leaf spring) is compressed and expanded with a sinusoidal force, the maximum mechanical force is supplied when the velocity is zero. In the case of maximal velocity the mechanical force is zero, see Figure 1.8.



Figure 1.8 Equivalent electrical and mechanical model (capacitance and spring): (a) starting point; (b) compressed – energy supply (imported); (c) expanded – energy generation (exported); (d) discharging the capacitor, voltage switched off (exported); (e) time course of current, voltage and power.

Mechanical force and velocity are characterized by a time shift of 90°, similar to the time shift of current and voltage at a capacitor. The mechanical system 'mass \leftrightarrow spring' and the electrical system 'inductance \leftrightarrow capacitor' can both oscillate with a defined frequency, namely the resonance frequency.

1.7 Ohmic and Reactive Current

An ohmic–inductive load with a sinusoidal waveform of current and voltage, such as in AC motors, transformers and reactors, can be modelled as the equivalent circuit of an ohmic resistance R in parallel with an inductive resistance X_L as in Figure 1.9a. The current can be represented in this equivalent model as two orthogonal components, see Figure 1.9b, one in phase with the voltage U, called the active current I_w , and the other with a phase shift of 90° lagging, called the inductive or reactive current I_b . The apparent current I has a phase shift against the voltage of phase angle φ . The active component I_w of the current describes the ohmic component and active power, while the reactive component I_b describes the inductive component, representing the reactive power. A line diagram of current, voltage and power is outlined in Figure 1.9c.

Electrical parameters such as voltage, current and power can be described by pointers (vectors) with rms values represented by the length of the pointer. Figure 1.10 indicates the relationship of active, reactive and apparent current and power in the orthogonal system, representing the same quantities and relations as the line diagram in Figure 1.9c. The phase shift (phase angle φ) depends on the amount of the reactive component in relation to the active component. With constant reactive power and increasing active power, the power factor and



Figure 1.9 Phase shift of current and voltage in the case of ohmic–inductive load: (a) equivalent circuit diagram; (b) orthogonal components of current; (c) line diagram of current, voltage and power.



Figure 1.10 Orthogonal components of current and power: (a) current; (b) power.

the apparent power are both increasing; in the case of constant active power and increasing reactive power, the power factor is decreasing and the apparent power increasing. For details see also Figure 1.5 and Equation 1.20.

1.8 Summary

The power in AC systems has an oscillating time course; the mean value is called the active power. The reactive power has a mean value of zero and is determined by the phase angle between voltage and current. One has to distinguish between the fundamental power factor $\cos \varphi$, sometimes called the displacement factor, which takes account of the active power and reactive power at the fundamental frequency, and the power factor λ , which takes the distortion power Q_d of the higher frequencies (harmonics and interharmonics) into account as well.

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