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Contributors

Kent Bach. Professor of Philosophy, San Francisco State University, California.

José A. Benardete. Professor of Philosophy, University of Syracuse, New York.

Ermanno Bencivenga. Professor of Philosophy, University of California, Irvine, California.

Emma Borg. Lecturer in Philosophy, University of Reading, UK.

E. P. Bos. University Lecturer in Ancient and Medieval Philosophy, Leiden University, The Netherlands.

Bryson Brown. Professor of Philosophy, University of Lethbridge, Lethbridge, Alberta, Canada.

James Cargile. Professor of Philosophy, University of Virginia, Virginia.

Robert Demolombe. Director of Research, ONERA, Department of Information Processing and Modeling, Toulouse Centre, Toulouse, France.

Maarten de Rijke. Professor, Institute for Logic, Language and Computation (ILLC), Department of Mathematics, Computer Science, Physics and Astronomy, University of Amsterdam, The Netherlands.

Branden Fitelson. Assistant Professor in the Department of Philosophy, San José State University, San José, California, and Scientific Associate, Mathematics and Computer Science Division, Argonne National Laboratory, Argonne, Illinois.

Melvin Fitting. Professor, Department of Mathematics and Computer Science of Lehman College, and Departments of Mathematics, Philosophy, Computer Science of the Graduate Center, City University of New York.

Peter A. Flach. Reader in Machine Learning, Department of Computer Science, University of Bristol, UK.

Peter Forrest. Professor of Philosophy, School of Social Science, University of New England, Armidale, New South Wales, Australia.

D. M. Gabbay. FRSC, Augustus De Morgan Professor of Logic, Group of Logic and Computation, Department of Computer Science, King’s College, London, UK.
Rolf George, Professor of Philosophy, University of Waterloo, Ontario, Canada.

Stephen Glaister, Lecturer, Department of Philosophy, University of Washington, Seattle, Washington.

Paul Gochet, Professor Emeritus, Department of Philosophy, and Ministère de l’Éducation et de la Recherche scientifique de la Communauté Française, University of Liège, Belgium.

Richard Grandy, McManis Professor of Philosophy, Rice University, Houston, Texas.

Petr Hájek, Professor and Head of the Department of Theoretical Computer Science, Institute of Computer Science, Academy of Sciences of the Czech Republic, Prague, Czech Republic.

Risto Hilpinen, Professor of Philosophy, University of Miami, Florida.

Herbert Hochberg, Professor of Philosophy, University of Texas, Austin, Texas.

Andrew Hodges, Wadham College, Oxford University, UK.

Dale Jacquette, Professor of Philosophy, Pennsylvania State University, University Park, Pennsylvania.

Andrew J. I. Jones, Professor, Department of Philosophy and Norwegian Research Centre for Computers and Law, University of Oslo, Norway.

Michael Jubien, Professor of Philosophy, University of California, Davis, California.

Matt Kaufmann, Senior Member of the Technical Staff, Advanced Micro Devices, Inc., Austin, Texas.

Gregory Landini, Associate Professor of Philosophy, University of Iowa, Ames, Iowa.

Ernest Lepore, Professor and Director, Center for Cognitive Science, Rutgers University, New Jersey.

Grzegorz Malinowski, Professor and Head of the Department of Logic, University of Łódź, Poland.

Edwin D. Mares, Senior Lecturer and Department Head, Department of Philosophy, Victoria University of Wellington, New Zealand.

J. Strother Moore, Admiral B. R. Inman Centennial Chair in Computing Theory, Department of Computer Sciences, University of Texas, Austin, Texas.

Gary Ostertag, Adjunct Assistant Professor, Department of Philosophy, New York University, New York City, New York, and Instructor, Department of Philosophy, Nassau Community College.

Graham Priest, Boyce Gibson Professor of Philosophy, University of Melbourne, Australia, and Arché Professorial Fellow, Department of Logic and Metaphysics, University of St. Andrews, Scotland.

Greg Ray, Associate Professor of Philosophy, University of Florida, Gainesville, Florida.
CONTRIBUTORS

Nicholas Rescher, University Professor of Philosophy, University of Pittsburgh, Pennsylvania.

Nathan Salmon, Professor of Philosophy, University of California, Santa Barbara, California.

Gerhard Schurz, Professor of Philosophy, Chair of Theoretical Philosophy, University of Düsseldorf, Germany, and International Research Center, Moenchsberg, University of Salzburg, Austria.

Stewart Shapiro, Professor of Philosophy, Department of Philosophy, Ohio State University, Columbus, and Professorial Fellow, Department of Logic and Metaphysics, University of St. Andrews, Scotland.

Gila Sher, Professor of Philosophy, University of California, San Diego, California.

Keith Simmons, Professor of Philosophy, University of North Carolina at Chapel Hill, North Carolina.

Robin Smith, Professor of Philosophy, Texas A&M University, College Station, Texas.

Roy A. Sorensen, Professor of Philosophy, Dartmouth College, Hanover, New Hampshire.

B. G. Sundholm, Professor of Philosophy and History of Logic, Leiden University, The Netherlands.

Mary Tiles, Professor of Philosophy, University of Hawaii at Manoa, Hawaii.

Alasdair Urquhart, Professor of Philosophy, University of Toronto, Ontario, Canada.

Mark van Atten, Institute of Philosophy, Catholic University, Louvian, Belgium.

Johan van Benthem, Professor of Mathematical Logic and its Applications, University of Amsterdam, The Netherlands, and Bonsall Chair of Philosophy, Stanford University, California.

Dirk van Dalen, Professor, History of Logic and Philosophy of Mathematics, University of Utrecht, The Netherlands.

James Van Evra, Associate Professor of Philosophy, University of Waterloo, Ontario, Canada.

Heinrich Wansing, Professor of Logic and Philosophy of Science, Dresden University of Technology, Germany.

Scott Weinstein, Professor of Philosophy, University of Pennsylvania, Philadelphia, Pennsylvania.

Jan Woleński, Professor of Philosophy, Institute of Philosophy, Department of Epistemology, Jagiellonian University, Cracow, Poland.

Larry Wos, Senior Mathematician, Mathematics and Computer Science Division, Argonne National Laboratory, Argonne, Illinois.
The essays collected in this volume are previously unpublished contributions to philosophical logic from some of the most respected researchers in the field. In inviting these specialists to write on their specialities, I have sought to combine a representative breadth of coverage with an accessible depth of philosophical and mathematical sophistication that offers a clear picture of the historical development and current state of the art in philosophical logic. To whatever extent the book succeeds in meeting its objective, credit is due to the superb work of the logicians and philosophers who agreed to be part of this immoderate editorial undertaking.

My strategy has been to identify what I consider to be the most important topic areas in philosophical logic from the standpoint of students as well as professional scholars, and then in each case to recruit three or more of the best experts I could find who I thought were likely to disagree in interesting ways, encouraging each to address the questions they believe most important in their own way and in their own voice, without concern for what any of their co-contributors have to say. The result is a remarkable testimony to a thriving industry in contemporary philosophical logic, and, despite some detractors’ premature eulogies of its imminent demise, the vitality of contemporary analytic philosophy.

With the exception of my introductory essay, the papers are clustered thematically, although the order is not always obvious. The first invisible division in the text proceeds from milestones in the history of logic to the relation of symbolic logic to ordinary language. Logical paradoxes and their philosophical implications are then introduced as essential for understanding Tarski’s truth semantics and responses especially to the liar paradox which have been so fundamental in shaping the theory of meaning in modern philosophical logic. A discussion of selected paradoxes is accordingly followed by a choice of topics involving Tarski’s concept of truth and Russell’s theory of definite description in classical semantics that continue to play an essential role in current discussions in philosophical logic. The stage is thereby set for investigations of more recent trends in logic, emphasizing alternative concepts of logical consequence, and questions of existence presuppositions and ontology in logic. Metatheoretical considerations about the scope and limits of logic come next, advances that are naturally complemented by a suite of papers on the logical foundations of set theory and mathematics. Here another invisible threshold is attained, after which nonclassical logics begin to
appear, starting with modal logics in several categories, a larger section than most, because of the importance of modal logics in the development of set theoretical semantics and their many applications, followed by intuitionistic, free and many-valued logics, inductive, fuzzy and quantum logics, relevance, and paraconsistent logics. In the final grouping of papers, two sections complete the book’s discussion of the implications for and practical applications of philosophical logic in machine theory and cognitive science, and the mechanization of logical inference and automated theorem and proof discovery.

Although some of the papers are more technical than others, all are intended for an introductory audience, and can be read with good understanding by beginning students in philosophy who have completed a first course in symbolic logic. This is especially true if the essays are read sequentially as they are presented within each section and from section to section. Inevitably, a full understanding of some topics treated at earlier stages of the book may require familiarity with principles and methods of logic that are considered in detail only in later sections, for which some looking ahead may occasionally be required. Additional background materials related to the study of philosophical logic can also be found in my simultaneously published Blackwell collections, *Philosophy of Logic: An Anthology* and *Philosophy of Mathematics: An Anthology*. The present volume will serve its purpose if it helps provide readers at all levels with a sufficient sense of interest in its subject to pursue advanced study of the concepts, methods, and problems of philosophical logic.
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Dale Jacquette
1 Philosophy as Logic

It has been many years since Bertrand Russell provocatively identified philosophy with logic. Although some logicians and philosophers continue to accept Russell’s thesis, not least because it bears the stamp of Russell’s authority in both fields, most commentators today prefer to describe the relationship between logic and philosophy as more complex. If logic remains important to philosophy, and philosophy to logic, it is undoubtedly because of what each can offer the other as an autonomous discipline.

Logic is no longer the monolithic edifice to which Russell could point in 1914, when in *Our Knowledge of the External World*, he made his famous observation that: “[E]very philosophical problem, when it is subjected to the necessary analysis and purification, is found either to be not really philosophical at all, or else to be, in the sense in which we are using the word, logical” (1914: 42). When contemporary philosophers speak of logic, they generally mean to refer to any of a variety of alternative formal symbolisms that can be used to formulate particular aspects of the formal inferential structures of language, including but not limited to languages in which philosophical ideas are conveyed. While logic is a useful tool in clarifying and perspicuously representing philosophical reasoning, many philosophers believe that there are areas, indeed, most parts, of legitimate philosophical inquiry, that have nothing directly to do with the specialized study of formal symbolic logic. Such a conclusion is especially plausible when philosophy is viewed broadly to include literary as well as scientific projects, particularly those that do not use or take any special notice of logic and mathematics, and that may even disclaim efforts to arrive at the truth about any philosophical subject, as in certain outgrowths of postmodern philosophy. Russell also feels the need to qualify the identification of philosophy with logic, adding immediately after his statement quoted above: “But as the word ‘logic’ is never used in the same sense by two different philosophers, some explanation of what I mean by the word is indispensable at the outset” (1914: 42).

The fact, as Russell observes, that philosophers have many different ideas of logic constitutes one of the most fundamental problems for philosophical logic and the philosophy of logic. To define the concept of logic, to understand the diverse kinds of systems that have been considered logics, and to arrive at a satisfactory definition of
the concept of logic that applies alike to Aristotelian syllogisms, Boolean algebras, Frege’s *Begriffsschrift*, Whitehead and Russell’s *Principia Mathematica*, and unlimitedly many nonstandard formal systems, and informal logic in several traditions, grading off into rhetoric, argumentation theory, and discourse analysis, is a formidable task. What makes all of these projects logical, a part or different forms of logic, or distinct logics? A working definition that may be correct if somewhat uninformative as far as it goes is to say that logic in any of its manifestations is the systematic study of principles of correct reasoning. The principles of logic can then be explored formally or informally, and by any of a number of different styles of exposition, some of which may be highly specialized in dealing with very particular areas of reasoning.

Logic is both a symbolism for the expression of the formal structures of thought and an inference mechanism for calculating and drawing conclusions from assumptions in reasoning. The dual nature of logic has figured prominently in the range of issues that have come to be associated with the problems of philosophical logic.

## 2 Logic and Philosophy of Language

A primary source of problems in philosophical logic is the analysis of language. Philosophers are interested in language and semantics or theory of meaning for a number of reasons. The problems and methods of applied logic in studying the philosophy of language are directly associated with the traditional domain of philosophical logic.

Language facility distinguishes human beings from other animals we know of, higher primates who have been taught by humans to make limited use of sign-language and computer push-button languages notwithstanding. Philosophers interested in human nature and what makes our species unique in the animal kingdom as a result are attracted to problems of understanding language as a way of gaining insight into the human condition. The complexity of language and the difficulty of formulating an adequate theory of meaning for ordinary and scientific language by itself is a sufficient invitation for many philosophers to answer the challenge of articulating a philosophical semantics. More importantly, logicians and philosophers in the analytic tradition have considered unclarities in the expression of philosophical ideas to be the foundation of philosophical puzzles and paradoxes, and have accordingly sought to solve, avoid, or at least gain a better perspective on the problems by way of the theory of meaning.

This is undoubtedly part of what Russell means in pronouncing all of philosophy properly so-called identical with logic. Symbolic logic has been the tool of choice for philosophers investigating the properties of language in philosophical logic, because it is itself a language whose syntax and semantics are at the disposal and under the control of the logician where they can be better studied in more ideal abstract terms. A formal system of logic considered as a language has definite advantages over colloquial discourse as a model of how language works, where its factors are more readily discerned and rigorously formulated independently of the ambiguities and etymological confusions that are endemic to natural language, which, as Ludwig Wittgenstein aptly remarks in the *Tractatus Logico-Philosophicus* (1922: 4.002), “is a part of the
human organism and is not less complicated than it.” Even for philosophical logicians who do not seek to replace ordinary language with an ideal language like Frege’s Begriffsschrift or Whitehead and Russell’s Principia Mathematica, but, like Wittgenstein, hope to understand how language generally is capable of expressing meaning, the use of symbolic logic has remained an indispensable instrument in philosophy of language. The fact that logic lends itself to more sharply and univocally defined distinctions makes it convenient for the analysis of concepts in philosophy, including the semantic principles by which logical formulas are themselves precisely interpreted. The usefulness of logic in philosophical applications has played a major role in the development of symbolic logic, which in turn has opened up new possibilities for logic’s use in refinements of philosophical techniques.

How, then, has the partnership between philosophical logic and philosophy of language taken shape? In too many ways for the story to be told in a summary that does not distort the true riches of ingenuity, invention, and discovery on the part of philosophers and logicians in the annals of recent and contemporary analytic philosophy. Nevertheless, something of the flavor of work in this exciting field can be conveyed from a brief discussion of a few well-chosen examples. We turn next to consider some instructive concrete possibilities.

3 Modes and Methods of Philosophical Logic

Logic is formal, and by itself has no content. It applies at most only indirectly to the world, as the formal theory of thoughts about and descriptions of the world. Logic can be used in many ways to state, clarify, and express ideas, and to authorize the derivation of consequences, when its formulas are assigned substantive content in application. Although logic in its pure form is unfreighted with philosophical truths, it can contribute in definite ways to the clarification and solution of philosophical problems.

Philosophical logic often combines an application of logical symbolisms with a commitment to specific philosophical ideas. Symbolic logic, even in its purest form, is also not entirely free of philosophical ideology, although some logicians have made it their mission to try to make logic as neutral a vehicle as possible for the unbiased expression of the logical form of philosophical disagreements on every conceivable topic, including those most closely related to the conceptual presuppositions of classical logic. To the extent that substantive philosophical positions are built into the interpretation of symbolic logic, the use of logic in addressing philosophical problems may seem highly effective and convincing. In that case, of course, it is not logic alone that is doing the work, but whatever philosophical theses have been packed into its symbolism.

There is often a temptation to use philosophical logic in this way. A logical notation is loaded with philosophical cargo to enable it to appear at least to make progress against outstanding philosophical problems. Logic as a branch of mathematics deservedly carries a certain authority in intellectual disputes. We should recognize, however, that when a logical formalism appears to solve a philosophical problem, it seldom does so by itself, but only by virtue of the philosophical ideas it is used to express. That being the case, we need to question whether the philosophy shouldered by philo-
sophical logic is sound or faulty, just as we would need to do if we had set about con-
sidering the philosophical issues directly without the intervention of a symbolic logical
notation. If logic helps the cause of clarifying and solving or avoiding philosophical
problems, it does so thanks largely to the ability of its formal structures to sort out and
more clearly represent a choice of philosophical ideas, and not by means of substan-
tive philosophical assumptions hidden in the background of a particular logical system.

In his “Introduction” to Wittgenstein’s *Tractatus*, Russell recognizes the potential of
a logical symbolism to clarify philosophical concepts. He states: “a good notation has a
subtlety and suggestiveness which at times make it seem almost like a live teacher.
Notational irregularities are often the first sign of philosophical errors” (1922: 17–18).
The value of an adequate logical notation is that it provides information about the
logical form of the ideas it expresses. It can call attention to logical structures that might
otherwise be overlooked in informal expression, including tipoffs about conceptual
inconsistencies. This, after all, is a primary pragmatic justification for the use of sym-
bolic logic. It teaches us things that we could not (or not as easily) learn without its for-
malisms. Such discoveries are often made as logicians explore the scope and expressive
flexibility of a formal system. They emerge in the study of a formalism’s mathematical
multiplicity, in Wittgenstein’s terminology, its shared isomorphism or lack thereof with
the features of thought or discourse it is supposed to formalize, together with its intern-
al logical interrelations and deductive consequences.

Russell, in his own celebrated application of philosophical logic in the analysis of
definite descriptions, in his essay “On Denoting” (Mind 1905), seems nevertheless to
have decanted a significant amount of philosophy into a logical vessel in order to gain
philosophical mileage from what appears to be purely logical distinctions. Russell’s
theory of descriptions has been enormously influential in the rise of analytic philoso-
phy, to such a degree that F. P. Ramsey in his essay “Philosophy” was moved to eulogize
it as “that paradigm of philosophy.” The theory has indeed been a model for some of
the best work in philosophical logic for over a century. It is worthwhile, therefore, to
consider the theory in detail, to understand how it combines philosophy with logic, and
the amount of labor borne by logic as opposed to the prior philosophical commitments
deeply integrated into Russell’s logic.

4 Logic as Philosophy in Philosophical Logic

We can identify at least three characteristics of Russell’s theory that provide enduring
guidelines for philosophical logic. Russell’s breakdown of definite descriptions into an
existence clause, uniqueness clause, and predication of a property to a uniquely
denoted entity, using the devices of symbolic logic to conjoin these three formalized con-
ditions, demonstrate the power of symbolic logic to present the analysis of a complex
concept into more basic components for philosophical purposes. Russell’s method has
very properly been compared to that of an optical prism that takes a single beam of
white light and breaks it up into its constituent spectrum of colors. The colors are not
added or produced by the prism, but are there all along, inherent in the white light,
although it takes a special instrument to reveal their presence. The same is true of
definite descriptions, to which Russell applies symbolic logic in order to break apart
and discover by reflection the three conditions concealed within the apparently simple word ‘the.’

This observation leads to the second noteworthy feature of Russell’s analysis. Russell makes an inestimable contribution to the flowering of analytic philosophy by suggesting that the logical form of a proposition, as judged in terms of its superficial grammatical structure, is not necessarily its real, underlying form, appreciated by means of logical analysis. I cannot put the point better than Wittgenstein in *Tractatus* (1922: 4.0031), when he declares: “Russell’s merit is to have shown that the apparent logical form of the proposition need not be its real form.” Wittgenstein no doubt puts his finger on a major ingredient in the appeal of Russell’s theory of descriptions. By suggesting that philosophical logic has as part of its project to uncover the real underlying or ulterior logical form of sentences in ordinary thought and language, Russell inspired generations of philosophers with a vision of logical analysis excavating the subterranean logical structures beneath the surface of colloquial discourse.

Third, Russell’s theory is rightly dignified as a wellspring of contemporary analytic philosophy because of its dramatic use of logical methods in disambiguating philosophically equivocal linguistic expressions. Russell considers among others the problem of interpreting the sentence, ‘The present king of France is not bald.’ The dilemma he intuits is that if the sentence is taken to mean that there is a present king of France who is not bald, then the sentence should be false. To declare the sentence false, at least when we are operating within the parameters of ordinary language, wrongly seems to entail that there is a present hirsute king of France. Russell’s genius in the theory of definite descriptions is partly seen in his recognition that symbolic logic permits the exact disambiguation of the scope of the negation operator that is blurred in everyday speech. He accordingly distinguishes between saying ‘There exists one and only one present king of France and it is not the case that he is bald,’ versus ‘It is not the case that there exists one and only one present king of France and he is bald (or, it is not the case that he is bald).’ The first sentence is false, but its proper negation is the second sentence, which does not commit the speaker to the existence of a hirsute present king of France.

Although the distinction can also be indicated as here in a modified form of ordinary English, Russell finds that it is only in symbolic logic that the full force of placing the negation sign externally, with the entire proposition in its scope, as opposed to internally, governing only the predication of the property of being bald in the third clause of the formal analysis of the definite description, can be fully and unequivocally appreciated. In standard logical notation, the difference is formalized as that between \( \neg(\exists x)(Kxf \& (\forall y)((Kfy \equiv x = y) \& Bx)) \) as opposed to \( (\exists x)(Kxf \& (\forall y)((Kfy \equiv x = y) \& \neg Bx)) \). The difference in the scope of the negation, and the difference it makes in the truth values of the two propositions, is so immediately apparent as to powerfully iconically recommend the use of symbolic logic as a general method of clarifying logical obscurities and circumventing conceptual confusions.

Having acknowledged the strength of Russell’s analytic paradigm, it may also be worthwhile to consider its underlying philosophical assumptions. Russell is interested not only in the truth value of sentences ostensibly designating nonexistent objects like the present king of France, but also in understanding predications of properties to fictional creatures, like Pegasus, the flying horse of ancient Greek mythology. Russell
regards proper names like ‘Pegasus’ as disguised definite descriptions, which he inter-
prets according to his three-part analysis as consisting of an existence claim, a unique-
ness claim, and the predication of a property to the uniquely designated entity. If I say,
then, that ‘Pegasus is winged,’ Russell interprets this sentence as falsely asserting that
there exists a flying horse, there is only one flying horse, and it is winged. From this it
appears to follow that something of metaphysical significance has been derived from
Russell’s skillful use of philosophical logic; namely, that it is false to say of any nonex-
istent object like Pegasus that the object has any of the properties attributed to it in
myths, legends, or storytelling contexts.

If we look at the logical symbolism Russell employs, we see that in this case it reads:
(∃x)(Fx & (∀y)(Fy ≡ x = y) & Wx). The formula, it must be said, is supposed to be judged
false only because the quantifier in (∃x)(Fx . . .) is interpreted as meaning that there
actually exists such an object in the logic’s semantic domain that truly possesses the
property F, of being a flying horse. Russell as a matter of fact has no way to construe
an object like Pegasus in his logic other than as the value of an existentially loaded
quantifier-bound variable. This is probably not the place to dispute with Russell about
whether such a logical treatment of names like ‘Pegasus’ is philosophically justified or
not. It is nevertheless important to recognize that Russell’s evaluation of such sentences
as false is predetermined by his existence presuppositional semantics for the ‘existen-
tial’ quantifier, and by the fact that his logic permits no alternative means of considering
the semantic status of sentences ostensibly containing proper names for nonexistent
objects. This makes it an altogether philosophically foregone conclusion that sentences
like ‘Pegasus is winged,’ which many logicians would otherwise consider to be true
propositions of mythology, are false. The point is that Russell is able to produce this
philosophical result from his logical analysis of the meaning of the sentence only
because the position is already loaded into the presuppositions of the syntax and
semantics of his interpretation of formal symbolic logic. The interesting philosophical
question that Russell would be hard-pressed to answer satisfactorily is whether his logic
is philosophically adequate to the proper analysis of problematic sentences in this cat-
egory. It is not a conclusion of logic alone that Russell advocates, whether correct or
incorrect, but of an applied philosophical logic that is heavily but not inevitably imbued
with a prior metaphysical commitment to an existence-presuppositional extensional
syntax and semantics.

A good logical notation, as Russell says, can function philosophically much like a
living teacher. As a pure formalism, however, logic is not an autonomous authority on
any matter of philosophical truth. It has, in itself, no philosophical implications, and
in its applications in philosophical logic, as Russell’s example illustrates, it is capable of
supporting only those philosophical conclusions with which it is deliberately or inad-
vertently invested by logicians. This, then, is another sense in which Russell in his most
important contributions to philosophical logic identifies logic with philosophy.

5 On Philosophical Presuppositions and Copia of Logical Systems

The perspective we have arrived at in understanding the relation between logic and phi-
losophy can help to answer a difficult question about the nature of logic and the status
of multiple logical systems. Why are there so many different systems of logic? Is there just one underlying logic, of which all the various systems are alternative partial expressions? Or are there many different logics that are related to one another by a network of partially overlapping family resemblances?

If we consider work in contemporary theoretical logic at face value, there seem to be indefinitely many logics. Alethic modal logics are concerned with matters of necessity and possibility; doxastic logics are designed to explain the logical structures of belief states; epistemic logics are offered to formalize valid inferences about knowledge. There are specialized logics of quantum physical phenomena, deontic logics of obligation and permission, and many others. An important source of the proliferation of logical systems in contemporary logic and philosophy is in philosophical issues arising from dissatisfaction with classical logics in dealing with specific aspects of scientific and everyday reasoning. This is the basis for work in many-valued logics, free logic, relevance, and paraconsistent logics, and logics of beingless intended objects, that do not limit logical inference to existent entities in referring to and truly predicating properties of objects, and for the paraconsistent stance that logical inconsistencies need not explosively entail any and every proposition, but that contradictions can be tolerated without trivializing all inferences.

Applications of logic to philosophical problems of these kinds are a continuing basis for innovations in formal symbolic logic and the development of new nonstandard systems of logic. Logic is also concerned with abstract theoretical matters concerning its own formal symbolisms and the properties, such as the scope and limits of logical and mathematical systems considered as a whole, in the study of logical metatheory. The advance of logic has been nourished by its theoretical and practical applications in set theory, computer engineering, artificial intelligence modeling, formal semantics and linguistic analysis of scientific theory, philosophical argument, and colloquial language. There is valuable feedback between logical theory and practice, much as there is in pure and applied mathematics. The need for new formalisms is sometimes made urgent by the limitations of received systems that are only discovered when we try to apply them to real problems. At the same time, developments in symbolic logic that are undertaken purely for the sake of their theoretical interest frequently suggest new applications of logical analysis for which no need had previously been perceived.

The number of distinct logical systems inevitably raises the philosophical question of how the multiplicity of logics should be understood. Some logicians are partisan defenders of particular logical formalisms as the ideal single correct logic. Others are tolerant of many logics, adopting an attitude according to which particular formal systems may be appropriate for particular analytic tasks, but that no single logic or cluster or family of logics deserves to be called the one and only correct system of logic. Those who favor a single correct system of logic must either regard alternative logics as incorrect, however formally interesting, or else interpret them as representing conflicting incompatible opinions about the best and uniquely correct logical system. Such a contrast of philosophical positions about the nature of logic and the uniquely correct logic or plurality of alternative logics has positive analogies in the opposition of moral absolutism and moral relativism, and in questions of privileged objective truth versus subjectivism, perspectivalism, and syncretism in the theory of knowledge. It would not be surprising to find philosophers who incline toward relativism in ethics or epistemol-
ogy also to prefer a tolerant attitude about the peaceful coexistence of many different logical systems, and for their adversaries who think in terms of moral and epistemic absolutes to embrace a single correct logic that either defeats the ostensible alternatives, or resolves apparent conflicts between many if not all of them in a greater overarching synthesis.

Philosophy thrives on just such tensions and ambiguities, and philosophical logic is no exception. All of the diverse formal syntactical distinctions available in contemporary symbolic logic can be put to good use in clarifying philosophical ideas and drawing more precisely interpreted distinctions than are otherwise possible in ordinary language, or even in specialized but nonsymbolic philosophical terminologies. The methods of set theory, model set theoretical semantics, and axiomatizations of many types of philosophical concepts are among the widely used formalisms in present-day philosophical logic. The future will likely see more sophisticated logical machinery, and with it an even greater upsurge in the number and variety of logical systems and distinctive categories of logic and philosophical logics. If there is a logic of knowledge and a logic of moral obligation, then there can surely be multiple logics of deductively valid inference, each tailored to a particular philosophical conception of how even the most basic logical operations may be thought to function. We can nonetheless continue to expect that partisan champions in philosophical logic will want to refer to a preferred formalism as logic full stop, or as the one and only correct or underlying primary or essential logic. The awareness of philosophical commitment and presupposition even in the most rigorous abstract logical symbolisms, and of philosophical logic as an application of logic in which philosophical ideas are already deeply infused, can help to make logic a more powerful ally of philosophical analysis.

References

Part I

HISTORICAL DEVELOPMENT OF LOGIC
Ancient Greek logic was inseparable from ancient Greek philosophy. The formal theories developed by major logicians such as Aristotle, Diodorus Cronus, and Chrysippus were in large part influenced by metaphysical and epistemological concerns. In this brief essay, I will try to give some picture of this interrelationship. For reasons of space, I make no attempt to cover, or even to mention, every aspect of ancient Greek logic. I have preferred instead to concentrate on illustrating its philosophical aspects.

1 The Origins: Parmenides and Zeno

Greek philosophical logic originates with Parmenides (c. 510–c. 440 BCE). Though Parmenides cannot be said to have had a logic, or even an interest in studying the validity of arguments, his views did much to set the agenda out of which many things in Greek philosophy, including logic, later arose. His philosophical position is both simple and mystifying: being is, whereas not being is not and cannot either be thought or said. Consequently, any type of expression that implies that being is not or that not being is must be dismissed as nonsense. For Parmenides, this includes any reference to change (since it must involve the coming to be of what is not and the not being of what is) or multiplicity (since to say that there are two things is to say that something is not something else). The conclusion is that what is is one, unchanging, and uniform, without distinctions. Much of subsequent Greek philosophy is an effort to avoid these consequences and defend the coherence of talk of motion and multiplicity.

A second, and more explicitly logical, impact of Parmenides’ thought on Greek philosophy is through its defense by Parmenides’ follower Zeno of Elea (c. 490–c. 430 BCE). According to Plato’s *Parmenides*, Zeno’s goal was to defend Parmenides’ views from the objection that they were absurd or in contradiction to our ordinary beliefs. In response, Zeno argued that the beliefs that there is motion and that there is a multiplicity of entities have consequences that are even more absurd because self-contradictory. This was the point of his celebrated arguments against motion and multiplicity.

To consider one example, Zeno gives the following argument (paraphrased) that motion is impossible:
In order to move from point A to point B, you must first reach the point halfway between them. But before you can reach that point, you must reach the point halfway to it. Continuing in this way, we see that before you can reach any point, you must already have reached an infinity of points, which is impossible. Therefore, motion is impossible.

This argument rests only on the assumptions that motion is possible, that in order to move from one point to another one must first pass through the point halfway between, and that there is a point halfway between any two points.

Zeno’s arguments take a particular form: beginning with premises accepted by his opponent, they derive conclusions that the opponent must recognize as impossible. Aristotle says that in introducing this form of argument, Zeno was the originator of ‘dialectic’. The meaning of this word is contested by scholars, but we may note three features of Zeno’s argument: (1) it is directed at someone else; (2) it takes its start from premises accepted by that other party; (3) its goal is the refutation of a view of that other party. These three characteristics can serve as a rough definition of a dialectical argument.

2 Dialectic and the Beginnings of Logical Theory

In the later fifth century BCE, professional teachers of oratory appeared in Athens. These were most often the same people called (by us, by their contemporaries, and often by themselves) ‘Sophists’. We know that a number of the Sophists had interesting (and quite divergent) views on philosophical matters. Teaching oratory was a profitable occupation, and several Sophists seem to have amassed fortunes from it. The content of their instruction, to judge by later treatises on rhetoric, would have included such things as style and diction, but it would also have included some training in argumentation. That could have ranged from teaching set pieces of argument useful for specific situations, all the way to teaching some kind of method for devising arguments according to principles. One theme that emerges in several sophistic thinkers is a kind of relativism about truth. This is most forcefully put by Protagoras (c. 485–415 BCE), who began his treatise entitled Truth with the line, “Man is the measure of all things: of things that are, that they are, and of things that are not, that they are not.” Plato tells us in his Theaetetus that this meant “whatever seems to be true to anyone is true to that person”: he denied that there is any truth apart from the opinions of individuals. For Protagoras, this appears to have been connected with a thesis about the functioning of argument in a political situation. Whoever has the most skill at argument can make it seem (and thus be) to others however he wishes: in Protagoras’ world, persuasive speech creates not merely belief but also truth.

Even apart from this perhaps extreme view, we find the themes of the variability of human opinion and the power of argument widespread in fifth-century Athens. Herodotus’ history of the Persian Wars present a picture of opinions about right and wrong as merely matters of custom by displaying the variability in customs from one people to another. The treatise known as the Twofold Arguments (Dissoi Logoi) gives a series of arguments for and against each of a group of propositions; the implication is that argument can equally well support any view and its contradictory.
Contemporary with the Sophists was Socrates (469–399 BCE), whose fellow Athenians probably regarded him as another Sophist. Socrates did not teach oratory (nor indeed does he appear to have taught anything for a fee). Instead, he engaged people he encountered in a distinctive type of argument: beginning by asking them questions about matters they claimed to have knowledge of, he would lead them, on the basis of their own answers to further questions, to conclusions they found absurd or to contradictions of their earlier admissions. This process, which Plato and Aristotle both saw as a form of dialectical argument, usually goes by the name of ‘Socratic refutation.’ In overall form, it exactly resembles Zeno’s arguments in support of Parmenides. Socrates insisted that he knew nothing himself and that his refutations were merely a tool for detecting ignorance in others.

Plato (428/7–348/7 BCE) did not develop a logical theory in any significant sense. However, he did try to respond to some of the issues raised by Parmenides, Protagoras, and others. In his *Theaetetus*, he argues that Protagoras’ relativistic conception of truth is self-refuting in the sense that if Protagoras intends it to apply universally, then it must apply to opinions about Protagoras’ theory of truth itself; moreover, it implies that the same opinions are both true and false simultaneously. He also partially rejects Parmenides’ thesis that only what is can be thought or said by distinguishing a realm of ‘becoming’ that is not simply non-being but also cannot be said simply to be without qualification.

Plato’s most celebrated philosophical doctrine, his theory of Forms or Ideas, can be seen as a theory of predication, that is, a theory of what it is for a thing to have a property or attribute. In very crude outline, Plato’s response is that what it is for x (e.g. Socrates) to be F (e.g. tall) is for x to stand in a certain relation (usually called ‘participation’) to an entity, ‘the tall itself,’ which just is tall. In his *Sophist*, Plato begins to develop a semantic theory for predications. He observes that truth and falsehood are not properties of names standing alone but only of sentences produced by combining words. ‘Theaetetus’ and ‘is sitting’ are, in isolation, meaningful in some way but neither true nor false. We find truth or falsehood only in their combination: ‘Theaetetus is sitting.’ For Plato, a major achievement of this analysis is that it allows him to understand falsehoods as meaningful. In the sentence ‘Theaetetus is flying,’ both ‘Theaetetus’ and ‘is flying’ are meaningful; their combination is false, but it is still meaningful.

Aristotle (384–322 BCE), Plato’s student, developed the first logical theory of which we know. He follows Plato in analyzing simple sentences into noun and verb, or subject and predicate, but he develops it in far greater detail and extends it to sentences which have general or universal (katholou, ‘of a whole’: the term seems to originate with Aristotle) subjects and predicates.

Aristotle also gives an answer to Protagoras and to related positions. Specifically, in Book IV of his *Metaphysics*, he argues that there is a proposition which is in a way prior to every other truth: it is prior because it is a proposition which anyone who knows anything must accept and because it is impossible actually to disbelieve it. The proposition in question is what we usually call the principle of non-contradiction: “it is impossible for the same thing to be both affirmed and denied of the same thing at the same time and in the same way” (*Met.* IV.3, 1005b19–20). He argues that it follows from this principle itself that no one can disbelieve it. At the same time, since it is prior to every other truth, it cannot itself be proved. However, Aristotle holds that anyone who claims
to deny it (or indeed claims anything at all) already presupposes it, and he undertakes to show this through what he calls a “refutative demonstration” (Met. IV.4).

3 Aristotle and the Theory of Demonstration

When Aristotle says that the principle of non-contradiction cannot be proved because there is nothing prior from which it could be proved, he appeals to a more general thesis concerning demonstration or proof: no system of demonstrations can prove its own first principles. His argument for this appears in his Posterior Analytics, a work best regarded as the oldest extant treatise on the nature of mathematical proof. The subject of the Posterior Analytics is demonstrate sciences: a demonstrative science is a body of knowledge organized into demonstrations (proofs), which in turn are deductive arguments from premises already established. If a truth is demonstrable, then for Aristotle to know it just is to possess its demonstration: proofs are neither a means of finding out new truths nor an expository or pedagogical device for presenting results, but rather are constitutive of knowledge. Though he does not limit demonstrative sciences to mathematics, it is clear that he regards arithmetic and geometry as the clearest examples of them. Both historical and terminological affinities with Greek mathematics confirm this close association.

A demonstration, for Aristotle, is a deduction that shows why something is necessarily so. This at once imposes two critical limits on demonstrations: nothing can be demonstrated except what is necessarily so, and nothing can be demonstrated except that which has a cause or explanation (the force of the latter restriction will be evident shortly).

Since demonstrations are valid arguments, whatever holds of valid arguments in general will hold of them. Therefore, a natural place to begin the discussion of demonstrations would be with a general account of validity. Aristotle announces exactly that intention at the beginning of his Prior Analytics, the principal subject of which is the ‘syllogism’, a term defined by Aristotle as “an argument in which, some things being supposed, something else follows of necessity because of the things supposed.” This is obviously a general definition of ‘valid argument.’ However, Aristotle thought that all valid arguments could be ‘reduced’ to a relatively limited set of valid forms which he usually refers to as ‘arguments in the figures’ (modern terminology refers to these forms as ‘syllogisms’; this can lead to confusion in discussing Aristotle’s theory).

Aristotle maintained that a single proposition was always either the affirmation or the denial of a single predicate of a single subject: ‘Socrates is sitting’ affirms ‘sitting’ of Socrates, ‘Plato is not flying’ denies ‘flying’ of Plato. In addition to simple predications such as those illustrated here, with individuals as subjects, he also regarded sentences with general subjects as predications: ‘All Greeks are humans,’ ‘Dogs are mammals,’ ‘Cats are not bipeds.’ (Here he parts company from modern logic, which since Frege has seen such sentences as having a radically different structure from predications.) Aristotle’s logical theory is in effect the theory of general predications. In addition to the distinction between affirmation and denial, general predications can also be divided according as the predicate is affirmed or denied of all (universal) or only part (particular) of its subject. There are then four types of general predications: