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INTERFEROMETRY
AND SYNTHESIS
IN RADIO ASTRONOMY
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To
Sheila, Barbara, Janice,
Sarah, Susan, and Michael
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...truste wel that alle the conclusioums that han ben founde, or elles possibly might'en be founde in so noble an instrument as an Astrolabie, ben un-knowe perfitly to any mortal man...

Geoffrey Chaucer

A Treatise on the Astrolabe

circa 1391
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Half a century of remarkable scientific progress has resulted from the application of radio interferometry to astronomy. Advances since 1986, when this book was first published, have resulted in the VLBA (Very Long Baseline Array) which is the first array fully dedicated to very-long-baseline interferometry (VLBI), the globalization of VLBI networks with the inclusion of antennas in orbit, increasing importance of spectral line observations, and improved instrumental performance at both ends of the radio spectrum. At the highest frequencies, millimeter-wavelength arrays of the Berkeley–Illinois–Maryland Association (BIMA), the Institut de Radio Astronomie Millimétrique (IRAM), Nobeyama Radio Observatory (NRO) and Owens Valley Radio Observatory (OVRO), which were in their infancy in 1986, have been greatly expanded in their capabilities. The Submillimeter Array (SMA), and the Atacama Large Millimeter Array (ALMA), which is a major international project at millimeter and submillimeter wavelengths, are under development. At low frequencies, with their special problems involving the ionosphere and wide-field mapping, the frequency coverage of the Very Large Array (VLA) has been extended down to 75 MHz, and the Giant Meter-wave Radio Telescope (GMRT), operating down to 38 MHz, has been commissioned. The Australia Telescope and an expanded Multielement Radio-linked Interferometer Network (MERLIN) have provided increased capability at centimeter wavelengths.

Such progress has led to this revised edition, the intent of which is not only to bring the material up to date but also to expand its scope and improve its comprehensibility and general usefulness. In a few cases symbols used in the first edition have been changed to follow the general usage that is becoming established in radio astronomy. Every chapter contains new material, and there are new figures and many new references. Material in the original Chapter 3 that was peripheral to the basic discussion has been condensed and moved to a later chapter. Chapter 3 now contains the essential analysis of the response of an interferometer. The section on polarization in Chapter 4 has been substantially expanded, and a brief introduction to antenna theory has been added to Chapter 5. Chapter 6 contains a discussion of the sensitivity for a wide variety of instrumental configurations. A discussion of spectral line observations is included in Chapter 10. Chapter 13 has been expanded to include a descrip-
tion of the new techniques for atmospheric phase correction, and site testing data and techniques at millimeter wavelengths. Chapter 14 has been added, and contains an examination of the van Cittert–Zernike theorem and discussions of spatial coherence and scattering, some of which is derived from the original Chapter 3.


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November 2000
PREFACE TO THE FIRST EDITION

The techniques of radio interferometry as applied to astronomy and astrometry have developed enormously in the past four decades, and the attainable angular resolution has advanced from degrees to milliarcseconds, a range of over six orders of magnitude. As arrays for synthesis mapping* have developed, techniques in the radio domain have overtaken those in optics in providing the finest angular detail in astronomical images. The same general developments have introduced new capabilities in astrometry and in the measurement of the earth's polar and crustal motions. The theories and techniques that underlie these advances continue to evolve, but have reached by now a sufficient state of maturity that it is appropriate to offer a detailed exposition.

The book is intended primarily for graduate students and professionals in astronomy, electrical engineering, physics, or related fields who wish to use interferometric or synthesis-mapping techniques in astronomy, astrometry, or geodesy. It is also written with radio systems engineers in mind and includes discussions of important parameters and tolerances for the types of instruments involved. Our aim is to explain the underlying principles of the relevant interferometric techniques but to limit the discussion of details of implementation. Such details of the hardware and the software are largely specific to particular instruments and are subject to change with developments in electronic engineering and computing techniques. With an understanding of the principles involved, the reader should be able to comprehend the instructions and instrumental details that are encountered in the user-oriented literature of most observatories.

The book does not stem from any course of lectures, but the material included is suitable for a graduate-level course. A teacher with experience in the techniques described should be able to interject easily any necessary guidance to emphasize astronomy, engineering, or other aspects as required.

The first two chapters contain a brief review of radio astronomy basics, a short history of the development of radio interferometry, and a basic discussion of the operation of an interferometer. Chapter 3 discusses the underlying relationships of interferometry from the viewpoint of the theory of partial coherence and may

*We define synthesis mapping as the reconstruction of images from measurements of the Fourier transforms of their brightness distributions. In this book the terms map, image, and brightness (intensity) distribution are largely interchangeable.
be omitted from a first reading. Chapter 4 introduces coordinate systems and parameters that are required to describe synthesis mapping. It is appropriate then to examine configurations of antennas for multielement synthesis arrays in Chapter 5. Chapters 6–8 deal with various aspects of the design and response of receiving systems, including the effects of quantization in digital correlators. The special requirements of very-long-baseline interferometry (VLBI) are discussed in Chapter 9. The foregoing material covers in detail the measurement of complex visibility and leads to the derivation of radio maps discussed in Chapters 10 and 11. The former presents the basic Fourier transformation method, and the latter the more powerful algorithms that incorporate both calibration and transformation. Precision observations in astrometry and geodesy are the subject of Chapter 12. There follow discussions of factors that can degrade the overall performance, namely, effects of propagation in the atmosphere, the interplanetary medium and the interstellar medium in Chapter 13, and radio interference in Chapter 14. Propagation effects are discussed at some length since they involve a wide range of complicated phenomena that place fundamental limits on the measurement accuracy. The final chapter describes related techniques including intensity interferometry, speckle interferometry, and lunar occultation observations.

References are included to seminal papers and to many other publications and reviews that are relevant to the topics of the book. Numerous descriptions of instruments and observations are also referenced for purposes of illustration. Details of early procedures are given wherever they are of help in elucidating the principles or origin of current techniques, or because they are of interest in their own right. Because of the diversity of the phenomena described, it has been necessary, in some cases, to use the same mathematical symbol for different quantities. A glossary of principal symbols and usage follows the final chapter.

The material in this book comes only in part from the published literature, and much of it has been accumulated over many years from discussions, seminars, and the unpublished reports and memoranda of various observatories. Thus we acknowledge our debt to colleagues too numerous to mention individually. Our special thanks are due to a number of people for critical reviews of portions of the book, or other support. These include D. C. Backer, D. S. Bagri, R. H. T. Bates, M. Birkinshaw, R. N. Bracewell, B. G. Clark, J. M. Cordes, T. J. Cornwell, L. R. D’Addario, J. L. Davis, R. D. Ekers, J. V. Evans, M. Faucherre, S. J. Franke, J. Granlund, L. J. Greenhill, C. R. Gwinn, T. A. Herring, R. J. Hill, W. A. Jeffrey, K. I. Kellermann, J. A. Klobuchar, R. S. Lawrence, J. M. Marcaide, N. C. Mathur, L. A. Molnar, P. C. Myers, P. J. Napier, P. Nisenson, H. V. Poor, M. J. Reid, J. T. Roberts, L. F. Rodriguez, A. E. E. Rogers, A. H. Rots, J. E. Salah, F. R. Schwab, I. I. Shapiro, R. A. Sramek, R. Stachnik, J. L. Turner, R. F. C. Vessot, N. Wax, and W. J. Welch. The reproduction of diagrams from other publications is acknowledged in the captions, and we thank the authors and the publishers concerned for permission to use this material. For major contributions to the preparation of the manuscript, we wish to thank C. C. Barrett, C. F. Burgess, N. J. Diamond, J. M. Gillberg, J. G. Hamwey, E. L. Haynes, G. L. Kessler, K. I. Maldonis, A. Patrick, V. J. Peterson, S. K. Rosenthal, A. W. Shepherd, J. F. Singarella, M. B. Weems, and C. H. Williams. We are grateful to M. S. Roberts and P. A. Vanden Bout, for-
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1 Introduction and Historical Review

The subject of this book can be broadly described as the principles of radio interferometry applied to the measurement of natural radio signals from cosmic sources. The uses of such measurements lie mainly within the domains of astronomy, astrometry, and geodesy. As an introduction we consider in this chapter the applications of the technique, some basic terms and concepts, and the historical development of the instruments and their uses.

1.1 APPLICATIONS OF RADIO INTERFEROMETRY

Radio interferometers and synthesis arrays, which are basically ensembles of two-element interferometers, are used to make measurements of the fine angular detail in the radio emission from the sky. The angular resolution of single radio antennas is insufficient for many astronomical purposes. Practical considerations limit the resolution to a few tens of arcseconds. For example, the beamwidth of a 100-m-diameter antenna at 7 mm wavelength is approximately 17 arcsec. In the optical range the diffraction limit of large telescopes (diameter ~8 m) is about 0.015 arcsec, but the angular resolution achievable from the ground by conventional techniques is limited to about one arcsec by turbulence in the troposphere. For progress in astronomy it is particularly important to measure the positions of radio sources with sufficient accuracy to allow identification with objects detected in the optical and other parts of the electromagnetic spectrum. It is also very important to be able to measure parameters such as intensity, polarization, and frequency spectrum with similar angular resolution in both the radio and optical domains. Radio interferometry enables such studies to be made.

Precise measurement of the angular positions of stars and other cosmic objects is the concern of astrometry. This includes the study of the small changes in celestial positions attributable to the parallax introduced by the earth's orbital motion, as well as those resulting from the intrinsic motions of the objects. Such measurements are an essential step in the establishment of the distance scale of the universe. Astrometric measurements have also provided a means to test the general theory of relativity and to establish the dynamical parameters of the solar system. In making astrometric measurements it is essential to establish a reference frame for celestial positions. A frame based on extremely distant large-mass
objects as position references is close to ideal. Radio measurements of distant, compact, extragalactic sources presently offer the best prospects for the establishment of such a system. Radio techniques provide an accuracy of the order of $10^{-3}$ arcsec for absolute positions and $10^{-5}$ arcsec or less for the relative positions of objects closely spaced in angle. Optical measurements of stellar images, as seen through the earth's atmosphere, allow the positions to be determined with a precision of about 0.05 arcsec. However, stellar positions have been measured to $\sim 1$ milliarcsecond (mas) with the Hipparcos satellite, and optical measurements with the National Aeronautics and Space Administration (NASA) Space Interferometry Mission hold promise of position measurements to $\sim 4 \mu$arcsec.

As part of the measurement process, astrometric observations include a determination of the orientation of the instrument relative to the celestial reference frame. Ground-based observations therefore provide a measure of the variation of the orientation parameters for the earth. In addition to the well-known precession and nutation of the direction of the axis of rotation, there are irregular shifts of the earth's axis relative to the surface. These shifts, referred to as polar motion, are attributed to the gravitational effects of the sun and moon on the equatorial bulge of the earth, and to dynamic effects in the earth's mantle, crust, oceans, and atmosphere. The same causes give rise to changes in the angular rotation velocity of the earth, which are manifest as corrections that must be applied to the system of universal time. Measurements of the orientation parameters are important in the study of the dynamics of the earth. During the 1970s it became clear that radio techniques could provide an accurate measure of these effects, and in the late 1970s the first radio programs devoted to the monitoring of universal time and polar motion were set up jointly by the U.S. Naval Observatory and the U.S. Naval Research Laboratory, and also by NASA and the National Geodetic Survey. Polar motion can also be studied by observation of satellites, in particular the Global Positioning System, but distant radio sources provide the best standard for measurement of earth rotation.

In addition to revealing angular changes in the motion and orientation of the earth, precise interferometer measurements entail an astronomical determination of the vector spacing between the antennas, which for spacings of $\sim 100$ km or more, is usually more precise than can be obtained by conventional surveying techniques. Very-long-baseline interferometry (VLBI) involves antenna spacings of hundreds or thousands of kilometers, and the uncertainty with which these spacings can be determined has decreased from a few meters in 1967, when VLBI measurements were first made, to a few millimeters. Average relative motions of widely spaced sites on separate tectonic plates lie in the range 1–10 cm per year, and have been tracked extensively with VLBI networks. Interferometric techniques have also been applied to the tracking of vehicles on the lunar surface and the determination of the positions of spacecraft. In this book, however, we limit our concern mainly to measurements of natural signals from astronomical objects. The attainment of the highest angular resolution in the radio domain of the electromagnetic spectrum results in part from the ease with which radio frequency signals can be processed electronically. Also, the phase variations induced by the earth's neutral atmosphere are less severe than at shorter wavelengths. Fu-
BASIC TERMS AND DEFINITIONS

This section is written for readers who are unfamiliar with the basics of radio astronomy. It presents a brief review of some background information that is useful when approaching the subject of radio interferometry.

Cosmic Signals

The voltages induced in antennas by radiation from cosmic sources are generally referred to as signals, although they do not contain information in the usual engineering sense. Such signals are generated by natural processes and almost universally have the form of Gaussian random noise. That is to say, the voltage as a function of time at the terminals of a receiving antenna can be described as a series of very short pulses of random occurrence that combine as a waveform with Gaussian amplitude distribution. In a bandwidth $\Delta v$ the envelope of the radio frequency waveform has the appearance of random variations with duration of order $1/\Delta v$. For most radio sources the characteristics of the signals are invariant with time, at least on the scale of minutes or hours typical of the duration of a radio astronomy observation. Gaussian waveforms of this type are assumed to be identical in character to the noise voltages generated in resistors and amplifiers. Such waveforms are usually assumed to be stationary and ergodic, that is, ensemble averages and time averages converge to equal values.

Most of the power is in the form of continuum radiation, the power spectrum of which shows slow variation with frequency and may be regarded as constant over the receiving bandwidth of most instruments. Figure 1.1 shows continuum spectra of three radio sources. Radio emission from the radio galaxy Cygnus A and from the quasar 3C48 is generated by the synchrotron mechanism [see, e.g., Rybicki and Lightman (1979), Longair (1992)], in which high-energy electrons in magnetic fields radiate as a result of their orbital motion. The radiating electrons are generally highly relativistic, and under these conditions the radiation emitted by each one is concentrated in the direction of its instantaneous motion. An observer therefore sees pulses of radiation from those electrons whose orbital motion lies in, or close to, a plane containing the observer. The observed polarization of the radiation is mainly linear, and any circularly polarized component is generally very small. The overall linear polarization from a source, however, is seldom large, since it is randomized by the variation of the direction of the magnetic field within the source and by Faraday rotation. The power in the electromagnetic pulses from the electrons is concentrated at harmonics of the orbital frequency, and a continuous distribution of electron energies results in a contin-
Figure 1.1 Continuum spectra of three discrete sources: Cygnus A, a radio galaxy; 3C48, a quasar; and NGC7027, an ionized nebula within our Galaxy. Data are from Conway, Kellermann, and Long (1963); Kellermann and Pauliny-Toth (1969); and Thompson (1974). [One jansky (Jy) = $10^{-20}$ W m$^{-2}$ Hz$^{-1}$.]

uum radio spectrum. The individual pulses from the electrons are too numerous to be separable, and the electric field appears as a continuous random process with zero mean. The variation of the spectrum as a function of frequency is related to the slope of the energy distribution of the electrons. In the quasar in Fig. 1.1, which is a very much more compact object than the radio galaxy, the electron density and magnetic fields are high enough to produce self-absorption of the radiation at low frequencies.

NGC7027, the spectrum of which is shown in Fig. 1.1, is a planetary nebula within our Galaxy in which the gas is ionized by radiation from a central star. The radio emission is a thermal process and results from free–free collisions between unbound electrons and ions within the plasma. At the low-frequency end of the spectral curve the nebula is opaque to its own radiation and emits a blackbody spectrum, for which the Rayleigh–Jeans law is a valid approximation. As the frequency increases, the absorptivity, and hence the emissivity, decrease approximately as $v^{-2}$ [see, e.g., Rybicki and Lightman (1979)], where $v$ is the frequency. This behavior counteracts the $v^2$ dependence of the Rayleigh–Jeans law, and thus the spectrum becomes flat when the nebula is no longer opaque to the radiation. Radiation of this type is unpolarized.

In contrast to continuum radiation, spectral line radiation is generated at specific frequencies by atomic and molecular processes. A fundamentally important