Brian H. Kaye

A Random Walk Through Fractal Dimensions

Second Edition

VCH Weinheim · New York · Basel · Cambridge · Tokyo

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VCH

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"This book is dedicated to my wife – Phyllis Dew Kaye, who has enriched my walk through the Dimensions of Life."

Biography

Dr. Brian Kaye was born in Hull, Yorkshire, England, in 1932. He obtained his B.Sc., M.Sc., and Ph.D. degrees from London University after studying at the University College of Hull, where he was a George Fredrick Grant Memorial Scholar. After working as a scientific officer at the British Atomic Weapons Research Establishment (Aldermaston) he taught physics at Nottingham Technical College from 1959 to 1963. He then moved to Chicago, where he was a Senior Physicist in the Chemistry Division of the IIT Research Institute (the Research Institute of the Illinois Institute of Technology). There he studied problems as different as why dirt sticks to the fibers of carpet to the design of better propellants for space rockets.

Since 1968 he has been Professor of Physics at Laurentian University in Sudbury, Ontario. He specializes in powder technology, which deals with the manufacture and properties of cosmetics, explosives, powdered metal pigments, drug powders, food powders, and abrasives. He has written a standard text on characterizing powders and authored over 100 scientific papers.

In 1977 his interest in the complex structure of soot involved him in the new subject of fractal geometry, an interest that led to the books "A Random Walk Through Fractal Dimensions" and "Chaos & Complexity. Discovering the Surprising Patterns of Science and Technology". The philosophical side of science has always interested him and has been complemented by his activities as a methodist local preacher in the Sudbury region of Ontario, Canada. He is just as likely to be found holding a service in a protestant church as he is to be lecturing on fractal geometry and chaos theory at the University.

Preface to the Second Edition

The response of readers to the first edition of this book on randomwalk modelling of fractal systems has been most gratifying. It has been a pleasure to receive letters from many people all over the world expressing interest in the subject matter and suggesting improvements for a second edition. Because of pressing demands on my time it was decided that in order to meet deadlines for a second edition the only feasible strategy was to add an extensive bibliography of the rapidly developing applications of fractal geometry.

Indeed, since the first edition of the book it is now becoming apparent that it is useful to differentiate between pure fractal geometry resplendent with Mandelbrot sets and Julian sets, the exotic coloured patterns of which make Joseph's coat of many colours look drab, and the prosaic down to earth measurement of the fractal dimensions of broken rocks and sooty fineparticles. Elsewhere I have explored the basic theory underlying Mandelbrot's exotic coloured set (*Chaos & Complexity,* VCH, 1993) and I continue to collect applications of fractal geometry to materials science and allied subjects, which hopefully will be described in detail at some future date. Accordingly I would like to invite people to write to me if they have new applications of fractal geometry that have escaped my attention. In structuring the bibliography I decided to order the list according to author name, since it would have been too big a task to identify appropriate key words for each paper.

I would like to thank Dr. Ute Anton and Dr. Peter Gregory for their enthusiasm and support in the development of the second edition of this book, and I hope that the topic continues to fascinate many people. As always my thanks go to Garry Clark for his enthusiastic support and work on the bibliography.

Sudbury
December 1993

B. H. Kaye

Preface to the First Edition

I am not sure whether one should write the preface of a book before or after one has completed the book. I have written this preface after I have written the book. Now that it is finished I am rather nervous of some of the simplifications that I have had to make in my presentation of complex ideas. I am reminded of the Italian proverb that "the translator is a traitor". When trying to simplify complex ideas one has to make simple statements which if one were to be making them in a professional context would be surrounded by cautionary statements and other comments on the limitations of the statements. Furthermore fractal geometry is growing so rapidly and ideas are developing so quickly that it is easy to make mistakes – if the reader detects mistakes please remember that "A man who never made a mistake never made anything". After using my first book (on the Characterization of Fineparticles) in a class, a student handed me two pages of corrections to be used in a future edition; hopefully there will not be too extensive a list of corrections to be made in a future edition of this book.

In retrospect, this book appears to be about randomwalk theory as much as about fractal geometry – this re-enforces the dictum I give my students – "scratch a fractal and you will usually find a randomwalk generative model underlying the fractal form".

As I look through some of the recent books on fractal geometry with their brilliant pictures of complex fractal sets my book appears pedestrian; this book is not glamorous, it is intended for a "first reader" in the nuts and bolts of applied fractal geometry. Hopefully readers will bear this intent in mind as they pursue a randomwalk through fractal dimensions.

Sudbury
January 1989

B. H. Kaye

Acknowledgments

First of all I would like to thank Dr. Helmut Grünewald of VCH Publishers who enthusiastically listened to my plans for a possible book an Fractal Dimensions during a visit to Heidelberg. In particular I was pleased that he encouraged me to write in the first person since I believe that this facilitated the presentation of the difficult ideas presented in this textbook. I would also like to thank Dr. Mandelbrot for inviting me to a conference on fractal geometry held in the conference centre of the University of Grenoble in Les Houches France in the early spring of 1985. This conference exposed me to the wide world of applied fractals. One of the pleasures of working in fractal geometry has been the development of a personal acquaintance with Dr. Mandelbrot and his wife Aliette who is a charming supporter of her husband's theories and endeavours.

Over the 10 years in which I have been active in fractal geometry, many students have participated in exploratory experiments to look at new applications of fractal geometry. Several of them are acknowledged in the text of the book. It is not possible to list all of the students who have helped me do develop my ideas on fractal geometry, however, I would like to especially thank student and Research Associate Mr. Garry Clark. Garry has always been very willing to carry out new experiments and has provided much of the data presented in the graphs of this book. He also drew most of the diagrams and co-ordinated the photographs required for this book. Without his sustained effort over a period of 4 years, it is doubtful that the book would have been written. Research Associates Remi Trottier and John Leblanc have been very active in helping me to define my ideas in various discussions and in carrying out experiments to test fractal theories. Barney MacFarlane and Robert Harrison also were of great assistance in generating data for the graphs. In the closing part of the activity Mr. Stephen Horodziejczyk was invaluable in preparing the final text on the word processor. Many typists have worked on converting dictated tapes to final scripts. My daughter Sharon was a major contributor to the editorial and typing process that has produced the book. My other daughter Alison has also contributed to the typing effort as have Mrs. June Talbot, Mrs. Linda Romas, Mrs. Donna Marshall, Mrs. Diane Hrytsak, Miss Vernice Thomas and Miss Leila Lindroos. My colleague Professor Ian Robb has been a constant supporter of my efforts to master fractal geometry and has been a great help in clarifying some of the more difficult aspects of the theory of fractals. Dr. Rizwan Haq of the Physics Department has also maintained a constant interest in the project. I would also like to thank the people who have sent me the literature over the years and who have helped me in my discussions of fractal geometry. In particular I would like to thank Dr. John Davidson of Avon Lake, Ohio, Mr. Alan Flook of Bedford, England, Dr. Englman of Israel and Professor Klinzing of the University of Pittsburgh, who stimulated my activities in fractal dimension through their comments and ideas.

The writing of this book was interrupted by a serious illness and I would like to thank Drs. E. MacCallum and A. Adegbite and the staff of the Neurosurgery ward of the Sudbury General Hospital for their care and attention which made it possible for me to return to my work on this book.

B. H. Kaye, January, 1989

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(Reproduced by permission Richard F. Voss/IBM Research).

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(Reproduced by permission of Richard F. Voss/IBM Research.)

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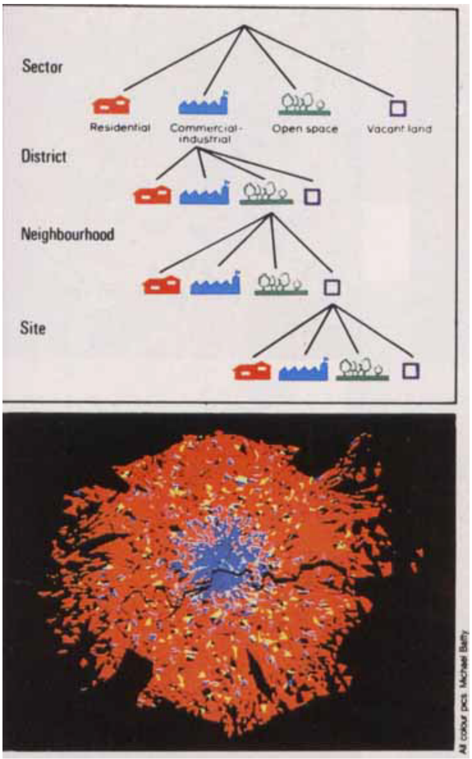

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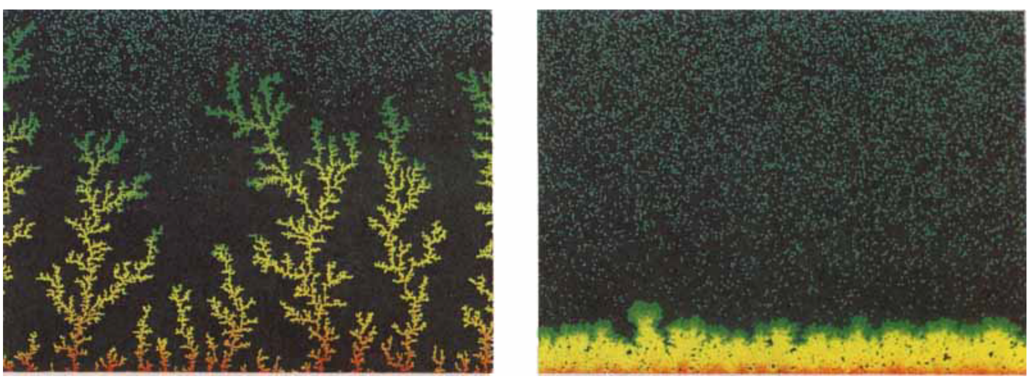

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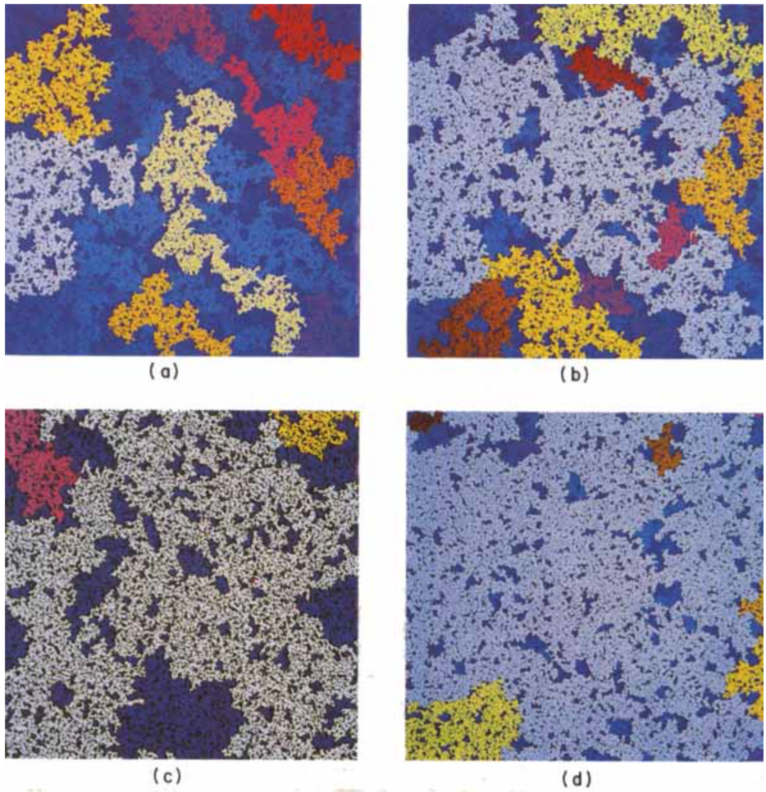

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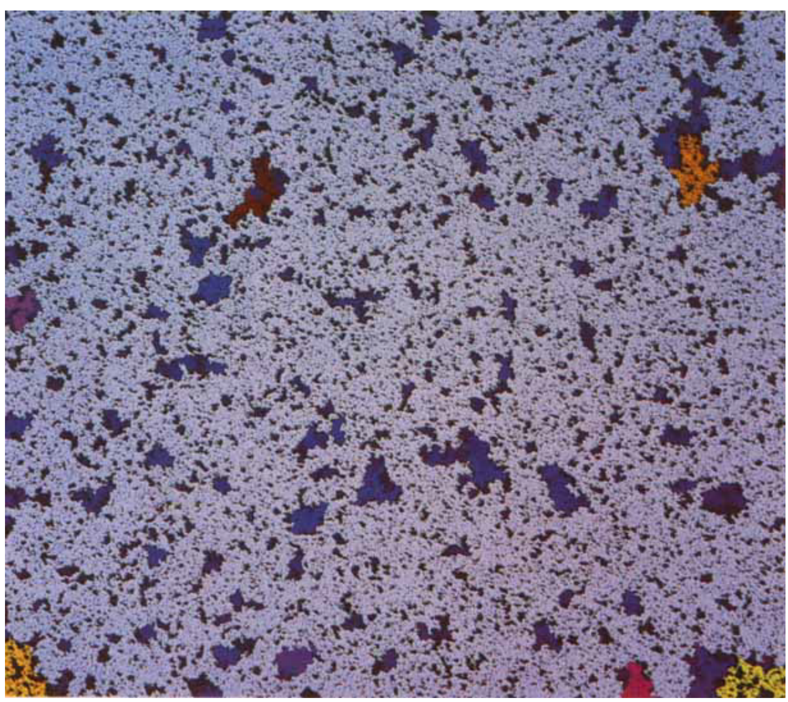

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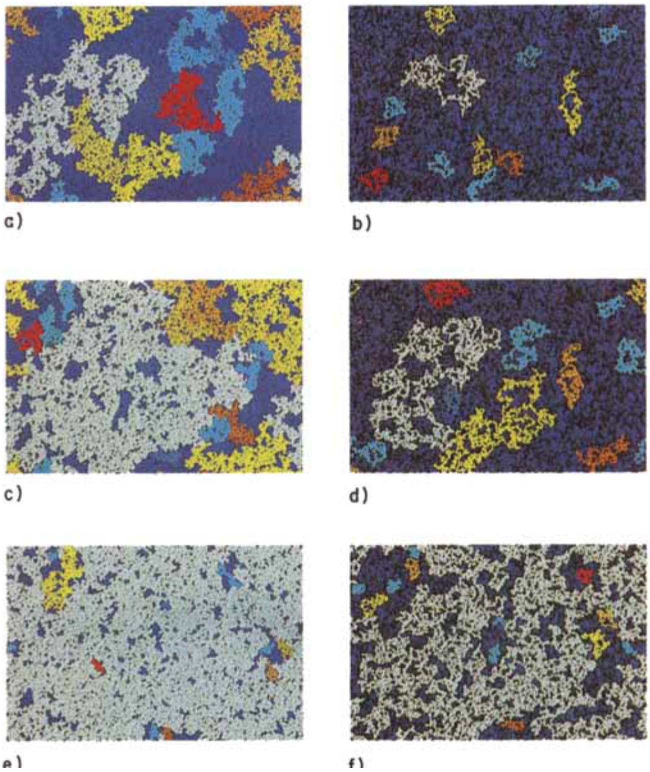

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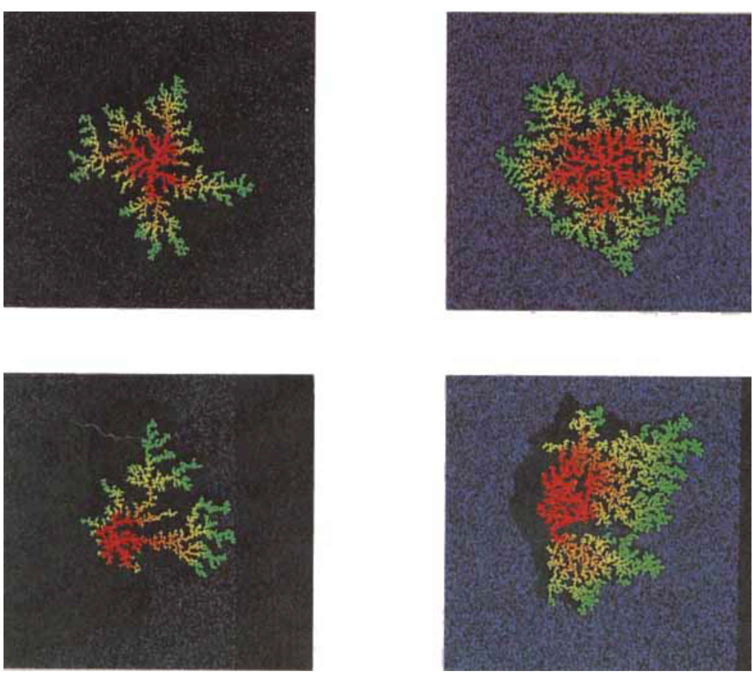

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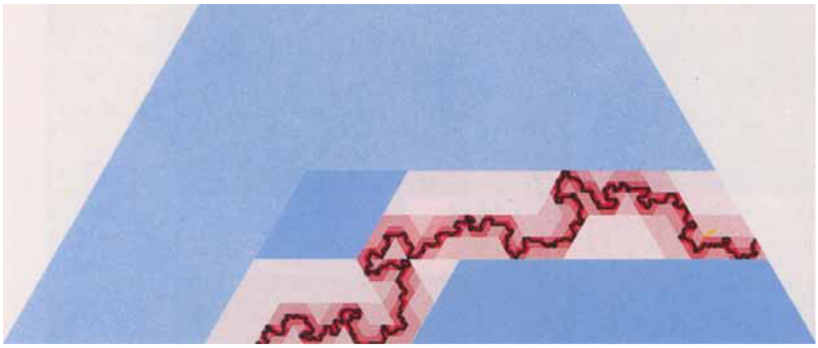

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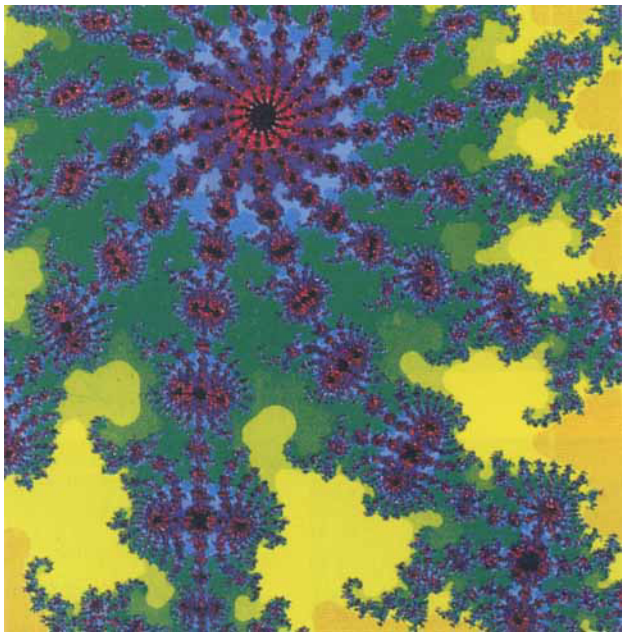

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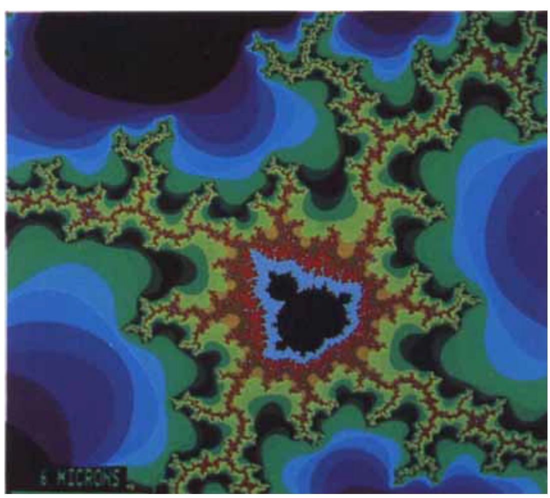

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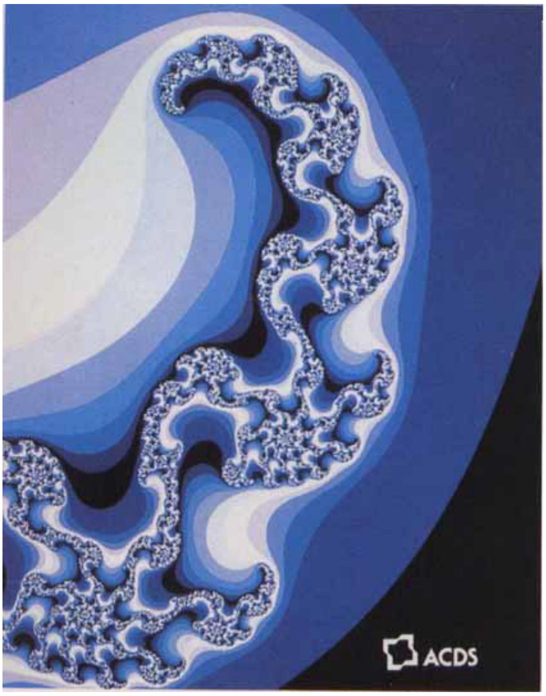

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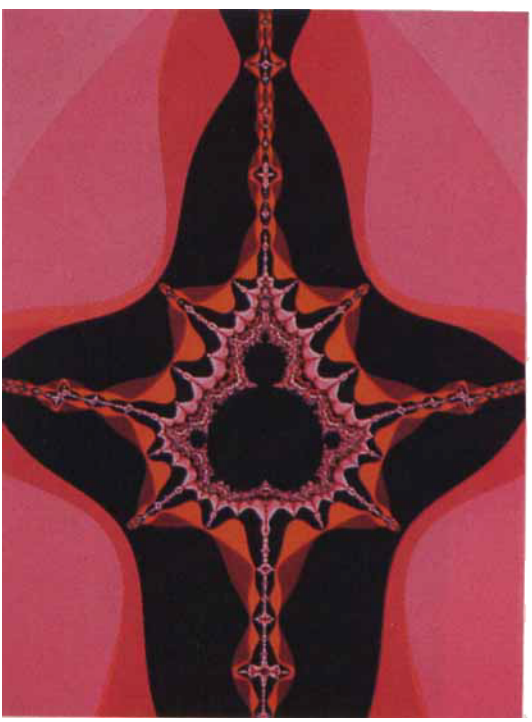

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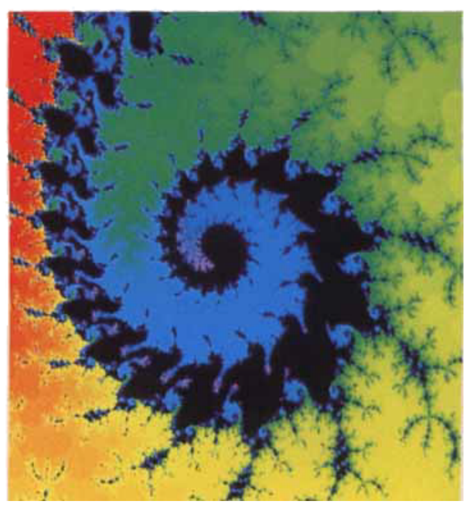

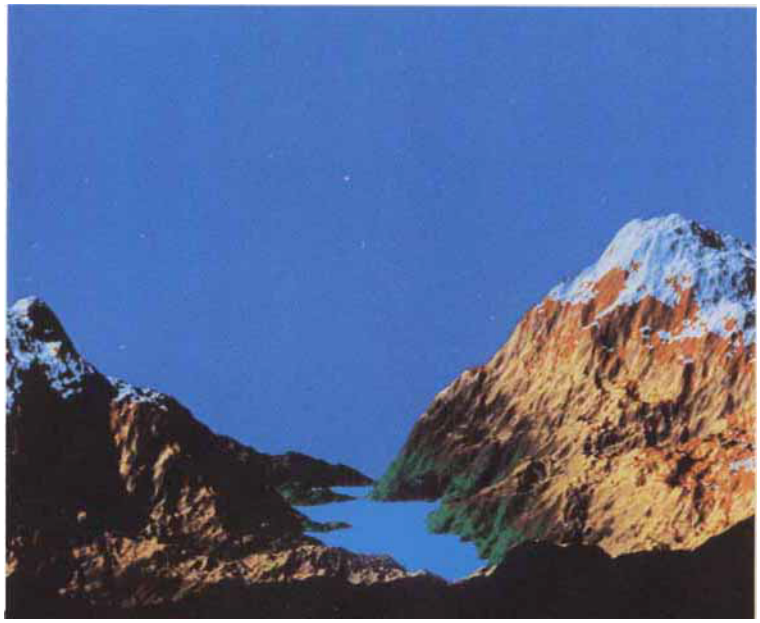

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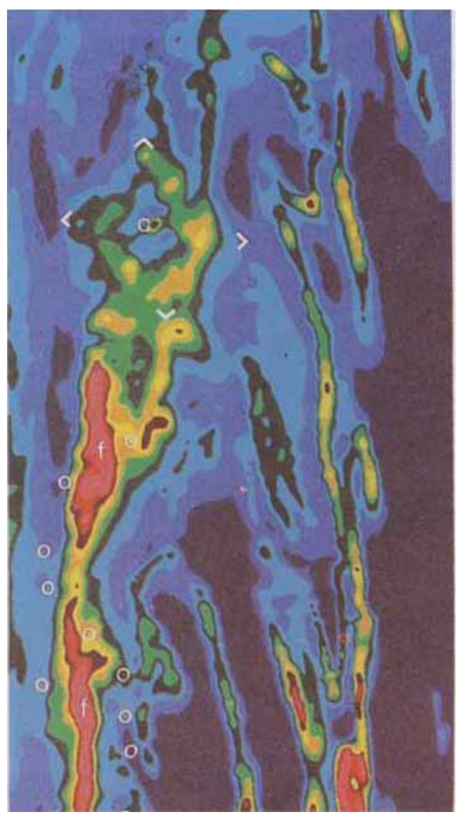

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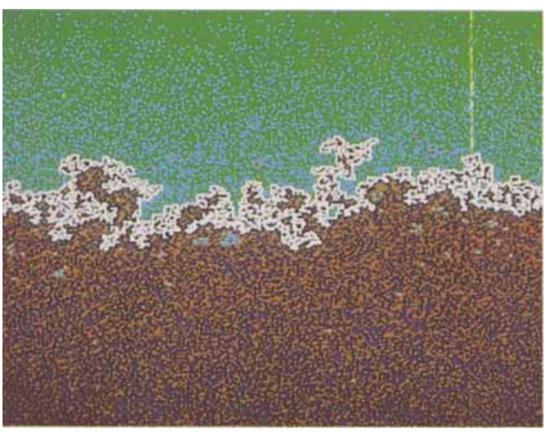

Plate 14

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1 A Starting Point for the Randomwalk

Fractal dimensions were introduced to the English-speaking scientific community in 1977 by Benoit Mandelbrot, their inventor, in a book entitled "Fractals: Form, Chance and Dimensions" [1]. I became aware of this book through a brochure sent to me by the Library of Science Book Club, which operates from New Jersey in the United States. In their description of the book, the reviewers for the Library of Science mentioned that Mandelbrot discussed in the book the problem of the length of the coastline of Great Britain. I did not know what a fractal was, but I did remember seeing an article by Mandelbrot on this problem when browsing through the journal *Science* in the 1960s. In his article in *Science* Mandelbrot had drawn attention to some earlier work by Louis Fry Richardson, who had pointed out that a simple question such as, "how long is the coastline of Great Britain?," has no answer apart from an operational description of how one estimates the length of the coastline [2]. For example, if one draws the coastline of Great Britain as shown in Figure 1.1, one can attempt to measure the length of the rugged coastline by striding around the coastline with steps λ to create a polygon whose perimeter is an estimate of the coastline, as shown in Figure 1.2. In Figure 1.2, three different estimates of the coastline of Great Britain, based on polygons of side length λ_1, λ_2 and λ_3, are shown. To obtain the estimates of the coast perimeter in dimension-

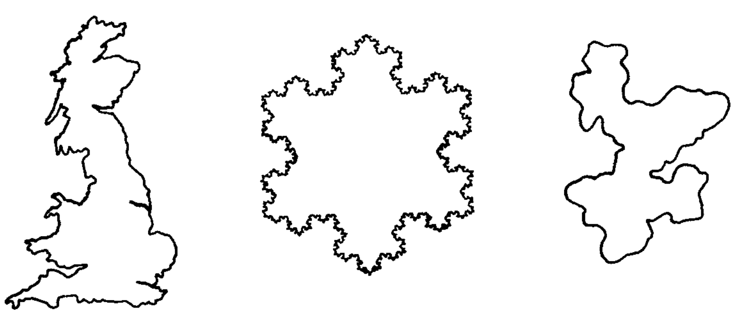

Figure 1.1. Richardson pointed out that the problem of measuring the length of a coastline leads to the paradox that all coastlines are infinite at infinitely small resolution. Mandelbrot linked this paradoxical conclusion to the structure of curves with infinite perimeter such as the *N*th-order Koch's triadic island, the structure of which he described by means of fractal dimensions. A visual comparison of the structure of a carbonblack agglomerate with the coastline of Great Britain and Koch's triadic island suggests that Mandelbrot's fractal dimensions may be useful in describing the structure of the carbonblack profile.
(a) Coastline of Great Britain.
(b) *N*th-order of Koch's triadic island.
(c) Carbonblack profile at high magnification.

less form, the perimeter estimates are normalized by dividing the perimeter of the polygon by the maximum projected length of the island, as illustrated in Figure 1.2.

In the bottom portion of Figure 1.2, the three estimates of the coastline are plotted on a graph having log-log scales. It can be seen that one can draw a straight line through the three points. If one extended the dataline, one would reach the conclusion that the

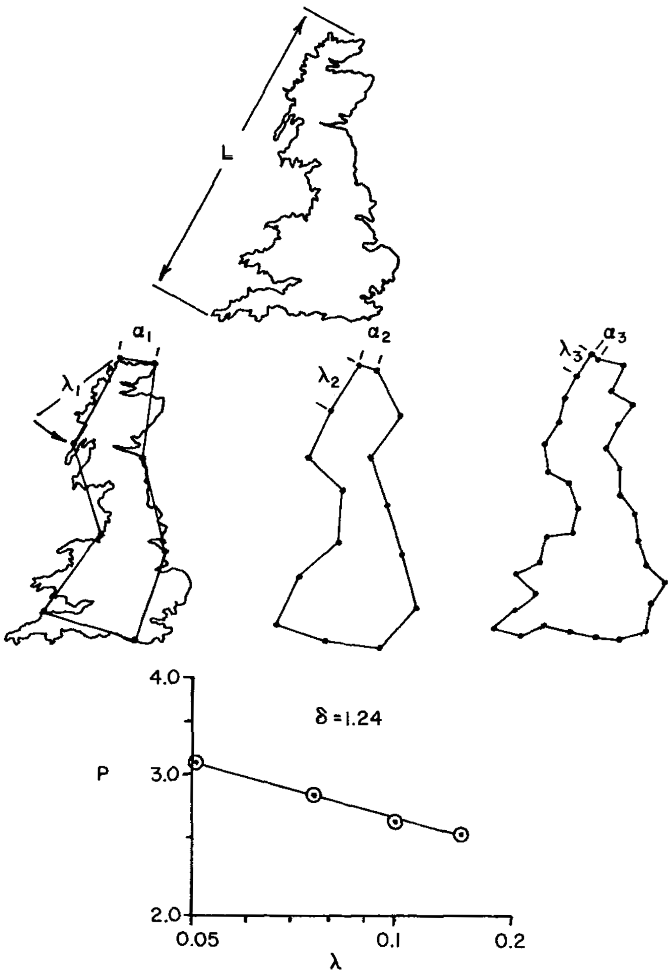

Figure 1.2. In the structured walk procedure for characterizing the fractal dimension of a rugged boundary, a series of polygons of side λ are constructed on the perimeter using a pair of compasses. A plot of P against λ on a log-log plot yields a dataline of slope m where

$$\delta = 1 + |m|$$

if the boundary can be described by a fractal dimension.

λ = side of polygon normalized with respect to maximum projected length of the profile.

P = polygon perimeter of side λ normalized with respect to maximum projected length of the profile.

δ = fractal dimension of the boundary.

L = maximum projected length of the island.