HILBERT TRANSFORM APPLICATIONS IN MECHANICAL VIBRATION

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Hilbert Transform Applications in Mechanical Vibration addresses recent advances in theory and applications of the Hilbert transform to vibration engineering, enabling laboratory dynamic tests to be performed more rapidly and accurately. The author integrates important pioneering developments in signal processing and mathematical models with typical properties of mechanical dynamic constructions such as resonance, nonlinear stiffness and damping. A comprehensive account of the main applications is provided, covering dynamic testing and the extraction of the modal parameters of nonlinear vibration systems including the initial elastic and damping force characteristics. This unique merger of technical properties and digital signal processing allows the instant solution of a variety of engineering problems and the in-depth exploration of the physics of vibration by analysis, identification and simulation. This book will appeal to both professionals and students working in mechanical, aerospace, and civil engineering, as well as naval architecture, biomechanics, robotics, and mechatronics.

Hilbert Transform Applications in Mechanical Vibration employs modern applications of the Hilbert transform time domain methods including:

- The Hilbert Vibration Decomposition method for adaptive separation of a multi-component non-stationary vibration signal into simple quasi-harmonic components; this method is characterized by high frequency resolution, which provides a comprehensive account of the case of amplitude and frequency modulated vibration analysis.
- The FREEVIB and FORCEVIB main applications, covering dynamic testing and extraction of the modal parameters of nonlinear vibration systems including the initial elastic and damping force characteristics under free and forced vibration regimes. Identification methods contribute to efficient and accurate testing of vibration systems, avoiding effort-consuming measurement and analysis.
- Precise identification of nonlinear and asymmetric systems considering high frequency harmonics on the base of the congruent envelope and congruent frequency.
- Companion website houses MATLAB® / Simulink® codes.

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Michael Feldman

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Preface

The object of this book, *Hilbert Transform Applications in Mechanical Vibration*, is to present a modern methodology and examples of nonstationary vibration signal analysis and nonlinear mechanical system identification. Nowadays the Hilbert transform (HT) and the related concept of an analytic signal, in combination with other time–frequency methods, has been widely adopted for diverse applications of signal and system processing.

What makes the HT so unique and so attractive?

- It solves a typical demodulation problem, giving the amplitude (envelope) and instantaneous frequency of a measured signal. The instantaneous amplitude and frequency functions are complementary characteristics that can be used to measure and detect local and global features of the signal – in the same way as for classical spectral and statistical signatures.

- The HT allows us to decompose a nonstationary complicated vibration, separating it into elementary time-varying components – preserving their shape, amplitude, and phase relations.

- It identifies and has an ability to capture – in a much faster and more precise way – the dynamic characteristics of system stiffness and damping, including their nonlinearities and the temporal evolution of modal parameters. This allows the development of more adequate mathematical models of tested vibration structures.

The information obtained can be further used in design and manufacturing to improve the dynamic behavior of the construction, to plan control actions, to instill situational awareness, and to enable health monitoring and preventive surplus maintenance procedures. Therefore, the HT is very useful for mechanical engineering applications where many types of nonlinear modeling and nonstationary parametric problems exist.

This book covers modern advances in the application of the Hilbert transform in vibration engineering, where researchers can now produce laboratory dynamic tests more quickly and accurately. It integrates important pioneering developments of signal processing and mathematical models with typical properties of mechanical construction, such as resonance, dynamic stiffness, and damping. The unique merger of technical properties and digital signal processing provides an instant solution to a variety of engineering problems, and an in-depth exploration of the physics
of vibration by analysis, identification, and simulation. These modern methods of
diagnostics and health monitoring permit a much faster development, improvement,
and economical maintenance of mechanical and electromechanical equipment.

The Hilbert Vibration Decomposition (HVD), FREEVIB, FORCEVIB, and con-
gruent envelope methods presented allow faster and simpler solutions for problems –
of a high-order and at earlier engineering levels – than traditional textbook approaches.
This book can inspire further development in the field of nonlinear vibration analysis
with the use of the HT.

Naturally, it is focused only on applying the HT and the analytic signal methods
to mechanical vibration analysis, where they have greatest use. This is a particular
one-dimensional version of the application of HT, which provides a set of tools for
understanding and working with a complex notation. HT methods are also widely
used in other disciplines of applied mechanics, such as the HT spectroscopy that
measures high-frequency emission spectra. However, the HT is also widely used
in the bidimensional (2D) case that occurs in image analysis. For example, the HT
wideband radar provides the bandwidth and dynamic range needed for high-resolution
images. The 2D HT allows the calculation of analytic images with a better edge and
envelope detection because it has a longer impulse response that helps to reduce the
effects of noise.

HT theory and realizations are continually evolving, bringing new challenges and
attractive options. The author has been working on applications of the HT to vibration
analysis for more than 25 years, and this book represents the results and achievements
of many years of research. During the last decade, interest in the topic of the HT
has been progressively rising, as evidenced by the growing number of papers on this
topic published in journals and conference proceedings. For that reason the author is
convinced that the interest of potential readers will reach its peak in 2011, and that
this is the right time to publish the book.

The author believes that this book will be of interest to professionals and stu-
dents dealing not only with mechanical, aerospace, and civil engineering, but also
with naval architecture, biomechanics, robotics, and mechatronics. For students of
engineering at both undergraduate and graduate levels, it can serve as a useful study
guide and a powerful learning aid in many courses such as signal processing, me-
chanical vibration, structural dynamics, and structural health monitoring. For in-
structors, it offers an easy and efficient approach to a curriculum development and
 teaching innovations.

The author would like to express his utmost gratitude to Prof. Yakov Ben-Haim
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research activities on the subject of Hilbert transforms.

The book summarizes and supplements the author’s investigations that have been
published in various scientific journals. It also reviews and extends the author’s recent

The author is very grateful to Donna Bossin and Irina Vatman who had such a difficult time reading, editing, and revising the text. Of course, any errors that remain are solely the responsibility of the author.

Michael Feldman