

Modern Physics for Engineers

Jasprit Singh



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PREFACE

Over the last few years there have been several important changes in the undergraduate curricula in both Engineering and Physics departments. In engineering schools there is an increased emphasis on design type courses which squeeze the time students have for *fundamental* courses. In physics departments there is an increased need to make a stronger connection between the material studied and modern technological applications. This book is motivated by these changes.

This book deals with important topics from the fields of quantum mechanics, statistical thermodynamics, and material science. It also presents a discussion of the special theory of relativity. Care is taken to discuss these topics not in a disjointed manner but with an intimate coupling. My own experience as a teacher is that applied science students are greatly motivated to learn basic and even esoteric concepts when these concepts are closely connected to applications from real-life technologies. This book strives to establish such connections wherever possible.

The material of the text can be covered in a single semester. The text is designed to be highly applied and each concept developed is followed with discussions of several applications in modern technology.

The emphasis in this text is not on tedious mathematical derivations for the solutions of various quantum problems. Instead we focus on the results and their physical implications. This approach allows us to cover several seemingly complex subjects which are of great importance to applied scientists.

I believe the level of the text is such that it can be taught in the physics, electrical engineering or material science departments of most schools. For physics students, the book may be suitable in the sophomore year, while in the engineering schools it may be used for seniors or even for entering graduate students.

I am extremely grateful to Greg Franklin, my editor, for his support and encouragement. He was able to get valuable input from a number of referees whose comments were most useful. I also wish to extend my gratitude to the following reviewers for their expert and valuable feedback: Professor Arthur Gossard of the Materials Department at the University of California, Santa Barbara; Professor Karl Hess of the Beckman Institute of Advanced Science and Technology at the University of Illinois; and Dr. Michael Strosio of the Army Research Office.

The figures, typing, cover design, and formatting of this book were done by Teresa Singh, my wife. She also provided the support without which this book would not be possible.

JASPRIT SINGH
Ann Arbor, MI

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INTRODUCTION

MODERN PHYSICS AND TECHNOLOGY

Modern technologies are changing our lives at an unprecedented pace. New materials, information technologies of communication and computation, and breakthroughs in medical technologies are bringing new opportunities to us. For students of applied physics and engineering these technologies offer both challenges and opportunities. On one hand, these developments offer solutions to complex problems—problems that seemed unsolvable a decade ago. On the other hand, the amount of knowledge needed to understand and contribute to new developments is also growing tremendously. This places a considerable burden on students.

Modern technologies seem so “magical” that they almost lull us into believing that one simply needs to wave a magic wand and new devices and systems will appear. It is easy to forget that these new inventions are based on well-established *fundamental* principles of physics derived through the application of the scientific method. Most of these principles have been known to humanity for only about a hundred years or less. Quantum mechanics, quantum statistics, theory of relativity, and structure of materials provide us the basis for modern technologies. Yet all of this understanding was developed in the twentieth century.

This book deals with the basic principles of modern physics that have led to revolutionary new technologies. Our focus is on the basic principles of quantum mechanics and material science and how these principles have been exploited to generate modern technologies. The emphasis in this text is not on tedious mathematical derivations for the solutions of various quantum problems. Instead we focus on the results and their physical implications. In Chapter 9 we present a treatment of the special theory of relativity along with some important applications.

In Figs. 1 and 2 we show an outline of some of the applications we link to the basic principles discussed in this text. As one can see, a number of different fields have benefited from modern physics. The connection between basic principles and applications is a very important one—one that is used by applied scientists as they go about inventing new technologies. It is important for students of applied physics and engineering to appreciate this connection.

I.1 Guidelines for the Instructor

This book can be used for engineering students as well as for physics majors. For engineering students there are two possible roles of this book, depending on the undergraduate curricula at a particular university. In some schools a modern physics course is offered in the physics department for undergraduate engineering students. This book should be particularly attractive for such a course, since there is a close

FIELD	APPLICATIONS
Electrical Engineering	<ul style="list-style-type: none">• Metals, semiconductors, insulators• Doping of semiconductors• Ohmic contact technology• Scanning tunneling microscope• Tunnel devices• Laser, LEDs, and display screens• Mobility and resistance• Quantum well devices• Quasi-Fermi level concept• Novel electron wave devices
Chemistry/ Chemical Engineering	<ul style="list-style-type: none">• Tailored drugs• Organic dyes• Catalytic converters
Medicine	<ul style="list-style-type: none">• Radioactivity in diagnostics• Radiation therapy• Nuclear magnetic resonance
Biological Sciences	<ul style="list-style-type: none">• Movements of cockroaches in sewer system• Progress of biological reaction pathways

Figure 1: Applied fields and applications discussed in this textbook.

FIELD	APPLICATIONS
Nuclear Engineering	<ul style="list-style-type: none">• Nuclear fission and fusion
Material Science	<ul style="list-style-type: none">• Superconducting materials• Surface characterization by electron diffraction, electron tunneling• Crystal structures• Tailoring of material stiffness
Anthropology	<ul style="list-style-type: none">• Carbon dating
Space Technology	<ul style="list-style-type: none">• Optical gyroscopes• Atomic clocks for global positioning system
Manufacturing Science	<ul style="list-style-type: none">• Radioactive materials in diagnostics of high-precision machine tool manufacturing

Figure 1: (Continued).

connection between basic principles and applications. For such students the instructor should cover the first six chapters and Chapter 9 in some detail and pick selected topics from Chapters 7 and 8. For example, the coupled well problem could be discussed from Chapter 7. A physical discussion of optical processes could be discussed from Chapter 8. Derivations of Fermi golden rule (Appendix A) or Boltzmann transport theory (Appendix B) should be avoided for students at this level.

The book would be very useful for students who are seniors or entering graduate students in the electrical engineering, material science, or chemical engineering departments. For such students the first couple of chapters could be covered rather quickly (some sections could be given as reading assignments). Topics in Chapters 7 and 8 as well as Appendices A and B could be covered in detail.

Finally, for physics majors the book could be used for a modern physics course. The book covers most topics of relevance to such a course and provides an important link to technical applications. Such a link is normally not provided to physics majors, but given the changes occurring in the physics profession, will most likely be appreciated by them.

The system of units used throughout the text is the SI system—a system that is widely used by technologists. The text contains nearly 100 solved examples, most of which are numerical in nature. It also contains about 200 end-of-chapter problems.

SOME IMPORTANT REFERENCES

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Special Relativity

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General

- R. P. Feynman, R. B. Leighton and M. Sands, *The Feynman Lectures of Physics*, Vol. III, Addison-Wesley Publishing Company, Reading, MA, 1964. This and the accompanying volumes are a *must read*.

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CHAPTER 1

CLASSICAL VIEW OF THE UNIVERSE

CHAPTER AT A GLANCE

- | | |
|---|--------------------|
| • What is the scientific method? | <i>Section 1.2</i> |
| • An overview of classical physics | <i>Section 1.3</i> |
| • Particles in classical physics | <i>Section 1.4</i> |
| • Waves in classical physics | <i>Section 1.5</i> |
| • An “uncertainty relation” in waves | <i>Section 1.6</i> |
| • Systems with large numbers of particles | <i>Section 1.7</i> |

1.1 INTRODUCTION

The term classical physics sounds old, but it is the term used for the physics in existence until the beginning of the twentieth century. This is the physics that was developed by scientists such as Archimedes, Newton, Maxwell, Boltzmann, . . . and even though the developments due to quantum physics and relativistic physics have ushered in the more accurate *modern* physics, classical physics continues to be used in a vast array of applications quite successfully.

Classical physics, it turns out, gives us an approximate description of how our world behaves. This approximation is usually adequate for a very large class of problems. However, on some levels, the inaccuracies of classical physics are manifested quite dramatically. Modern physics can be classified as a much better approximation and is suitable for cases where classical concepts fail.

In this chapter we will take a quick tour of important classical concepts. It is important to keep in mind that modern physics reduces to classical physics when certain conditions are met. These conditions may be stated as, “mass of the particle is large,” or “potential energy is slowly varying in space,” etc. Thus it is important to understand the classical concepts.

We start this chapter with an overview of the approach known as the scientific method. This approach must be followed in the understanding and development of any physics related concept. This is the method that has brought physics to the present level—a level where enormously complex physical phenomena are well understood.

1.2 SCIENTIFIC METHOD

Physics is the branch of science that attempts to answer the questions related to the ‘*hows*’ and ‘*whys*’ of our physical universe. In this great attempt, some very rigorous paths are to be taken. There must be a logical flow between one assertion and the next. It may be fine to explain the twinkling of stars to a child by claiming there are angels lowering and raising their lanterns. But this is not the approach we can follow in physics.

While there are still phenomena we cannot explain by simply invoking our understanding of physics, it is quite remarkable that a great range of observations can now be explained through modern physics. Physics has developed from explaining how cannon balls fly and pulleys work to what happens inside a white dwarf star or what happens to protons when they are smashed into each other at close to the speed of light.

It is important to understand the approach known as the *scientific method*, which has allowed us to understand our universe so well. This approach has been used by intellectual giants like Galileo, Newton, Maxwell, Einstein, Feynman, etc. It is also an approach used by thousands of basic and applied scientists around the world. The scientific method is used by the electrical engineer when he or she is trying to build a gigabit memory chip. It is used by the biophysicist who is assembling a special molecule to attack cancer cells. Or by the mechanical engineer developing a more crash-safe car. The scientific method allowed Galileo to do his

famous experiment to show that objects with different masses drop to the earth with the same velocity. It also allowed Copernicus to show that the earth went around the sun. It allowed Newton to describe how gravity worked and Einstein to come up with the theory of relativity. It ushered in the quantum age and the information age based on computers.

Let us examine the components of the scientific process. Fig. 1.1 gives an overview of this approach. Here are the building blocks:

- **Observation:** This is the core of the scientific method. Careful measurements of a physical phenomenon set the stage for most scientific endeavors. Sometimes the experiment may be motivated by a theoretical prediction based on existing knowledge. *In the scientific method no one can argue against a fundamentally sound experiment.* The experiment can never be subverted to fit an existing line of thought.
- **Postulates:** The scientific method now starts, with the ultimate aim to explain experiments and to predict phenomena. The method sets up some postulates which seem reasonable to the scientists. *These postulates are the starting points from which the theory describing the phenomena springs forth.* At the most fundamental level the postulates do not have any proofs, in the sense that they cannot be derived from other laws.
- **Physical Laws:** Next, physical laws are set up, which describe how physical quantities evolve from one value to another in space and time. These *equations of motion* along with the powerful language of mathematics allow the scientist to relate experiments and the scientific formalism. These laws not only should be able to explain existing observations, but also must predict phenomena that can then be observed by properly designed experiments. It is also very important to keep in mind what a good scientific method tries to do:
 - *Make assumptions that are absolutely necessary.* If we are building a formalism for how an airplane flies we must not assume anything about whether a cockroach evolved 10 million years ago or 100 million years ago!
 - *Accept the guidance of experiments.* A good scientist should not retain a formalism which explains everything except one lonely experiment. It is this lonely experiment that may ultimately advance our knowledge.
 - *Consider our present understanding as an approximation.* It is important to realize that when a scientist writes down certain laws or equations of motion, he or she is only describing how the real world behaves in an approximate way. This is because the laws may describe nature quite well in some regimes but may not be able to describe things well in a different regime. And even though we know a great deal of the rules of nature, we don't know all the rules. And some rules may manifest themselves under conditions that are almost impossible to realize in a controlled manner in a laboratory.

It sometimes happens that a certain set of laws or rules work quite well for a long period in our history. Then something is observed which does not fit into the scheme we have developed. One then looks for a broader set of rules. *These new rules must explain all the previous observations, along with the new ones.* One of the most dramatic instances of such a development is the subject of this book. Toward the end of the nineteenth century, a series of remarkable experiments were conducted that completely baffled existing physics. This created a tremendous excitement among

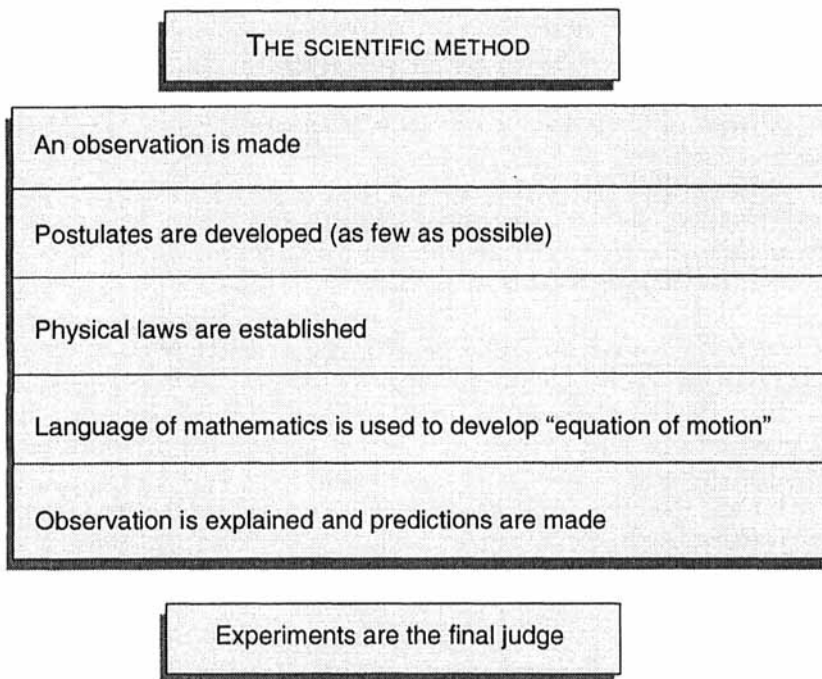


Figure 1.1: An overview of the scientific method.

the physicists who then gradually built what we now call *modern physics*. Before we start our discussions of modern physics we need to review *classical physics*, which is the name often given to physics existing up to the beginning of the twentieth century.

1.3 OVERVIEW OF CLASSICAL PHYSICS: BASIC INTERACTIONS

In classical physics our physical universe is distinctly divided into two categories—particles and waves. When we talk about particles, we do not necessarily refer to point particles or very tiny particles. The earth is a particle; so are a car, a refrigerator, and a tennis ball. The laws which describe particles and those which describe waves are very different, and we have no problem distinguishing a particle from a wave. In Fig. 1.2 we show an overview of classical concepts. We have particles which are represented by masses, momentum, position, etc., and waves which are described by amplitude, wavelength, phase, etc. We also have an important branch of physics which deals with systems of large numbers of particles. This field is known as thermodynamics.

In classical physics particles interact with each other via two kinds of interactions. As shown in Fig. 1.2, the first kind of interaction is the gravitational interaction. If we have two particles of masses m_1 and m_2 placed at a separation of

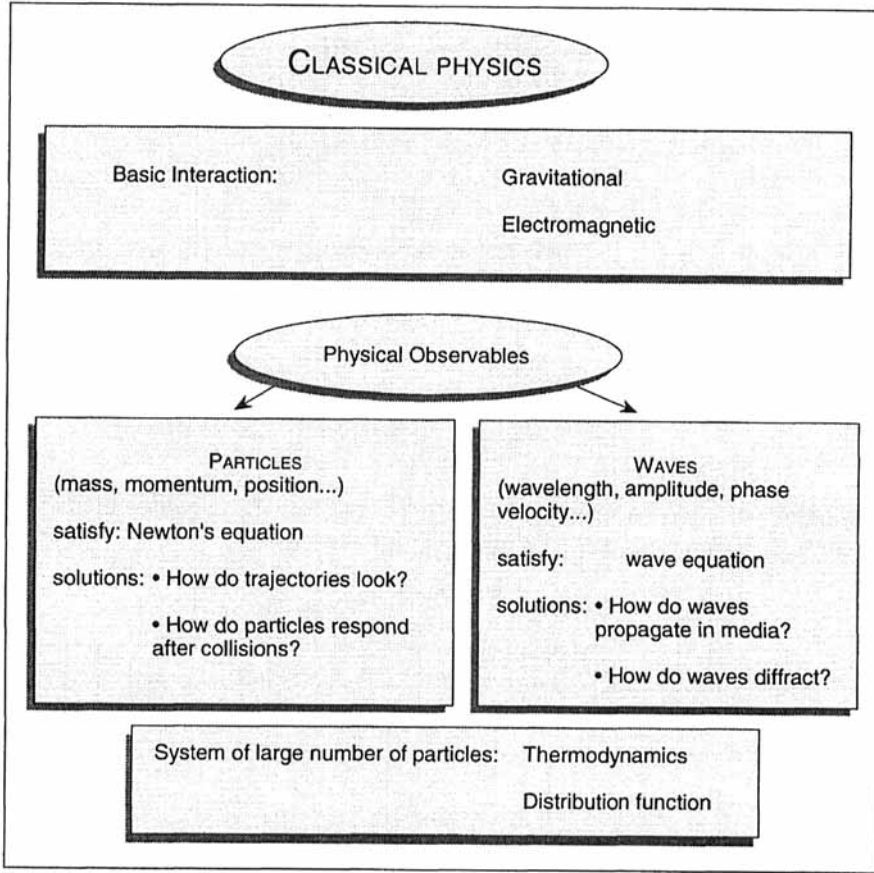


Figure 1.2: An overview of the classical concepts that govern our universe.

r , they have an attractive force given by

$$\mathbf{F} = G \frac{m_1 m_2}{r^2} \hat{r} \quad (1.1)$$

where G is a coefficient with a value $6.67 \times 10^{-11} \text{ N.m}^2/\text{kg}^2$.

The second form of interaction between particles is the electromagnetic interaction. A manifestation of this interaction is that if we have two particles with charges q_1 and q_2 separated by a distance r , there is a force felt by each particle given by (see Fig. 1.3)

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \quad (1.2)$$

where

$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9 \text{ N.m}^2/\text{C}^2$$

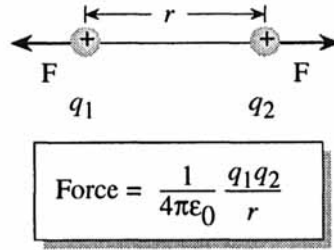


Figure 1.3: Coulombic force between two charged particles.

The potential energy of the system is

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \quad (1.3)$$

If a charged particle is moving in an electric field \mathbf{E} and a magnetic field \mathbf{B} with a velocity \mathbf{v} , it sees a force given by (see Fig. 1.4a)

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \quad (1.4)$$

Another manifestation of the electromagnetic interaction is that a magnetic field is produced by an electric current I . The current is just the motion of charged particles. If, for example, the current is flowing in a circular loop as shown in Fig. 1.4b, the magnetic field at the center is

$$\mathbf{B} = \frac{\mu_0 I}{2r} \hat{n} \quad (1.5)$$

with

$$\mu_0 = 4\pi \times 10^{-7} \text{ N s}^2/\text{C}^2$$

The direction of the field is given by the right-hand rule, i.e., if you hold the wire in the right hand with the thumb pointing along the current direction, the fingers point in the field direction.

A number of interesting interactions arise from the equations described above. These include the attraction or repulsion between current-carrying wires, torque on a current-carrying wire in a magnetic field, etc. All these disparate looking phenomena were placed into a single coherent picture by James Clark Maxwell. The classical electromagnetic phenomena is described completely by the Maxwell equations.

The properties of electromagnetic fields in a medium are described by the four Maxwell equations. Apart from the electric (\mathbf{E}) and magnetic (\mathbf{B}) fields and velocity of light, the effects of the material are represented by the dielectric constant, permeability, electrical conductivity, etc. We start with the four Maxwell equations

$$\begin{aligned} \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} &= 0 \\ \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} &= \mathbf{J} \end{aligned}$$

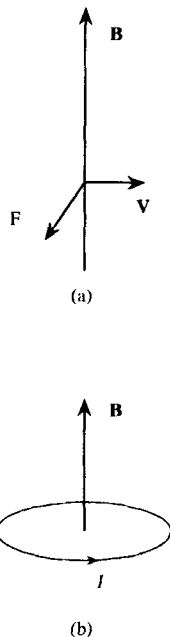


Figure 1.4: (a) Force on a charged particle moving with velocity v in a magnetic field. (b) Magnetic field produced by a current.

$$\begin{aligned}\nabla \cdot \mathbf{D} &= \rho \\ \nabla \cdot \mathbf{B} &= 0\end{aligned}\tag{1.6}$$

where \mathbf{E} and \mathbf{H} are the electric and magnetic fields, $\mathbf{D} = \epsilon \mathbf{E}$, $\mathbf{B} = \mu \mathbf{H}$, and \mathbf{J} and ρ are the current and charge densities. In electromagnetic theory it is often convenient to work with the vector and scalar potentials \mathbf{A} and ϕ , respectively, which are defined through the equations

$$\begin{aligned}\mathbf{E} &= -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi \\ \mathbf{B} &= \nabla \times \mathbf{A}\end{aligned}\tag{1.7}$$

The first and fourth Maxwell equations are automatically satisfied by these definitions. The potentials \mathbf{A} and ϕ are not unique but can be replaced by a new set of potentials \mathbf{A}' and ϕ' given by

$$\begin{aligned}\mathbf{A}' &= \mathbf{A} + \nabla \chi \\ \phi' &= \phi - \frac{\partial \chi}{\partial t}\end{aligned}\tag{1.8}$$

The new choice of potentials does not have any effect on the physical fields \mathbf{E} and \mathbf{B} .

It can be shown that the equations for the vector and scalar potential have

the form (for $\mu = \mu_0$)

$$\begin{aligned}\frac{1}{\mu_0} \nabla^2 \mathbf{A} - \epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} &= \mathbf{J} \\ \nabla^2 \phi - \frac{\partial^2 \phi}{\partial t^2} &= -\frac{\rho}{\epsilon}\end{aligned}\tag{1.9}$$

It is useful to establish the relation between the vector potential \mathbf{A} and the optical power. The time-dependent solution for the vector potential solution of Eqn. 1.9 with $\mathbf{J} = 0$ is

$$\mathbf{A}(\mathbf{r}, t) = \mathbf{A}_0 \{ \exp [i(\mathbf{k} \cdot \mathbf{r} - \omega t)] + \text{c.c.} \} \tag{1.10}$$

with

$$k^2 = \epsilon \mu_0 \omega^2$$

Note that in the MKS units $(\epsilon_0 \mu_0)^{-1/2}$ is the velocity of light c ($3 \times 10^8 \text{ ms}^{-1}$). The electric and magnetic fields are

$$\begin{aligned}\mathbf{E} &= -\frac{\partial \mathbf{A}}{\partial t} \\ &= 2\omega \mathbf{A}_0 \sin(\mathbf{k} \cdot \mathbf{r} - \omega t) \\ \mathbf{B} &= \nabla \times \mathbf{A} \\ &= 2\mathbf{k} \times \mathbf{A}_0 \sin(\mathbf{k} \cdot \mathbf{r} - \omega t)\end{aligned}\tag{1.11}$$

The Poynting vector \mathbf{S} representing the optical power is

$$\begin{aligned}\mathbf{S} &= \mathbf{E} \times \mathbf{H} \\ &= \frac{4}{\mu_0} v k^2 |\mathbf{A}_0|^2 \sin^2(\mathbf{k} \cdot \mathbf{r} - \omega t) \hat{k}\end{aligned}\tag{1.12}$$

where v is the velocity of light in the medium ($= c/\sqrt{\epsilon}$) and \hat{k} is a unit vector in the direction of \mathbf{k} . Here ϵ is the relative dielectric constant. The time-averaged value of the power is

$$\begin{aligned}\langle \mathbf{S} \rangle_{\text{time}} &= \hat{k} \frac{2v k^2 |\mathbf{A}_0|^2}{\mu_0} \\ &= 2v \epsilon \omega^2 |\mathbf{A}_0|^2 \hat{k}\end{aligned}\tag{1.13}$$

since

$$|\mathbf{k}| = \omega/v \tag{1.14}$$

The energy density is then

$$\left| \frac{\mathbf{S}}{v} \right| = 2\epsilon \omega^2 |\mathbf{A}_0|^2 \tag{1.15}$$

The electromagnetic spectrum spans a vast array of wavelengths. In Fig. 1.5 we identify the important regimes of this spectrum.