RADIATIVE PROCESSES
IN ASTROPHYSICS
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To Verena and Jean
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PREFACE

This book grew out of a course of the same title which each of us taught for several years in the Harvard astronomy department. We felt a need for a book on the subject of radiative processes emphasizing the physics rather than simply giving a collection of formulas.

The range of material within the scope of the title is immense; to cover a reasonable portion of it has required us to go only deeply enough into each area to give the student a feeling for the basic results. It is perhaps inevitable in a broad survey such as this that inadequate coverage is given to certain subjects. In these cases the references at the end of each chapter can be consulted for further information.

The material contained in the book is about right for a one-term course for seniors or first-year graduate students of astronomy, astrophysics, and related physics courses. It may also serve as a reference for workers in the field. The book is designed for those with a reasonably good physics background, including introductory quantum mechanics, intermediate electromagnetic theory, special relativity, and some statistical mechanics. To make the book more self-contained we have included brief reviews of most of the prerequisite material. For readers whose preparation is less than ideal this gives an opportunity to bolster their background by studying the material again in the context of a definite physical application.
A very important and integral part of the book is the set of problems at the end of each chapter and their solutions at the end of the book. Besides their usual role in affording self-tests of understanding, the problems and solutions present important results that are used in the main text and also contain most of the astrophysical applications.

We owe a debt of gratitude to our teaching assistants over the years, Robert Moore, Robert Leach, and Wayne Roberge, and to students whose penetrating questions helped shape this book. We thank Ethan Vishniac for his help in preparing the index. We also want to thank Joan Verity for her excellence and flexibility in typing the manuscript.

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Cambridge, Massachusetts
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1

FUNDAMENTALS OF RADIATIVE TRANSFER

1.1 THE ELECTROMAGNETIC SPECTRUM; ELEMENTARY PROPERTIES OF RADIATION

Electromagnetic radiation can be decomposed into a spectrum of constituent components by a prism, grating, or other devices, as was discovered quite early (Newton, 1672, with visible light). The spectrum corresponds to waves of various wavelengths and frequencies, related by $\lambda \nu = c$, where $\nu$ is the frequency of the wave, $\lambda$ is its wavelength, and $c = 3.00 \times 10^{10}$ cm s$^{-1}$ is the free space velocity of light. (For waves not traveling in a vacuum, $c$ is replaced by the appropriate velocity of the wave in the medium.) We can divide the spectrum up into various regions, as is done in Figure 1.1. For convenience we have given the energy $E = h\nu$ and temperature $T = E/k$ associated with each wavelength. Here $h$ is Planck's constant $= 6.625 \times 10^{-27}$ erg s, and $k$ is Boltzmann's constant $= 1.38 \times 10^{-16}$ erg K$^{-1}$. This chart will prove to be quite useful in converting units or in getting a quick view of the relevant magnitude of quantities in a given portion of the spectrum. The boundaries between different regions are somewhat arbitrary, but conform to accepted usage.
1.2 RADIATIVE FLUX

Macroscopic Description of the Propagation of Radiation

When the scale of a system greatly exceeds the wavelength of radiation (e.g., light shining through a keyhole), we can consider radiation to travel in straight lines (called rays) in free space or homogeneous media—from this fact a substantial theory (transfer theory) can be erected. The detailed justification of this assumption is considered at the end of Chapter 2. One of the most primitive concepts is that of energy flux: consider an element of area $dA$ exposed to radiation for a time $dt$. The amount of energy passing through the element should be proportional to $dA \cdot z$, and we write it as $FdA \cdot dt$. The energy flux $F$ is usually measured in erg $s^{-1} \cdot cm^{-2}$. Note that $F$ can depend on the orientation of the element.

Flux from an Isotropic Source—the Inverse Square Law

A source of radiation is called isotropic if it emits energy equally in all directions. An example would be a spherically symmetric, isolated star. If we put imaginary spherical surfaces $S_1$ and $S$ at radii $r_1$ and $r$, respectively, about the source, we know by conservation of energy that the total energy passing through $S_1$ must be the same as that passing through $S$. (We assume no energy losses or gains between $S_1$ and $S$.) Thus

$$F(r_1) \cdot 4\pi r_1^2 = F(r) \cdot 4\pi r^2,$$
or

\[ F(r) = \frac{F(r_i) r_i^2}{r^2}. \]

If we regard the sphere \( S_1 \) as fixed, then

\[ F = \text{constant} \frac{\text{F}}{r^2}. \]  \hspace{1cm} (1.1)

This is merely a statement of conservation of energy.

1.3 THE SPECIFIC INTENSITY AND ITS MOMENTS

Definition of Specific Intensity or Brightness

The flux is a measure of the energy carried by all rays passing through a given area. A considerably more detailed description of radiation is to give the energy carried along by individual rays. The first point to realize, however, is that a single ray carries essentially no energy, so that we need to consider the energy carried by sets of rays, which differ infinitesimally from the given ray. The appropriate definition is the following: Construct an area \( dA \) normal to the direction of the given ray and consider all rays passing through \( dA \) whose direction is within a solid angle \( d\Omega \) of the given ray (see Fig. 1.2). The energy crossing \( dA \) in time \( dt \) and in frequency range \( dv \) is then defined by the relation

\[ dE = I_s \, dA \, dt \, d\Omega \, dv, \]  \hspace{1cm} (1.2)

where \( I_s \) is the specific intensity or brightness. The specific intensity has the

\[ \text{Figure 1.2 Geometry for normally incident rays.} \]
dimensions

\[ I_\nu(\nu, \Omega) = \text{energy (time)}^{-1} \text{(area)}^{-1} \text{(solid angle)}^{-1} \text{(frequency)}^{-1} \]
\[ = \text{ergs s}^{-1} \text{cm}^{-2} \text{ster}^{-1} \text{Hz}^{-1}. \]

Note that \( I_\nu \) depends on location in space, on direction, and on frequency.

**Net Flux and Momentum Flux**

Suppose now that we have a radiation field (rays in all directions) and construct a small element of area \( dA \) at some arbitrary orientation \( \mathbf{n} \) (see Fig. 1.3). Then the differential amount of flux from the solid angle \( d\Omega \) is (reduced by the lowered effective area \( \cos \theta dA \))

\[ dF_\nu(\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}) = I_\nu \cos \theta d\Omega. \quad (1.3a) \]

The net flux in the direction \( \mathbf{n}, F_\nu(\mathbf{n}) \) is obtained by integrating \( dF \) over all solid angles:

\[ F_\nu = \int I_\nu \cos \theta d\Omega. \quad (1.3b) \]

Note that if \( I_\nu \) is an isotropic radiation field (not a function of angle), then the net flux is zero, since \( \int \cos \theta d\Omega = 0 \). That is, there is just as much energy crossing \( dA \) in the \( \mathbf{n} \) direction as the \( -\mathbf{n} \) direction.

To get the flux of momentum normal to \( dA \) (momentum per unit time per unit area = pressure), remember that the momentum of a photon is \( E/c \). Then the momentum flux along the ray at angle \( \theta \) is \( dF_\nu/c \). To get

![Figure 1.3 Geometry for obliquely incident rays.](image-url)
the component of momentum flux normal to \( dA \), we multiply by another factor of \( \cos \theta \). Integrating, we then obtain

\[
p_r (\text{dynes cm}^{-2} \text{ Hz}^{-1}) = \frac{1}{c} \int I_r \cos^2 \theta \, d\Omega.
\] (1.4)

Note that \( F_r \) and \( p_r \) are moments (multiplications by powers of \( \cos \theta \) and integration over \( d\Omega \)) of the intensity \( I_r \). Of course, we can always integrate over frequency to obtain the total (integrated) flux and the like.

\[
F (\text{erg s}^{-1} \text{ cm}^{-2}) = \int F_r \, dv
\] (1.5a)

\[
p (\text{dynes cm}^{-2}) = \int p_r \, dv
\] (1.5b)

\[
I (\text{erg s}^{-1} \text{ cm}^{-2} \text{ ster}^{-1}) = \int I_r \, dv
\] (1.5c)

**Radiative Energy Density**

The specific energy density \( u_r \) is defined as the energy per unit volume per unit frequency range. To determine this it is convenient to consider first the energy density per unit solid angle \( u_r(\Omega) \) by \( dE = u_r(\Omega) \, dV \, d\Omega \, dv \) where \( dV \) is a volume element. Consider a cylinder about a ray of length \( ct \) (Fig. 1.4). Since the volume of the cylinder is \( dAc \),

\[
dE = u_r(\Omega) \, dAc \, dt \, d\Omega \, dv.
\]

Radiation travels at velocity \( c \), so that in time \( dt \) all the radiation in the cylinder will pass out of it:

\[
dE = I_r \, dA \, d\Omega \, dt \, dv.
\]

*Figure 1.4 Electromagnetic energy in a cylinder.*
Equating the above two expressions yields

\[ u_\nu(\Omega) = \frac{I_\nu}{c}. \]  

(1.6)

Integrating over all solid angles we have

\[ u_\nu = \int u_\nu(\Omega) d\Omega = \frac{1}{c} \int I_\nu d\Omega, \]

or

\[ u_\nu = \frac{4\pi}{c} J_\nu, \]  

(1.7)

where we have defined the mean intensity \( J_\nu: \)

\[ J_\nu = \frac{1}{4\pi} \int I_\nu d\Omega. \]  

(1.8)

The total radiation density (erg cm\(^{-3}\)) is simply obtained by integrating \( u_\nu \) over all frequencies

\[ u = \int u_\nu dv = \frac{4\pi}{c} \int J_\nu dv. \]  

(1.9)

**Radiation Pressure in an Enclosure Containing an Isotropic Radiation Field**

Consider a reflecting enclosure containing an isotropic radiation field. Each photon transfers twice its normal component of momentum on reflection. Thus we have the relation

\[ p_\nu = \frac{2}{c} \int I_\nu \cos^2 \theta d\Omega. \]

This agrees with our previous formula, Eq. (1.4), since here we integrate only over \( 2\pi \) steradians. Now, by isotropy, \( I_\nu = J_\nu \) so

\[ p = \frac{2}{c} \int J_\nu dv \int \cos^2 \theta d\Omega. \]

The angular integration yields

\[ p = \frac{1}{3} u. \]  

(1.10)
The radiation pressure of an isotropic radiation field is one-third the energy density. This result will be useful in discussing the thermodynamics of blackbody radiation.

**Constancy of Specific Intensity Along Rays in Free Space**

Consider any ray $L$ and any two points along the ray. Construct areas $dA_1$ and $dA_2$ normal to the ray at these points. We now make use of the fact that energy is conserved. Consider the energy carried by that set of rays passing through both $dA_1$ and $dA_2$ (see Fig. 1.5). This can be expressed in two ways:

$$dE_1 = I_r, dA_1 dt d\Omega_1 dv_1 = dE_2 = I_r, dA_2 dt d\Omega_2 dv_2.$$ 

Here $d\Omega_1$ is the solid angle subtended by $dA_2$ at $dA_1$ and so forth. Since $d\Omega_1 = dA_2 / R^2$, $d\Omega_2 = dA_1 / R^2$ and $dv_1 = dv_2$, we have

$$I_{r_1} = I_{r_2}.$$ 

Thus the intensity is constant along a ray:

$$I_r = \text{constant.} \quad (1.11)$$

Another way of stating the above result is by the differential relation

$$\frac{dI_r}{ds} = 0, \quad (1.12)$$

where $ds$ is a differential element of length along the ray.

**Proof of the Inverse Square Law for a Uniformly Bright Sphere**

To show that there is no conflict between the constancy of specific intensity and the inverse square law, let us calculate the flux at an arbitrary

![Figure 1.5 Constancy of intensity along rays.](image-url)
distance from a sphere of uniform brightness $B$ (that is, all rays leaving the sphere have the same brightness). Such a sphere is clearly an isotropic source. At $P$, the specific intensity is $B$ if the ray intersects the sphere and zero otherwise (see Fig. 1.6). Then,

$$ F = \int I \cos \theta \, d\Omega = B \int_0^{\pi} d\phi \int_0^{\theta_c} \sin \theta \cos \theta \, d\theta, $$

where $\theta_c = \sin^{-1} R/r$ is the angle at which a ray from $P$ is tangent to the sphere. It follows that

$$ F = \pi B (1 - \cos^2 \theta_c) = \pi B \sin^2 \theta_c $$

or

$$ F = \pi B \left( \frac{R}{r} \right)^2. \quad (1.13) $$

Thus the specific intensity is constant, but the solid angle subtended by the given object decreases in such a way that the inverse square law is recovered.

A useful result is obtained by setting $r = R$:

$$ F = \pi B. \quad (1.14) $$

That is, the flux at a surface of uniform brightness $B$ is simply $\pi B$.

### 1.4 RADIATIVE TRANSFER

If a ray passes through matter, energy may be added or subtracted from it by emission or absorption, and the specific intensity will not in general remain constant. "Scattering" of photons into and out of the beam can also affect the intensity, and is treated later in §1.7 and 1.8.
Emission

The spontaneous emission coefficient $j$ is defined as the energy emitted per unit time per unit solid angle and per unit volume:

$$dE = j dV d\Omega dt.$$ 

A monochromatic emission coefficient can be similarly defined so that

$$dE = j_\nu dV d\Omega dt dv,$$

where $j_\nu$ has units of erg cm$^{-3}$ s$^{-1}$ ster$^{-1}$ Hz$^{-1}$.

In general, the emission coefficient depends on the direction into which emission takes place. For an isotropic emitter, or for a distribution of randomly oriented emitters, we can write

$$j_\nu = \frac{1}{4\pi} P_\nu,$$

where $P_\nu$ is the radiated power per unit volume per unit frequency. Sometimes the spontaneous emission is defined by the (angle integrated) emissivity $\epsilon_\nu$, defined as the energy emitted spontaneously per unit frequency per unit time per unit mass, with units of erg gm$^{-1}$ s$^{-1}$ Hz$^{-1}$. If the emission is isotropic, then

$$dE = \epsilon_\nu \rho dV dt dv \frac{d\Omega}{4\pi},$$

where $\rho$ is the mass density of the emitting medium and the last factor takes into account the fraction of energy radiated into $d\Omega$. Comparing the above two expressions for $dE$, we have the relation between $\epsilon_\nu$ and $j_\nu$:

$$j_\nu = \frac{\epsilon_\nu \rho}{4\pi},$$

holding for isotropic emission. In going a distance $ds$, a beam of cross section $dA$ travels through a volume $dV = dA ds$. Thus the intensity added to the beam by spontaneous emission is:

$$dl_\nu = j_\nu ds.$$

Absorption

We define the absorption coefficient, $\alpha_\nu$ (cm$^{-1}$) by the following equation, representing the loss of intensity in a beam as it travels a distance $ds$ (by
Fundamentals of Radiative Transfer

convention, \( \alpha_r \), positive for energy taken out of beam):

\[ dI_r = - \alpha_r I_r ds. \quad (1.20) \]

This phenomenological law can be understood in terms of a microscopic model in which particles with density \( n \) (number per unit volume) each present an effective absorbing area, or cross section, of magnitude \( \sigma_r (\text{cm}^2) \). These absorbers are assumed to be distributed at random. Let us consider the effect of these absorbers on radiation through \( dA \) within solid angle \( d\Omega \) (see Fig. 1.7). The number of absorbers in the element equals \( n dA ds \). The total absorbing area presented by absorbers equals \( n\sigma_r dA ds \). The energy absorbed out of the beam is

\[ - dI_r, dA d\Omega dt dv = I_r (n\sigma_r dA ds) d\Omega dt dv; \]

thus

\[ dI_r = - n\sigma_r I_r ds, \]

which is precisely the above phenomenological law (1.20), where

\[ \alpha_r = n\sigma_r. \quad (1.21) \]

Often \( \alpha_r \) is written as

\[ \alpha_r = \rho \kappa_r, \quad (1.22) \]

where \( \rho \) is the mass density and \( \kappa_r (\text{cm}^2 \text{g}^{-1}) \) is called the mass absorption coefficient; \( \kappa_r \) is also sometimes called the opacity coefficient.

\[ \text{Figure 1.7a Ray passing through a medium of absorbers.} \]

\[ \text{Figure 1.7b Cross sectional view of 7a.} \]
There are some conditions of validity for this microscopic picture: The most important are that (1) the linear scale of the cross section must be small in comparison to the mean interparticle distance $d$. Thus $a_{\nu}^{1/2} \ll d \sim n^{-1/3}$, from which follows $a_{\nu}d \ll 1$ and (2) the absorbers are independent and randomly distributed. Fortunately, these conditions are almost always met for astrophysical problems.

As is shown in §1.6, we consider "absorption" to include both "true absorption" and stimulated emission, because both are proportional to the intensity of the incoming beam (unlike spontaneous emission). Thus the net absorption may be positive or negative, depending on whether "true absorption" or stimulated emission dominates. Although this combination may seem artificial, it will prove convenient and obviate the need for a quantum mechanical addition to our classical formulas later on.

**The Radiative Transfer Equation**

We can now incorporate the effects of emission and absorption into a single equation giving the variation of specific intensity along a ray. From the above expressions for emission and absorption, we have the combined expression

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu.$$  

(1.23)

The transfer equation provides a useful formalism within which to solve for the intensity in an emitting and absorbing medium. It incorporates most of the macroscopic aspects of radiation into one equation, relating them to two coefficients $\alpha_\nu$ and $j_\nu$. A primary task in later chapters of this book is to find forms for these coefficients corresponding to particular physical processes.

Once $\alpha_\nu$ and $j_\nu$ are known it is relatively easy to solve the transfer equation for the specific intensity. When scattering is present, solution of the radiative transfer equation is more difficult, because emission into $d\Omega$ depends on $I_\nu$ in solid angles $d\Omega'$, integrated over the latter (scattering from $d\Omega'$ into $d\Omega$). The transfer equation then becomes an integrodifferential equation, which generally must be solved partly by numerical techniques. (See §1.7 and 1.8.)

A formal solution to the complete radiative transfer equation will be given shortly. Here, we can give solutions to two simple limiting cases:

1—**Emission Only**: $\alpha_\nu = 0$. In this case, we have

$$\frac{dI_\nu}{ds} = j_\nu.$$