Ingo Dierking

Textures of Liquid Crystals
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Demus, D., Goodby, J. W., Gray, G. W., Spiess, H. W., Vill, V.
Handbook of Liquid Crystals
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Ferroelectric and Antiferroelectric Liquid Crystals
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Textures of Liquid Crystals
Liquid crystals were discovered well over 100 years ago, and since that time texture analysis by polarizing microscopy has been employed as a primary tool for the characterization of different liquid crystalline phases. More than 20 years have passed since the first monograph on liquid crystal textures by Dietrich Demus and Lothar Richter was published (Textures of Liquid Crystals, Verlag Chemie, Weinheim, 1979), with its emphasis on nematic, cholesteric, and fluid smectic phases, as many other phases were still to be discovered. Some years later, George W. Gray and John W. Goodby published their monograph on smectic liquid crystal textures (Smectic Liquid Crystals – Textures and Structures, Leonard Hill, Glasgow, 1986), with its obvious emphasis, as indicated by the title, on phases with positional order. These two books have served the liquid crystal community and generations of students extremely well in their efforts to identify and characterize the liquid crystalline phases of newly synthesized mesogenic compounds. The past decade, give or take a few years, has seen the discovery of a number of novel liquid crystalline phases, many of them inherently connected to the chirality of the constituent molecules. The time thus seems to be right, not to revise, but rather to complement, the existing range of literature on the textures of liquid crystals, especially when considering the fact that both of the above monographs have long been out of print.

The present publication does not claim to cover all of the different possible aspects of liquid crystal texture appearance, and it is not a text about the wealth of topological defects exhibited by these materials. Rather, I have tried to present a collection of the most commonly observed textures, placing a clear emphasis on chiral materials and especially on those phases which have raised increasing scientific, but also to some extent applicational, interest over the past decade. These are namely the frustrated twist grain boundary (TGB) phases, the chiral tilted smectic phases (ferroelectric liquid crystals), including the variety of SmC\textsuperscript{*} subphases, and the so-called “banana phases”. As experimental investigations on many of these novel liquid crystalline phases are still in progress, and some of their structures and textures are still the subject of intense discussion, obviously it cannot be excluded that some of the remarks made in the present text may be subject to revision in years to come. The textures of lyotropic as well as polymer liquid crystals are generally quite similar to those of the calamitic mesogens, although often
much less pronounced and characteristic. For this reason, they were neglected in the present text. Obviously, I could not include all of the references that might have been relevant to cover certain subjects exhaustively, but I hope to have provided a representative overview and guide to further literature. I apologize to all those who are not mentioned, but who nevertheless have greatly contributed to the understanding of liquid crystals and their textures.

During years of teaching liquid crystal physics, in both lectures and the laboratory, I have gained the impression that, to many students, the textures of liquid crystals are very interesting and appealing on an aesthetic level, while at the same time they find it quite hard to relate the observed optic appearance to respective phases. On the other hand, graduate students and post-doctoral fellows are really the ones who spend numerous hours in the laboratory, performing much of the actually published research. For this reason, the present collection of liquid crystalline textures, their basic explanation, as well as short discussions of some related electrooptic effects, is designed for graduate students, post-doctoral fellows, and scientists who are generally new to the field of liquid crystal research, to be used in everyday laboratory work for the characterization of novel materials. The coverage of textures is presented as a tour through the polarizing microscopic world of a virtual liquid crystal on cooling all the way from the isotropic to the crystalline state. Although this text is primarily written for students and younger researchers, not for specialists in liquid crystal textures and their defects, I sincerely hope that it will also attract the interest of those members of the liquid crystal community who are familiar with the topic.

Completion of this texture collection would not have been possible without contributions from a number of colleagues, who provided their beautiful photographs and whom I would like to thank sincerely for their ready cooperation: G. Andersson, D. Demus, H. F. Gleeson, E. Grelet, W. Kuczynski, M. H. Li, J. G. Meier, H. T. Nguyen, B. Pansu, G. Pelzl, S. K. Prasad, P. Rudquist, and H. Stegemeyer. Most of all I would like to thank Dietrich Demus and Helen Gleeson for taking on the burden of carefully and critically reading the manuscript prior to publication, as well as D. Ganzke for his comments on parts of the manuscript. Finally, I am indebted to Jörn Ritterbusch and the staff at Wiley-VCH, Weinheim, for their continuous support and guidance throughout this project.

Ingo Dierking

Manchester, April 2003
Foreword

In the past 40 years, liquid crystal research has developed from a scientific curiosity to a topic of great scientific and technical importance. In the course of this development, more than 80,000 chemical compounds have been synthesized. The first characterization of newly synthesized materials is generally done by investigation with a polarizing microscope equipped with a hot stage, in order to observe the textures of liquid crystalline phases. Because of the number of different textures, in certain cases even for the same phase type, an extensive atlas showing and explaining the most important examples is urgently needed, even for experienced researchers.

In the more recent period of liquid crystal research, only two attempts have been undertaken to provide comprehensive collections of textures of thermotropic liquid crystals: Textures of Liquid Crystals by D. Demus and L. Richter in 1979, and Smectic Liquid Crystals by G. W. Gray and J. W. Goodby in 1986. In the meantime, several types of novel liquid crystals have been developed, e.g. discotic compounds, ferroelectric and antiferroelectric phases, twist grain boundary (TGB) phases, and banana-shaped compounds, showing characteristic textures with many unexpected features. Therefore, I warmly welcome the present up-to-date approach by Ingo Dierking, who is an active and experienced scientist in the field of chiral materials with their stimulating and complicated textural features.

In addition to the scientific and practical value of a collection of texture photographs, I would point out the aesthetic side of this study. The beauty of liquid crystal textures has excited not only researchers in this field, but also professional and amateur artists, who use photographs of special features or design motifs derived from textural studies.

I wish Ingo Dierking and his book deserved success. May this beautifully equipped volume help to inspire the scientific community in continuing research on liquid crystals.

Dietrich Demus

Halle, April 2003
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1
Introduction

1.1 States of Matter

Liquid crystals are partially ordered, anisotropic fluids, thermodynamically located between the three dimensionally ordered solid state crystal and the isotropic liquid. Their discovery is generally dated back to the year 1888, when the Austrian botanist Friedrich Reinitzer reported on the observation of compounds (cholesteryl benzoate and cholesteryl acetate) with apparently two melting points [1]. In his publication, originally written in German, he wrote reluctantly: “Es fiel mir jedoch auf, dass die Substanz dabei nicht zu einer klaren, durchsichtigen, sondern stets zu einer trüben, nur durchscheinenden Flüssigkeit schmolz … [bis] beim Erwärmen auf höhere Temperaturen plötzlich die Trübung verschwindet … Diese merkwürdige Erscheinung des Vorhandenseins von zwei Schmelzpunkten, wenn man sich so ausdrücken darf, und das Auftreten der Farberscheinungen war es hauptsächlich, welche mich auf den Gedanken brachte, dass hier und beim Acetat physikalische Isomerie vorliegen müsse …” (“I noticed, however, that the compound did not melt to a clear, transparent, but always to a muddy, only translucent, liquid … [until] by further temperature increase suddenly this muddiness disappears … This strange phenomenon of the existence of two melting points, if one may say so, and the occurrence of coloring was the main reason to conclude that here, and for the acetate, physical isomerism should be present …”). Friedrich Reinitzer had observed the typical light scattering of a liquid crystal and the phenomenon of selective reflection of the helical cholesteric phase. Without really being able to explain the impact of his discovery, Reinitzer nevertheless realized that for the compounds he had investigated the observed behavior was different from the generally accepted view on the states of matter at his time. This should not detract from the fact that liquid crystalline phases had (possibly) been observed before [2–6], but the fundamental realization of a novel behavior of matter was missing from those reports. For further characterization, Reinitzer sent samples to Otto Lehmann, the successor of Heinrich Hertz in Karlsruhe, an accepted specialist in polarizing microscopy, who only one year later coined the term “flüssiger Kristall” (liquid crystal) [7], not without receiving massive resistance from his colleagues.

Liquid crystalline materials are generally divided into two categories (Fig. 1.1), the thermotropic and the lyotropic mesophases. The term “mesophase” originates
from the Greek word *meso*, meaning “in between” (the crystal and the liquid phases). Thermotropic liquid crystal phases are observed by a change of temperature, while lyotropic phases form in the presence of a suitable (isotropic) solvent. An additional variable of state in the latter case is the concentration. Thus lyotropic and thermotropic phases; these materials are named *amphotropic*. To date, some 80000 different compounds, low molecular mass as well as polymeric materials, exhibiting liquid crystal properties have been reported. Most of these are collected in the database LiqCryst [8], together with their phase behavior, some physical properties and literature references. For an introductory text on the chemistry and physical properties of thermotropics, one may refer to books by Stegemeyer [9] and Collings and Hird [10]. An example of a more advanced text is that by Chandrasekhar [11], while the theoretical aspects are covered in the classic book by de Gennes and Prost [12]. Recently, a four volume compendium, summarizing the state of liquid crystal research up to approximately 1998, has been published as the *Handbook of Liquid Crystals* [13]. As lyotropic liquid crystals will not be covered within the scope of this book, we refer the reader to the monograph of Petrov [14].

Thermotropic liquid crystals are generally further distinguished with respect to the molecular shape of the constituent molecules, being called *calamitic* for rod-like, *discotic* for disk-like, and *sanidic* for brick- or lath-like molecules. In Fig. 1.2 a small selection of typical calamitic and discotic mesogens is shown. A common structural feature of calamitic mesogens is a relatively rigid core, often incorporating phenyl and biphenyl groups, and two flexible endgroups, often alkyl or alkoxy chains. In discotic mesogens six flexible endgroups are commonly attached to a rigid, disk-like core. Obviously, numerous variations to these general molecular structures may be implemented, leading to the large variety of mesogenic compounds we know today.
Calamitic mesogens:

Cholesteryl benzoate

\[
\text{Cr}^* \ 145 \ \text{N}^* \ 178 \ \text{BP}^* \ 179 \ \text{Iso}^*
\]

4-butyl-N-[methoxy-benzylidene]-aniline (MBBA)

\[
\text{Cr} \ 22 \ \text{N} \ 47 \ \text{Iso}
\]

D-2-methylbutyl 4-[4-n-decyloxybenzylideneamino]-cinnamate (DOBAMBC)

\[
\text{Cr}^* \ 76 \ \text{SmC}^* \ 95 \ \text{SmA}^* \ 117 \ \text{Iso}^*
\]

Discotic mesogens:

Fig. 1.2. A small selection of typical calamitic and discotic mesogens. Cholesteryl benzoate was the first liquid crystal reported by Friedrich Reinitzer in 1888, DOBAMBC the first mesogenic material where ferroelectric properties were demonstrated in the SmC* phase.

1.2

Liquid Crystal Nomenclature

The nomenclature of liquid crystals is probably a field of research in itself. As in any field of active research, the terminology of liquid crystal (LC) phases is subject to more or less continual change. It started with the classification of phases into
nematic, cholesteric, and smectic, as introduced by Friedel [15] in 1922. At that
time not very many mesogenic compounds were known and the classification
was meaningful, referring to systems with only long range orientational order (ne-
matic), the helical nematic phase observed for compounds derived from choles-
terin (cholesteric), and systems with additional positional order, thus phases
with a layered structure (smectic). Already at that time those phases were distin-
guished by their typical textures in polarizing microscopy. It is also worth noting
at this point that the nematic and the cholesteric phases are essentially equivalent
from the thermodynamic point of view. The latter is simply a nematic phase com-
posed of chiral molecules, which exhibits a helical superstructure.

During the decades following the classification by Friedel, especially in the 1950s
and 1960s, a variety of different smectic phases were discovered, many of them by
the liquid crystal group at Halle University, which has a very long tradition in
liquid crystal research, dating back to the days of Daniel Vorländer. It was here
that the nomenclature of assigning letters to different phases was first introduced
[16], although their scheme does not follow any systematic classification as is the
case for solid state crystal structures, but instead the phases were basically named
alphabetically in the order of their discovery. While offering an unequivocal
scheme of classification, this procedure had led to some confusion in the early
days, as different groups named different phases with the same letter, or the
same phases with different letters, which should have been assigned to an already
known phase type. Many of these problems were resolved when Sackmann and
Demus proposed their rule of miscibility [17] for the identification of liquid crystal
phases: if an unknown phase is continuously miscible with a reference phase over
the entire concentration range, then they are equivalent. On the other hand, if they
are not miscible, no conclusion can be drawn. This implies that the complete
phase diagram between the unknown and the reference compound has to be deter-
mained, as is exemplified in Fig. 1.3 (data after Ref. [18]), taken from an investiga-
tion of the Halle group. This procedure involves the preparation of a number of
mixtures, which is quite time consuming. A less work intensive, but at the same
time only qualitative, method is a contact preparation. The compound A with
the higher crystallization temperature is melted on a hot stage and drawn between
a glass substrate and a microscope coverslip by capillary action. Then the sample is
cooled to recrystallize it and the procedure repeated with the other, lower melting,
compound B. This yields a sample with a contact region between both compounds,
varying their concentration between 100 % A and 100 % B (Fig. 1.4). Variation of
temperature then simulates the whole phase diagram, although without knowl-
edge of the absolute concentration of both compounds. An illustrative example
is given in Plate 1, where the unidentified highly ordered phase of a compound
at the bottom left is in contact with an extensively investigated reference compound
at the upper right. For elevated temperatures a clear phase boundary is observed,
but within the temperature region of interest, that of the highly ordered phase to
be identified, the texture of the reference and the unidentified compound are the
same, without any phase boundary. This indicates that the low temperature phase
of the two materials are equivalent and we have thus identified the low tempera-
ture phase of the novel compound by comparison with that of the known reference material.

Additional confusion about the nomenclature of phases arose with the discovery of several classes of novel chiral LCs, namely the twist grain boundary (TGB) phases and the smectic C* subphases. In several early papers the TGBA* phase was named SmA*, with the asterisk (*) denoting a helical superstructure. This was somewhat misleading, because we should already make a distinction between SmA, the phase composed of achiral molecules, and SmA*, a smectic A phase composed of chiral molecules, but without any helical superstructure. The reason for this distinction lies in the different properties of the two phases. SmA* exhibits effects that are simply absent in SmA, like molecular optical activity or the electroclinic effect [19], somewhat analogous to piezoelectricity in solid state materials. Here we will follow the recommendation of Lagerwall [20] and use the asterisk to denote a phase composed of chiral molecules, regardless of any possible helical superstructure. This means that we will not use the often found notation “Ch.” for cholesteric, but rather N*. Nevertheless, we will for historic reasons still refer to the “cholesteric” phase, when talking about the “chiral nematic” phase. Again, the notation N* will be used regardless of a helical superstructure, a situation that can be observed for twist in-

Fig. 1.3. Typical phase diagram of two (very similar) compounds A (left) and B (right). (Data after Ref. [18].) Considering A to be the reference compound and B the one to be identified, application of the miscibility rule allows one to identify the nematic phase of compound B, as this is miscible for all concentrations with the nematic phase of compound A.

Fig. 1.4. Schematic illustration of a contact preparation to “simulate” the phase diagram of a binary system of compounds A and B. For an experimental example, see Plate 1.
version compounds, which show a temperature interval where a nematic director
configuration is observed with infinite pitch, even though the structure is composed
of chiral molecules. To indicate this situation the notation N\textsubscript{c}* will be used. Consis-
tency requires that also phases which appear only in chiral materials, like the Blue
Phases, should be denoted with an asterisk, BP*. Even the isotropic phase of chiral
molecules should be abbreviated as Iso* to distinguish it from the isotropic phase of
achiral mesogens, because the former shows optical activity, while the latter does
not. (The notation “Iso” instead of “I” will be used to avoid confusion with the
SmI phase, which is sometimes simply called “I” in the literature.)

Summarizing, we will mainly follow the nomenclature recommendations for
thermotropic liquid crystals of the International Liquid Crystal Society (ILCS)
and the International Union of Pure and Applied Chemistry (IUPAC) [21]:

- Crystalline phases
  - Cr for the crystalline phase (Cr\textsubscript{1}, Cr\textsubscript{2}, … if several crystal modifications are
    observed).
  - Cr* for crystalline phases composed of chiral molecules.

- Soft crystals (with long range positional order)
  - B, E, G, H, J, K instead of the formerly used notation SmB\textsubscript{cryst}, SmE, SmG, …. 
  - B*, E*, G*, H*, J*, K* for the soft crystal phases composed of chiral molecules.

- Smectic liquid crystals
  - SmA, SmB, SmC, SmI, SmF for the achiral smectic phases.
  - SmA*, SmB*, SmC*, SmI*, SmF* for the chiral smectic phases.

- Smectic C* “subphases” (which appear only in chiral materials)
  - SmC\textsubscript{u}* for a still controversially discussed, but presumably incommensurate,
    phase appearing just below the SmA* phase of some materials.
  - SmC* for the ordinary helielectric chiral smectic C phase, which is often
    referred to as “ferroelectric”.
  - SmC\textsubscript{1/3}* and SmC\textsubscript{1/4}* for the intermediate, often misleadingly termed
    “ferrielectric” phases Sm\textsubscript{FI1}* and Sm\textsubscript{FI2}*. 
  - SmC\textsubscript{A}* for the antiferroelectric chiral smectic C phase.

- Other antiferroelectric phases
  - SmI\textsubscript{A}* for the antiferroelectric chiral smectic I phase, etc.

- Twist grain boundary phases (which appear only in chiral materials)
  - TGBA* for the twist grain boundary smectic A phase.
  - TGBC* for the ferroelectric twist grain boundary smectic C phase.
  - TGBC\textsubscript{A}* for the antiferroelectric twist grain boundary smectic C phase.

Already here, we face problems with the nomenclature, as there are apparently
different TGBC* phases. At this point it seems to be too early to define a nota-
tion, and we refer to the structural models shown in Chapter 6.

- Nematic phase
  The nematic phase will be denoted as N, its chiral version as N*, also being re-
ferred to as the cholesteric phase.

- Blue Phases (which appear only in chiral materials)
  - BPI*, BPII*, BPIII* for the blue phases.
• Isotropic phase
The isotropic phase will be denoted as Iso for achiral and Iso* for chiral constituent molecules.

• Other phases
There are also some phases that will not play a great role in our further discussion, although some texture illustrations will be given. Their nomenclature will follow the ordinary scheme: SmQ* etc. Also the cubic D phase (CubD) and the recently discovered “banana phases” fall into this category and will be covered briefly in this context.

1.3 General Structure of Liquid Crystal Phases

1.3.1 The Nematic Phase

From the general liquid crystal phases, leaving the chiral ones aside to be treated individually in later chapters, the nematic phase is the one with the least order and the highest symmetry. It exhibits solely orientational order of the long molecular axis, i.e. an angular distribution of the long molecular axis around a particular direction, the director \( n \), while the molecules’ centers of mass are isotropically distributed in all three dimensions, as schematically depicted in Fig. 1.5. The director \( n \) is the average local direction of the long molecular axis and represents the direction of the optic axis of the system. Orientational order is generally described by a second rank symmetric traceless tensor with elements

\[
Q_{\alpha\beta} = \langle a_\alpha a_\beta \rangle - \frac{1}{3} \delta_{\alpha\beta}
\]

where \( a \) is a unit vector along the long axis of a molecule at position \( r \), \( \alpha, \beta = x, y, z \) is the fixed laboratory coordinate frame, and \( \delta_{\alpha\beta} \) is the Kronecker tensor, which is equal to one when \( \alpha = \beta \) and zero for \( \alpha \neq \beta \). Brackets \( \langle \rangle \) denote the temporal and spatial average. For an isotropic director distribution the average \( \langle a_\alpha^2 \rangle \) is equal to \( 1/3 \), and therefore \( Q_{\alpha\beta} = 0 \) in the isotropic phase. The order parameter tensor can be diagonalized by a suitable choice of the coordinate system, so that the five different ten-

![Fig. 1.5. Model structure of the nematic (N) phase. The spatial and temporal average of the long molecular axis is called the director \( n \). The angle \( \theta_i \) denotes the deviation of the long molecular axis of an individual mesogen \( i \) from the director.](image-url)
isor elements are reduced to the three diagonal elements. These are all different in the case of a biaxial phase, while any two of them are equal in the case of a uniaxial phase, like the nematic. In the latter case, we rewrite Eq. (1.1) for cylindrical molecules with rotational freedom around their long molecular axis as

\[ Q_{\alpha\beta} = S_2 (n_{\alpha} n_{\beta} - \frac{1}{3} \delta_{\alpha\beta}) \]  

with

\[ S_2 = \frac{1}{2} \langle 3 \cos^2 \theta_i - 1 \rangle \]  

the often used Hermans orientational order parameter [22], which was introduced as a scalar order parameter for liquid crystals by Tsvetkov [23]. If we choose the director \( \mathbf{n} \) along the z direction of the coordinate system, the diagonal elements of the tensor order parameter become \( S_{xx} = S_{yy} = -\frac{1}{3} S_2 \) and \( S_{zz} = \frac{2}{3} S_2 \). The brackets of Eq. (1.3) denote a statistical (spatial and temporal) average over an ensemble of molecules with their individual long molecular axis deviating from the director \( \mathbf{n} \) by an angle \( \theta_i \). \( S_2 \) is equal to the average of the second Legendre polynomial, \( S_2 = \langle P_2 \rangle \), the first non-trivial term in a Legendre expansion of the order distribution function \( f(\theta) \), which is itself generally not accessible from experiments. The scalar order parameter \( S_2 \) describes the phase transition from the isotropic to the liquid crystalline nematic phase: \( S_2 = 0 \) for random orientational order (the isotropic phase) and \( S_2 = 1 \) for perfectly aligned molecules. Experimentally, it is found that \( S_2 \approx 0.3 \) at the clearing point \( T_C \) (N–Iso transition), increasing to values of \( S_2 \approx 0.6 \) far below \( T_C \) (Fig. 1.6, data after Ref. [24]), with a temperature dependence that is described by the famous Maier–Saupe theory [25–27]. Note that these values for \( S_2 \) imply average values of \( \langle \theta_i \rangle \) in the order of 30°, which means that the deviation of the long molecular axis from perfect orientational order, as would be expected for a solid state crystal, is considerable.

![Fig. 1.6. Typical illustration of the temperature dependence of the scalar nematic order parameter \( S_2 \) for PAA (\( p \)-azoxyanisole). (Data after Ref. [24].)](image)
1.3.2
The Fluid Smectic Phases

On lowering the temperature of a nematic material, additional features of order may appear, namely positional order of the molecules’ centers of mass. This can at the least be one dimensional, leading to the layered structure of the SmA and SmC phases, which can in fact be regarded as two dimensional liquids. In addition to the orientational order of the long molecular axis, the molecules’ centers of mass arrange in layers. Looking at their projection onto the smectic layer plane, the molecules’ centers of mass are isotropically oriented, with no positional correlation being observed within or across the layer planes.

1.3.2.1 The Smectic A Phase

In the SmA phase the director $n$ and thus the optic axis are perpendicular to the smectic layer plane, as shown schematically in Fig. 1.7. At the same time, orientational order is far from being perfect. From a discussion with Woiteck Kuczyński some years back I remember his pointed description of the SmA phase: “... it’s more like a field of wheat in the wind”. The degree of translational order with periodicity $d$, equal to the smectic layer spacing, i.e. the length $L$ of a mesogen, is described by a smectic order parameter

$$\sigma = \frac{3 \cos^2 \theta_i - 1}{2} \cos \left( \frac{2\pi d}{d} z_i \right)$$

with the positions of individual molecule’s centers of mass at $z_i$, if the smectic layer normal is chosen in the $z$ direction. The smectic order parameter $\sigma$ was first introduced by Kobayashi and McMillan [28–30]. It contains the orientational order term $S_2$ multiplied by the positional order term, which represents the first term of a Fourier expansion of the positional order distribution function. Equation (1.4) is directly related to x-ray experiments on respective phases. The smectic layer spac-

![Fig. 1.7. Model structure of the smectic A (SmA) phase. The director $n$ is oriented parallel to the smectic layer normal $k$, while within a smectic layer the molecules’ centers of mass are isotropically distributed.](image)
ing can be evaluated by small angle x-ray scattering (SAXS), for example with a Kratky camera, according to the Bragg equation

\[ n \lambda = 2d \sin \alpha \]  

(1.5)

where \( n \) is the diffraction order, \( \lambda \) is the x-ray wavelength (often \( \lambda = 1.54 \text{ Å, Cu K}_{\alpha} \) radiation), \( d \) is the periodicity, and \( \alpha \) is the diffraction angle. For most fluid smectic liquid crystals, except amphiphilic molecules, mesogenic sugars, etc., generally only the first order diffraction peak is observed. As the x-ray diffraction pattern is in principle proportional to the Fourier transform of the electron density, we can readily deduce that in reality the fluid smectic structure is not perfectly layered (this would lead to reflections of higher diffraction order), but rather a sinusoidal distribution of the molecules’ centers of mass. Thus, a more realistic view of the SmA phase is schematically given by Fig. 1.8 (after Ref. [11], p. 321). Some materials also form a double layer structure with the identity period \( d = 2L \), or that of interdigitated molecules with \( L < d < 2L \) (Fig. 1.9).

### 1.3.2.2 The Smectic C Phase

Another smectic phase with one dimensional positional order is the SmC phase, which looks similar to SmA, only with the director being tilted by an angle with respect to the smectic layer normal. The tilt of an individual molecule can be described by a vector with amplitude \( \theta_i \) and azimuthal tilt direction \( q_{i} \). The average molecular tilt over an ensemble of molecules is given by

\[ \theta = \left\langle \theta_i \left( \frac{\cos q_i}{\sin q_i} \right) \right\rangle \]  

(1.6)

While the amplitude of the tilt \( \theta \) is determined by thermodynamic variables (temperature, pressure), the tilt direction is not predetermined at all. As we will discuss later in greater detail, the director tilt will have a tremendous impact on not only the physical properties but also the textures, if the constituent molecules are chiral. Nevertheless, the general structure of the SmC phase is schematically depicted in Fig. 1.10. With a uniform tilt direction the director tilt angle \( \theta = \langle \theta_i \rangle \) is considered to be the order parameter for the SmA to SmC transition and is a function of temperature \( T \), with increasing tilt for decreasing temperature. In the low temperature range of the SmC phase the majority of single component materials exhibit values for the director tilt in the order of \( \theta \approx 25^\circ -35^\circ \). Commercial SmC* mixtures are generally optimized to \( \theta \approx 22.5^\circ \) at room temperature, due to applicational aspects. The interesting case of \( \theta > 45^\circ \) has not yet seemed to be observed.

Two general groups of SmC materials may be distinguished: those with a high temperature SmA phase, and those with a high temperature nematic or isotropic phase. In the first case, the transition is (most often) of second order and can be described by a Landau theory, an expansion of the free energy density \( F \) at the phase transition temperature \( T_C \) with respect to the order parameter \( \theta \).