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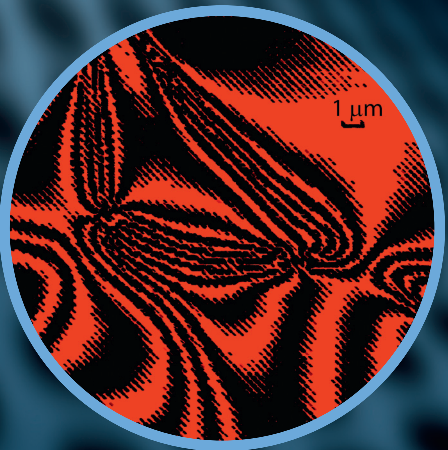
Experimental Mechanics **of Solids**



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EXPERIMENTAL MECHANICS OF SOLIDS

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This book is dedicated to:
Esther & Stephanie our loving wives and great supporters
Eduardo a great son and older brother
Sasha and Lhasa – faithful companions

Contents

About the Authors

Cesar A. Sciammarella was Director of the world renowned Experimental Mechanics Laboratory at the Illinois Institute of Technology for more than 30 years. Over that time he made pioneering developments in applying moiré, holography, and speckle interferometry methodologies as an experimental tool to solve industrial problems around the world. He recently completed a five year project funded by the Italian government to help the Politecnico of Bari develop its experimental mechanics lab and increase its future talent. Currently he is Research Professor at Northern Illinois University where he is working on various industrial projects involving optical contouring and experimental mechanics down at the nanometric level. This effort has taken him beyond the Rayleigh limit that traditionally was considered as the maximum resolution that could be obtained in optics in far field observations. His recent work has yielded measurements in the far field of nanocrystals and nanospheres with accuracies on the order of ± 3.3 nm. His recent discoveries will no doubt lead this field as he has done in the past. He has been an active member in the Society of Experimental Mechanics where he has received almost every honor possible.

Federico M. Sciammarella joined the College of Engineering and Engineering Technology at Northern Illinois University in 2007 and is an assistant professor in the Mechanical Engineering Department. His two research areas are laser materials processing and experimental mechanics. One of several projects involves laser assisted machining (LAM) of ceramics through NIU's Rapid Optimization of Commercial Knowledge (ROCK) Project. The ROCK project enhances the capabilities of small companies by working through supply chains and with experts to improve their productivities and process. He has now spent some time using a novel optical method developed with his father and colleague Dr. Lamberti, Advanced Digital Moiré Contouring to measure surface roughness of the ceramic bars after the LAM process. Through its mission, the ROCK project, working with local companies, strive to develop niche technologies that will directly benefit the U.S. and by providing higher quality parts at reduced costs, improving supply logistics, and creating new manufacturing tools and methods that are critical to the continued growth of this nation.

Preface

The aim of this book, *Experimental Mechanics of Solids*, is to provide a comprehensive and in depth look at the various approaches possible to analyze systems and materials via experimental mechanics. This field has grown mostly through ideas, chance and pure intuition. This field is now mature enough that a comprehensive analysis on the nature of material properties is possible. Often we do things without too much thought and experimental mechanics is no exception.

The approach of this book is to break down each chapter into specific categories and provide some historical context so that the reader can understand how we have reached a certain level in the respective fields. The first two chapters provide some insight into the fundamental issues with regards to continuum mechanics and stress analysis that must be clear to the reader so that they may then make the appropriate decisions when performing field measurements. The next three chapters deal with the use and application of strain gages. There has been a lot of work done in this field so the aim was to provide some basic and practical information for the reader to be able to make sound choices with regards to a selection of gage and understanding the conditions for measurements. The remaining chapters deal with optical methods. Here for the first time ever the reader will see the unifying nature behind all these methods and should walk away with a more complete understanding of the various optical techniques. Most importantly, all the various examples that we have done over our careers are shared so that the reader can understand the advantages of one method over another in a given application.

Ultimately this book should serve as both a learning tool and a resource for industry when faced with difficult problems that only experimental mechanics can help solve. It is our hope that the students who read this book will understand what it takes to perform research in this field and provide inspiration for the future generations of experimentalists.

Our thanks go to Kristina Young M.S. who kindly rendered our illustrations.

Foreword

It is a great honor for me to write the foreword of *Experimental Mechanics of Solids* authored by Prof. Cesar A. Sciammarella and Dr. Federico M. Sciammarella. I have been involved with the authors for the past 10 years. Professor C.A. Sciammarella has taught me optics and made me familiar with the use of optics in that wonderful field called Experimental Solid Mechanics. Dr. F.M. Sciammarella, my friend, was a PhD student when I visited Prof. C.A. Sciammarella's lab at the Illinois Institute of Technology. We took the class on Experimental Solid Mechanics taught by Prof. C.A. Sciammarella. Since then Fred and I collaborated on many pioneering studies carried out by the Professor.

I always asked Prof. Sciammarella to write a book with the purpose to disclose his enormous knowledge to young "fellows" who are interested in Experimental Solid Mechanics. In his five years at the Politecnico in Bari, the Professor was very busy carrying out frontier research and organizing international conferences that brought world renowned scientists to Bari. In spite of all of this hard work, Prof. Sciammarella found the time for conceiving the general organization of his book. In October 2008, when Prof. Sciammarella moved back to US we promised to continue working together. I am glad to say that Prof. Sciammarella, Dr. Sciammarella and myself still work together and will work together in the future, always investigating new exciting topics.

I have seen this book being developed day by day, chapter by chapter. Prof. Sciammarella and Dr. Sciammarella have shown me several chapters of their work. I remember the discussions we had in Chicago. There is no doubt that the quality of the book is outstanding. Apart from the technical content that is excellent in view of the high scientific reputation of the two authors, what has impressed me at the first reading is the clarity of the presentation which has plenty of useful examples. At the second reading, one realizes that the clarity is the obvious result of a total knowledge of the subject presented in the book. I now teach experimental mechanics and I am eager to suggest this new book to my students.

Thank you very much Professor and Fred for having given this book to us!

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1

Continuum Mechanics – Historical Background

The fundamental problem that faces a structural engineer, civil, mechanical or aeronautical is to make efficient use of the materials at their disposal to create shapes that will perform a certain function with minimum cost and high reliability whenever possible. There are two basic aspects of this process selection of materials, and then selection of shape. Material scientists, on the basis of the demand generated by applications, devote their efforts to creating the best possible materials for a given application. It is up to the designer of the structure or mechanical component to make the best use of these materials by selecting shapes that will simultaneously provide the transfer of forces acting on the structure or component in an efficient, safe and economical fashion. Today, a designer has a variety of tools to achieve these basic goals.

These tools have evolved historically through a heritage that can be traced back to the great builders of structures in 2700 BC Egypt, Greece and Rome, to the builders of cathedrals in the Middle Ages. Throughout the ancient and medieval period structural design was in the hands of master builders, helped by artisan masons and carpenters. During this period there is no evidence that structural theories existed. The design process was based on empirical evidence, founded many times in trial and error procedures done at different scales. The Romans achieved great advances in structural engineering, building structures that are still standing today, like the Pantheon, a masonry semi-spherical vault with a bronze ring to take care of tension stresses in the right place. It took many centuries to arrive at the beginning of a scientific approach to structures. It was the universal genius of the Renaissance Leonardo Da Vinci (1452–1519) one of the first designers that gives us evidence that scientific observations and rigorous analysis formed the basis of his designs. He was also an experimental mechanics pioneer and many of his designs were based on extensive materials testing.

The text that follows will introduce the names of the most outstanding contributors to some of the basic ideas of the mechanics of the continuum that we are going to review in this chapter. The next chapter provides background on those who contributed further in the nineteenth century and early twentieth. In the twentieth century many of the basic ideas were reformulated in a more rigorous and comprehensive mathematical framework. At the same time basic principles were developed to formulate solid mechanics problems in terms of approximate solutions through numerical computation: Finite Element, Boundary Element, Finite Differences.

The birth of the scientific approach to the design of structures can be traced back to Galileo Galilei. In 1638 Galileo published a manuscript entitled *Dialogues Relating to Two New Sciences*. This book can be considered as the precursor to the discipline Strength of Materials. It includes the first attempt

Experimental Mechanics of Solids, First Edition. Cesar A. Sciammarella and Federico M. Sciammarella.

to develop the theory of beams by analyzing the behavior of a cantilever beam. A close successor of Galileo was Robert Hook, curator of experiments at the Royal Society and professor of Geometry at Gresham College, Oxford. In 1676, he introduced his famous Hooke's law that provided the first scientific understanding of elasticity in materials.

At this point it is necessary to mention the contribution of Sir Isaac Newton, with the first systematic approach to the science of Mechanics with the publication in 1687 of Philosophiae Naturalis Principia Mathematica. There is another important contribution of Newton and Gottfried Leibniz that helped in the development of structural engineering; they established the basis of Calculus, a fundamental mathematical tool in structural analysis.

From the eighteenth century, we must recall Leonard Euler, the mathematician who developed many of the tools that are used today in structural analysis. He, together with Bernoulli, developed the fundamental beam equation around 1750 by introducing the Euler-Bernoulli postulate of the plane sections which remain plane after deformation. Another important contribution of Euler was his developments concerning the phenomenon of buckling.

From the nineteenth century we recognize Thomas Young, English physicist and Foreign Secretary of the Royal Institute. Young introduced the concept of elastic modulus, the Young's modulus, denoted as E, in 1807. The complete formulation of the basis of the theory of elasticity was done by Simon-Denis Poisson who introduced the concept of what is called today Poisson's ratio.

Ausgustin-Louis Cauchy (1789–1857) the French mathematician, besides being an outstanding contribution to mathematics was one of the early creators of the field of what we call continuum mechanics, both through the introduction of the concept of stress tensor as well his extensive work on the theory of deformation of the continuum.

Claude-Louis Navier (1785–1836), a French engineer, professor of the Ecole de Ponts et Chaussées in Paris, is considered to be the founder of structural analysis by developing many of the equations required for the solution of structural problems and applying them to the construction of bridges.

Another contributor to the basic equations of the continuum is Gabriel Lamé (1795–1870) French mathematician, professor of physics at L'École Polytechnique and professor of probability at the Sorbonne and member of the French Academy. He made significant contributions to the elasticity theory (the Lamé constants and Lamé equations). He was one of the first authors to publish a book on the theory of elasticity. In 1852 he published *Leçons sur la théorie mathématique de l'élasticité des corps solides*. Another outstanding contributor to the foundations of the mechanics of solids is the French engineer and mathematician Adhemar-Jean- Claude Barré de Saint Venant (1797–1880). His major contributions were in the field of torsion and the bending of bars and the introduction of his principle that is key to the formulation of the solutions in the continuum. The original statement was published in French by Saint-Venant in 1852. The statement concerning his principle is to be found in *Mémoires sur la torsion des prismes*. The Saint-Venant's principle has made it possible to solve elasticity problems with complicated stress distributions, by transforming them into problems that are easier to solve.

G. B. Airy (1801–1892) mathematician and professor of Astronomy at Cambridge, introduced in 1862 the concept of stress function. The idea of stress function was applied by Lamé in his work on thick walled vessels, by Boussineq in his work of contact stresses and by Charles Edward English, professor at the Department of Engineering at Cambridge University who applied the idea of stress functions to the solution of problems of stress concentration (1913). August Edward Hough Love (1863–1940), English Mathematician Professor of Natural Philosophy at Oxford author of many papers on the field of Elasticity, author of, A treatise in the Mathematical Theory of Elasticity, first published in 1892.

Tulio Levi-Civita (1871–1941), professor of Rational Mechanics at the University of Padova. He was one of the outstanding mathematicians of the 19th century. He introduced the idea of tensors and tensor calculus that played a fundamental role in the field of mechanics of solids and in the Theory of Relativity. The contributors to the mechanics of solids includes the names of many outstanding mathematicians and physicists of the nineteenth century: James Clerk Maxwell, H. Herzt, Eugenio Beltrami, John Henri Mitchell, Carlo Alberto Castigliano, Luigi Federico Menabrea.

Let us start with a basic approach to see how these different schools of thought are utilized. Here is the scenario: Given a certain body subjected to given loads and given form of support what are the stresses? In strength of materials (i.e., buckling of columns, late eighteenth century) assumptions are made on how body deformations occur and from that stress distributions are obtained. For this approach intuition and experimental measurements are necessary in order to provide an educated guess of how the body deforms. From deformations strains are obtained and then, by using elastic law, stresses are obtained.

Theory of elasticity, a mathematical model of the behavior of materials subjected to deformations (formalized in the late nineteenth and early twentieth century) has a different approach. In theory of elasticity there is no need to make any assumptions in the way the body deforms. All that is needed to solve the problem is:

1. Certain differential equations; and
2. The postulated boundary conditions for the body.

If the solution meets all the conditions of the theory it is possible to say that an exact solution was achieved. At this stage the following question may be asked: What value does this solution have? If experiments are performed using (experimental mechanics) the solution that was obtained using the theory of elasticity will be in agreement with the experiment within a certain number of significant figures. It should be noted that using the theory of elasticity is more complicated than using the strength of materials approach, but it is worth understanding.

The main reason why the theory of elasticity is worth using is because it yields solutions that would not be possible to get using strength of materials. A very simple example of this concept is the case of bending a beam. Strength of material gives the strain and the stress distribution of a section of a beam but these distributions are the correct answers under special conditions: pure bending and away from the applied load. If we have a beam with a concentrated load the stress distribution in the section where the load is applied will be quite different from that given in strength of materials. In many cases the solution of theory of elasticity agrees with strength of materials solutions, but the understanding that comes from theory of elasticity allows us to have a good grasp of the validity of the solutions. In particular it is possible to know when the solutions can be applied to a particular problem.

Today, numerical techniques (i.e., Finite Element Analysis "FEA") are used in almost all applications. A FEA practically provides the solution for any possible problem of the theory of elasticity. One may go so far as to say that FEA is all that is necessary to solve problems. However, it should be mentioned at this point that the ability of numerical analysis to provide the solutions is due to the understanding gained through theory of elasticity and continuum mechanics. Another very important distinction should be made between the solution obtained by theory of elasticity and one that is obtained by a numerical method. The theory of elasticity solution provides the answer for all possible solutions of a given problem. The numerical solution provides the answer for specific dimensions and loads. For example, if one wants to analyze what influence a given variable has on a given problem, this can be done in FE but it will require continual computations for all the range of values of interest of the variable. If one knows the theory of elasticity solution the effect of a variable can be deduced directly from this solution. At this stage of our knowledge the possibility of obtaining solutions directly from the theory of elasticity is limited and hence numerical techniques such as FE allow us to solve numerically any possible problem of the theory of elasticity if we have correct information concerning the boundary conditions and the initial conditions in time if we have dynamic problems.

What follows is a review of the basic concepts upon which the theory of the continuum is built. Continuum mechanics is a branch of classical mechanics. It deals with the analysis of the kinematics and the mechanical behavior of materials modeled as a continuous rather than as an aggregate of discrete particles such as atoms. The French mathematician Augustin Louis Cauchy was the first to formulate this model in the early nineteenth century. The continuum model is not only utilized in mechanics, but also in many branches of physics. It is a very powerful concept that helps in the mathematical modeling of complex problems. A continuum can be continually sub-divided into infinitesimal elements whose properties are those of the bulk material. The continuum hypothesis has at its basis the concepts of a representative volume element. What is a representative volume element? It is an actual volume, with given dimensions. To this volume we can apply continuum mechanics and get results that can be verified by experimental mechanics. It is a concept that depends on scales, for example, when we consider a large structure like a dam, the representative volume may be in the order of centimeters, if we consider a metal the representative volume will be of the order of 10 microns or less. What we measure in experimental mechanics is a certain statistical average of what occurs at the level of the microstructure. This characteristic of the continuum model leads us to ambiguities in language, for example, when we talk of properties at a point of the continuum we are in reality referring to the representative volume that has a definite size.

1.1 Definition of the Concept of Stress

The concept of stress is one of the building blocks of continuum mechanics. The stress vector at a point is defined as a force per unit area as in

$$\vec{\sigma} = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.1}$$

where ΔF denotes the force acting on ΔA, this vector depends on the orientation of the surface defined by its normal. This vector is not necessarily normal to the surface.

The stress vector does not characterize the state of stress at a given point of the space in the continuum. The state of stress is characterized by a more complex quantity know as the stress tensor σ_{ij}. The stress tensor has nine components, of which only six are independent. The stress components are represented in a Cartesian system of coordinates by the stress Cartesian tensor that was originally introduced by Cauchy.

$$[\sigma] = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix} \tag{1.2}$$

The cube shown in Figure 1.1 represents the stress tensor at a point with its nine components $(\sigma_{ij} = \sigma_{ji})$.

This definition has the ambiguity in language we have pointed out before. Figure 1.1 represents a cube in the continuum, but as we said before ideally it represents a system of three mutually perpendicular planes that go through a point. Each of these planes are defined by their normals, in this case the base vector of an orthogonal Cartesian system x, y, z. At each face of the cube there is a resultant stress vector that we have represented by $\overset{\rightarrow}{\sigma}^{(\hat{e}_i)}$ with $i = x, y, z$. As can be seen these vectors are not perpendicular to the faces of the cube. The components of the stress tensor are the projections of the stress vectors in the direction of the coordinate axis. Mathematically, the tensor is a point function that, according to continuum mechanics, is continuous and has continuous derivatives up to the third order. However, when we want to measure it we need to make the measurement in a finite volume. If the finite volume is too small compared to the representative volume, what we measure will appear to us as a random quantity. The fact that we have talked about measuring a stress tensor is again an ambiguity in language. There is no way to measure stresses directly, we will be able to measure deformations and changes of geometry from which we will compute the values of stresses.

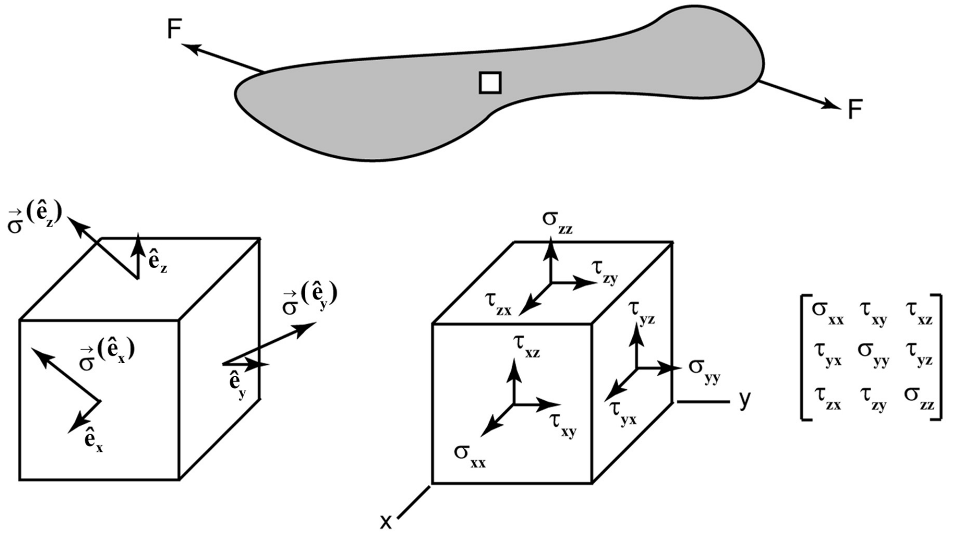

Figure 1.1 (a) Elementary cube with stress vectors for the faces of the cube. (b) Components of the stress vectors of the faces. (c) Loaded body showing the elementary cube inside the volume.

1.2 Transformation of Coordinates

All our measurement procedures will require us to define a coordinate system that we need to specify. But to handle this information in posterior manipulations it may be necessary to switch coordinate systems. A tensor is an entity that mathematically is defined by the way it transforms. In the following derivations we are going to go in an inverse way, define the components and then find out how they transform. It is a classical way through which historically the stress tensor was defined. We consider the equilibrium of a tetrahedron, as in Figure 1.2.

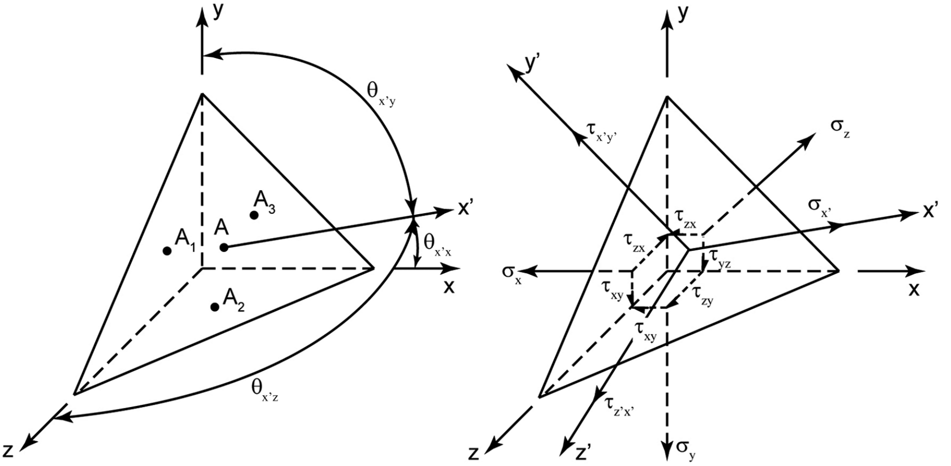

Figure 1.2 Equilibrium of a tetrahedron at point P of a continuum. (a) Component of the stress vectors acting on the different elementary areas. (b) Angular orientation of the rotated axis.

Introducing an arbitrary oblique plane, where it intersects the three mutually perpendicular reference planes creates a tetrahedron. A tetrahedral element about a point P is defined. The axis x′ of the rotated Cartesian coordinates system is perpendicular to oblique plane whereas y′ and z′ are tangent to the plane orientation of the axis x′ and can be established by the angles shown in Figure 1.2 (b). Areas for the triangular elements formed by the coordinates axis and by the intersection of the oblique plane with the coordinates planes are given by,

$$A_x = A_o n_{x'x},\ A_y = A_o n_{x'y},\ A_z = A_o n_{x'z} \tag{1.3}$$

Where $n_{x'i}$ are the direction cosines of the normal $\vec{n}$ with respect to the coordinate axis. The projection equations of static equilibrium can be applied to get the components shown in Figure 1.2. To utilize the projection equations, the first step is to obtain the summation of forces in the x′ direction. Recall that the force corresponding to each stress is: $\sigma \times A_\sigma$. Next it is important to obtain the component of the force in x′ direction. Force due to σ_x is $\sigma_x A_x = \sigma_x A_o n_{x'x}$. The component of force in x′ is given as $(\sigma_x A_o n_{x'x})\, n_{x'x}$.

The same procedure is utilized for the other components and the summation of forces in x′ direction gives,

$$\sigma_{x'} = \sigma_x n_{x'x}^2 + \sigma_y n_{x'y}^2 + \sigma_z n_{x'z}^2 + 2\tau_{xy} n_{x'x} n_{x'y} + 2\tau_{yz} n_{x'y} n_{x'z} + 2\tau_{zx} n_{x'z} n_{x'x} \tag{1.4}$$

For a complete transformation of the stress components with respect to the arbitrary oblique surface, the shear stresses $\tau_{x'y'}$ and $\tau_{x'z'}$ must be computed. Directional cosines for y′ and z′ as in x′ are defined as,

$$\begin{aligned}\tau_{x'y'} &= \sigma_x n_{x'x} n_{y'x} + \sigma_y n_{x'y} n_{y'y} + \sigma_z n_{x'z} n_{y'z} + \tau_{xy}(n_{x'x} n_{y'y} + n_{x'y} n_{y'x}) + \tau_{yz}(n_{x'y} n_{y'z} + n_{x'z} n_{y'y}) \\ &\quad + \tau_{zx}(n_{x'x} n_{y'z} + n_{x'z} n_{y'x})\end{aligned} \tag{1.5}$$

$$\begin{aligned}\tau_{z'x'} &= \sigma_x n_{x'x} n_{z'x} + \sigma_y n_{x'y} n_{z'y} + \sigma_z n_{x'z} n_{z'z} + \tau_{xy}(n_{x'x} n_{z'y} + n_{x'y} n_{z'x}) + \tau_{yz}(n_{x'y} n_{z'z} + n_{x'z} n_{z'y}) \\ &\quad + \tau_{zx}(n_{x'x} n_{z'z} + n_{x'z} n_{z'x})\end{aligned} \tag{1.6}$$

These equations are sufficient for the determination of the stress components on any internal surface in which an arbitrarily selected tangential set of coordinates is used (y′z′). For a complete transformation of the stress tensor shown earlier to that of a rectangular element oriented by the x′y′z′ coordinate system, the six stresses on the two surfaces with normals in the y′ and z′ must also be determined. The component $\sigma_{y'}, \sigma_{z'}, \tau_{y'z'}$ are:

$$\sigma_{y'} = \sigma_x n_{y'x}^2 + \sigma_y n_{y'y}^2 + \sigma_z n_{y'z}^2 + 2\tau_{xy} n_{y'x} n_{y'y} + 2\tau_{yz} n_{y'y} n_{y'z} + 2\tau_{zx} n_{y'z} n_{y'x} \tag{1.7}$$

$$\begin{aligned}\tau_{y'z'} &= \sigma_x n_{y'x} n_{z'x} + \sigma_y n_{y'y} n_{z'y} + \sigma_z n_{y'z} n_{z'z} + \tau_{xy}(n_{y'x} n_{z'y} + n_{y'y} n_{z'x}) + \tau_{yz}(n_{y'y} n_{z'z} + n_{y'z} n_{z'y}) \\ &\quad + \tau_{zx}(n_{y'x} n_{z'z} + n_{y'z} n_{z'x})\end{aligned} \tag{1.8}$$

$$\sigma_{z'} = \sigma_x n_{z'x}^2 + \sigma_y n_{z'y}^2 + \sigma_z n_{z'z}^2 + 2\tau_{xy} n_{z'x} n_{z'y} + 2\tau_{yz} n_{z'y} n_{z'z} + 2\tau_{zx} n_{z'z} n_{z'x} \tag{1.9}$$

The above equations give all the components of the stress tensor when the Cartesian axis orientation is changed. Although these equations have been derived using a finite tetrahedron the postulation is that these relationships continue to be valid in the limit when the tetrahedron dimensions go to zero and the tetrahedron merges with the point P.

1.3 Stress Tensor Representation

The nine components of σ_{ij}, with i, j = x, y, z of the stress vectors are the components of a second-order Cartesian tensor called the Cauchy stress tensor, which completely defines the state of stresses at a given