Materials with Complex Behaviour II

Properties, Non-Classical Materials and New Technologies
Common engineering materials reach in many demanding applications such as automotive or aerospace their limits and new developments are required to fulfill increasing demands on performance and characteristics. The properties of materials can be increased for example by combining different materials to achieve better properties than a single constituent or by shaping the material or constituents in a specific structure. Many of these new materials reveal a much more complex behavior than traditional engineering materials due to their advanced structure or composition. Furthermore, the classical applications of many engineering materials are extended to new ranges of applications and to more demanding environmental conditions such as elevated temperatures. All these tendencies require in addition to the synthesis of new materials, proper methods for their manufacturing and extensive programs for their characterization. In many fields of application, the development of new methods and processes must be accomplished by accurate and reliable modeling and simulation techniques. Only the interaction between these new developments with regard to manufacturing, modeling, characterization, further processing and monitoring of materials will allow to meet all demands and to introduce these developments in safety-relevant applications.

The 4th International Conference on Advanced Computational Engineering and Experimenting, ACE-X 2010, was held in Paris, France, from 05 to 07 July 2010 with a strong focus on the above-mentioned developments. This conference served as an excellent platform for the engineering community to meet with each other and to exchange the latest ideas. This volume contains 45 revised and extended research articles written by experienced researchers participating in the conference. The book will offer the state-of-the-art of tremendous advances in engineering technologies of materials with complex behavior and also serve as an excellent reference volume for researchers and graduate students working with advanced materials. The covered topics are related to Materials and Properties, Non-classical Materials and Structures and New Technologies.
The organizers and editors wish to thank all the authors for their participation and cooperation which made this volume possible. Finally, we would like to thank the team of Springer-Verlag, especially Dr. Christoph Baumann, for the excellent cooperation during the preparation of this volume.

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Part I
Materials and Properties
A Finite Element Simulation of Longitudinal Impact Waves in Elastic Rods

Hesham A. Elkaranshawy and Nasser S. Bajaba

Abstract In this chapter, wave propagation in a thin rod struck by a rigid mass is considered and a finite element simulation of the system is developed. Both cases of free–free and fixed-free rods are considered. Though impact generates a propagating stress wave in both cases, the free–free rod is going to have a rigid-body motion. The analytical equations of motion are presented and the corresponding finite element equations are derived. A numerical scheme is constructed and solutions are obtained using Newmark implicit integration method and Newton–Raphson iterative technique. Solutions include time histories of displacement, velocity, stress, and contact force. The contact force is calculated, according to St. Venant’s impact model. Numerical results of the simulation are compared to traditional analytical results. A simulated visualization of the propagation of the stress wave in the rod is presented, which enhances the understanding of this complicated physical phenomenon. The achieved results are accurate enough to have confidence in using this model for practical applications in wave propagation simulation and analysis.

Keywords Longitudinal impact • Stress and strain analysis • Wave propagation • Finite element simulation
Abbreviations

$A$ Cross-sectional area (m$^2$)
$c$ Wave propagation velocity (m/s)
$E$ Young’s modulus (N/m$^2$)
$F$ Contact force (N)
$\{f\}$ Global force vector (N)
$\{f\}_e$ Force vector for element in contact with the rigid mass (N)
$[K]$ Global stiffness matrix (N/m)
$l$ Length (m)
$L$ Lagrangean (J)
$m$ Mass of the rod (kg)
$m_0$ Mass of the rigid mass (kg)
$[M]$ Global mass matrix (kg)
$[N]$ Finite element shape functions (m/m)
$q$ Displacement of the rigid mass (m)
$\dot{q}$ Velocity of the rigid mass (m/s)
$t$ Time (s)
$\tau_c$ Contact period (s)
$T$ Kinetic energy (J)
$U$ Displacement of the rod at position $x$(m)
$\{U\}$ Nodal displacement vector (m)
$\{\dot{U}\}$ Velocity vector (m/s)
$\{\ddot{U}\}$ Acceleration vector (m/s$^2$)
$\{U\}_e$ Nodal displacement vector element in contact with the rigid mass (m)
$U_s$ Strain energy (J)
$v_0$ Initial velocity of the rigid mass (m/s)
$W$ Work done (J)
$x$ Position in the rod (m)
$\varepsilon$ Strain in the rod (m/m)
$H$ Potential energy (J)
$\rho$ Density (kg/m$^3$)
$\sigma$ Stress in the rod (N/m$^2$)
$\tau$ Time for the wave to travel across the rod from one end to the other end (s)
$()_N$ Value at the previous time step ($N = 0,1,2,...$)
$()_{N+1}$ Value at the current time step ($N = 0,1,2,...$)

1 Introduction

Investigation of wave propagation in a rod due to impact has a long history. Bernolli, Navier, Poisson, and St. Venant are among the great researchers who investigated this problem. Good reviews of the treatments of longitudinal waves in
rods produced by impact are offered in [1, 2]. Recently, due to the presence of powerful computers, new computational methods are applied to solve this classical problem. An analytical simulation, symbolic solution, and a solution using time delay method have been developed [3–5]. Both theoretical and experimental researches were conducted [6–8] and a review of the experimental studies is offered in [9].

One practical device which utilizes longitudinal wave propagation in rods is the Hopkinson (or Davies) bar. The device is used to calibrate shock accelerometers under high acceleration levels and a wide frequency bandwidth. The bar is a long, thin, and elastic rod, in which a stress wave is generated at one end by a projectile impact. The projectile is a rigid mass or a striker bar. At the other end of the bar the generated wave can be used in many applications. The propagation of the shock wave in a Hopkinson bar is modeled [10, 11]. Insertion of a deformable disk between the projectile and the bar can decrease the wave dispersion, hence, a commercial finite element code was utilized to investigate dispersion in the bar and to find the optimum characteristics of the inserted deformable disk [12].

Wave propagation can be used in the determination of mechanical properties of materials. Some dynamic strength material constants were obtained using the split Hopkinson pressure bar [13]. The split tensile Hopkinson bar tests are interrupted to evaluate the damage in the materials at high strain rate [14]. The evaluation of the coefficient of restitution, through numerical simulation of impact of a rigid mass and a slender elastic rod, was investigated [15, 16]. Furthermore, there is an increasing interest in using wave propagation in crack detection, for example, wave propagations in cracked beams and plates were examined [17, 18].

Some machine elements are rod-like bodies that are subjected to impact loading during their functional operations. Examples are encountered in piling, percussive drilling and hydraulic hammering. Due to the elasticity of these axial elements, waves propagate through them while they are in translational motion. At the same time, it is obvious that wave propagation is gaining more potential in non-destructive testing methods. Therefore, reliable finite element models are needed to be used in the simulation of the propagation of waves. In this chapter, a finite element model is constructed to represent impact of a rigid mass on a flexible rod. The model overcomes the limitations in the previously reviewed works where the impact forces were assumed, see [17, 18], or calculated using methods that are highly time consuming, see [15, 16]. Hence, the contact force is calculated using an efficient approach utilizing the St.Venant’s classical impact model. The two cases of free–free and free-fixed elastic rods are investigated. A numerical scheme is formulated depending upon Newmark implicit time stepping method and Newton–Raphson iterative method. The contact force is calculated and the wave propagation in the rod is simulated. To enhance the understanding of the complicated physical phenomenon, a simulated visualization of the propagation of the impact wave through the bar is monitored.
2 Mathematical Modeling

It is assumed that the rod has mass $m$, Young’s modulus $E$, density $\rho$, cross-sectional area $A$ and length $l$. The rod is initially at rest and is struck on the right end $x = l$ at the initial time $t = 0$ by a moving rigid mass $m_0$ with initial velocity $v_0$. The displacement of the rigid mass at time $t$ is donated by $q(t)$ and the displacement of the rod at position $x$ and time $t$ is given by $u(x,t)$, as in Fig. 1.

The governing equation for the longitudinal wave in the rod is

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}$$

(1)

where $c$ is the wave propagation velocity

$$c = \sqrt{\frac{E}{\rho}}$$

(2)

The strain $\varepsilon(x,t)$ in the rod is given by

$$\varepsilon(x,t) = \frac{\partial u(x,t)}{\partial x}$$

(3)

For an elastic rod, the stress is proportional to strain, or

$$\sigma(x,t) = E \frac{\partial u(x,t)}{\partial x}$$

(4)

As contact is established between the mass and the rod, both the mass and the contact end of the rod ($x = l$) are assumed to have the same velocity $v_0$. Therefore, a compression wave is created in the rod. The wave travels along the rod and is reflected at the other end ($x = 0$). During the contact period, displacement $q(t)$ and velocity $\dot{q}(t)$ of the mass are the same as those of the contact end of the rod ($x = l$).

$$q(t) = u(l,t) \quad \text{and} \quad \dot{q}(t) = \frac{\partial u}{\partial t}(l,t), \quad 0 < t < t_c$$

(5)

where $t_c$ is the contact period.

The contact persists as long as the contact force between the mass and contact end of the rod does not vanish. The contact force equals the stress at the contact end times the rod’s cross-sectional area, i.e.

$$F(t) = EA \frac{\partial u(l,t)}{\partial x}$$

(6)

The motion of the rigid mass is governed by

$$m \frac{d\dot{q}}{dt} = F(t)$$

(7)
Equations (1), (6), and (7) are the equations of motion of the rod and the rigid mass during the impact period. After the cease of impact the motion of the rod is controlled by Eq. (1). In the same time, since $F(t)$ vanishes, Eq. (7) declares that the rigid mass moves with a constant velocity.

3 Finite Element Solutions

The pre-mentioned differential formulation of the equations of motion is equivalent to integral formulation, which requires the application of Lagrange’s equation of motion. First, one defines Lagrangean ‘$L$’ by

$$L = T - \pi$$

where ‘$T$’ is the kinetic energy and ‘$\pi$’ is the potential energy defined by

$$\pi = U_s - W$$

$U_s$ and $W$ are the strain energy and the work done, respectively, that are given by

$$U_s = \sum_e \frac{1}{2} \int_e EA \left( \frac{\partial u}{\partial x} \right)^2 \, dx$$

$$T = \sum_e \frac{1}{2} \int_e \rho A \left( \frac{\partial u}{\partial t} \right)^2 \, dx + \frac{1}{2} m_0 \left[ \frac{\partial u}{\partial t} (x = l) \right]^2$$

$$W = F(t)u(l, t)$$

The finite element shape functions $[N(x)]$ link the displacement ‘$u$’ to the nodal displacement vector $\{U\}$ through

$$u(x, t) = [N]\{U\}$$
Consequently,

\[
L = \frac{1}{2} \{\dot{U}\}^T [M]\{\dot{U}\} - \frac{1}{2} \{U\}^T [K]\{U\} + \{f(t)\}^T \{U\}
\]  

(14)

\([M]\) and \([K]\) are the global mass and stiffness matrices and \(\{f(t)\}\) is the global force vector. \(\{f(t)\}\) contains only the nodal forces of the last element, \(\{f(t)\}_{el}\), due to the contact force. The rest of the global force vector is full of zeros. \(\{f(t)\}_{el}\) is given by

\[
\{f(t)\}_{el} = F(t)[N(x = l)]^T
\]  

(15)

The Lagrange’s equation of motion is given by

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \{\dot{U}\}} \right) - \frac{\partial L}{\partial \{U\}} = 0
\]  

(16)

this leads to

\[
[M]\{\ddot{U}\} + [K]\{U\} = \{f(t)\}
\]  

(17)

Equations (6) and (13) give

\[
F(t) = EA \left[ \frac{\partial N}{\partial x} (x = l) \right] \{U\}_{el}
\]  

(18)
where \( \{U\}_{el} \) is the nodal displacement vector for the last element, which is the element in contact with the rigid mass.

Equations (17) and (18) are the finite element equations of motion during impact. These equations are applied for both cases of free-free rod and free-fixed rod, see Fig. 2. In the case of free-fixed bar both \([M]\) and \([K]\) are positive definite matrices. For the free-free bar, though \([M]\) is positive definite matrix, \([K]\) is positive semi-definite matrix due to the existence of rigid body modes.

Newmark implicit time stepping method (Bathe [19]) is used to express the current velocity \(\{\dot{U}\}_{N+1} \) and acceleration \(\{\ddot{U}\}_{N+1} \) in terms of the current displacement \(\{U\}_{N+1} \) and previously determined values of displacement \(\{U\}_N \), velocity \(\{\dot{U}\}_N \), and acceleration \(\{\ddot{U}\}_N \). Combining these equations with the equations of motion (17) and (18) yields a system of algebraic equations in terms of \(\{U\}_{N+1} \) and \(F(t)_{N+1} \). The Newton–Raphson iterative method (Bathe [19]) is used to solve the resulting equations to find the current displacement and contact force. The displacement and other variables' distributions in the rod at the end of impact serve as the initial conditions for the subsequent free vibrations of the bar, which are governed by the solution of Eq. (17) while Eq. (18) is no longer relevant.

4 Numerical Simulation

Numerical simulations, for a rigid mass collides with a free-free elastic rod and with a free-fixed elastic rod, are presented in this section, see Fig. 2. The rod in both cases is an aluminum rod with a 3 mm \( \times \) 25 mm cross section, 200 mm length, 70 GN/m\(^2\) Young’s modulus, and 2,710 kg/m\(^3\) mass density. The rigid mass has the same mass as the rod. The mass is moving towards the rod with a velocity of 1 m/s. Fifty elements are used to model the rod in the finite element model. The elements are two-nodes and one-dimensional linear elastic elements. The velocity of the created wave is \(c = \frac{v_0}{\sqrt{\rho E}} = 5082.35 \text{ m/s} \) and the time for the wave to travel across the rod from one end to the other end is \(\tau = \frac{l}{c} = 3.935 \times 10^{-5} \text{ s} \).

According to St. Venant’s principle, as contact starts the velocity of the contact end becomes immediately equals to the rigid mass velocity and right away a compression wave is created at the contact end and travels across the rod with velocity ‘c’. The initial compression stress at the contact end is \( \sigma_0 = v_0 \sqrt{\rho E} \) and the stress at that end starts to decrease with time until the reflected wave reaches that end.

For the bar with the other end free, the stress at the free end is always zero, therefore, the traveling compression stress wave is reflected at the free end as a tension wave and whenever that tension wave reaches the contact end at time \(\tau\), it cancels out the stress at that end and contact is terminated. Following the cease of contact, the wave is reflected from the contact end as compression wave and periodic cycles start with a period equals to \(\tau\).
The finite element solutions successfully predict this phenomenon as can be seen in Figs. 3, 4, 5, 6, 7.

Figures 4, 5, 6, 7 show the dimensionless displacement $\frac{c}{v_0}u$, velocity $\frac{v}{v_0}$, stress $\frac{E}{\mu_0} \sigma$, and contact force $\frac{1}{v_0A\sqrt{\rho}} F$, respectively, with respect to dimensionless time $\frac{c}{l}t$. Slight numerical damping is introduced to reduce the oscillations in solutions. Dimensionless analytical solutions are given in [4]. Very good agreement is found between the solutions of the proposed finite element model and the analytical solutions [1, 2]. Figure 3 shows the distribution of the dimensionless stress over the dimensionless length $\frac{x}{l}$ at equal dimensionless time steps of 0.125. Therefore, the wave propagation can be visualized in that figure. Figures 3 and 7 show that the arrival of the reflected tension wave into the contact end nullifies the contact

**Fig. 3** Stress wave propagation in the free–free bar; waves of dimensionless stress $\frac{E}{\mu_0} \sigma$ versus dimensionless length $\frac{x}{l}$ are shown at times 0.125 $\frac{c}{l}t$ apart
force. Therefore, it marks the end of impact. Most of the time, a portion of the rod is in tension while the other portion is in compression, as can be seen in Fig. 3. Therefore, the mid-point stress alternate between compression and tension marked by the arrival of the wave at that point, see Figs. 3 and 6. Though the displacements of the bar ends are continuous, see Fig. 4, the slope of each displacement history suffers discontinuity corresponding to the arrival of the wave at that end,
which is reflected in the discontinuity of the velocities, Fig. 5. The time history of velocity in Fig. 5 indicates that after the end of impact, the striking mass does not change its original moving direction and the bar starts a continued free vibration. The bar has an average rigid body motion velocity and for each end, the velocity is varying between two limits. The arrival of the wave at each bar end increases the velocity of that end impulsively to its maximum value. The analytical solutions given by Goldsmith [1] predict that the final dimensionless velocity of the rigid
mass to be 0.1353 and the present simulation predicts 0.1469, see Fig. 5. At the arrival of the reflected wave to the contact end, the analytical dimensionless stress is 0.1353 and in Fig. 6 the finite element calculates 0.1325.

Fig. 8 Stress wave propagation in the fixed-free bar; waves of dimensionless stress $\frac{c}{v_0} \sigma$ versus dimensionless length $\frac{x}{l}$ are shown at times $0.125 \frac{c}{l} t$ apart

mass to be 0.1353 and the present simulation predicts 0.1469, see Fig. 5. At the arrival of the reflected wave to the contact end, the analytical dimensionless stress is 0.1353 and in Fig. 6 the finite element calculates 0.1325.
Using the present finite element simulation, a visualization of the wave motion is illustrated in Fig. 8 for the bar with one fixed end. The figure illustrates the distribution of the dimensionless stress over the dimensionless length $\frac{c}{l}$, at equal dimensionless time steps of 0.125. It shows that the traveling compression stress wave is reflected at the fixed end as a compression wave, as expected. Since the contact is not terminated yet, the contact end operates as a fixed end and the compression wave is reflected from that end as a compression wave again. Once more, the wave is reflected as a compression wave at the fixed end, but shortly after that the contact is terminated. For that reason, during the contact period, the whole rod is under compression all the time, see Fig. 8. Without presenting a similar figure, analytical solutions given in [1, 2] predict the same phenomena. After the end of impact, the subsequent free vibration of the bar has periodic cycles.
with a period equals to $2\tau$, as can be seen in Figs. 8, 9, 10, and 11. As anticipated, the free end reflects the wave with opposite polarity. Therefore, during these periods, most of the time a part of the rod is in compression while the other part is in tension, see Fig. 8. The time histories of the bar displacement, velocity, stress, and contact force in dimensionless forms are shown in Figs. 9, 10, 11, and 12. The figures illustrate that the displacement is continuous while velocity, stress, and contact force suffer discontinuities. At any location in the bar, the discontinuities occur at intervals corresponding to the arrival of the waves to that location; see
Figs. 8, 9, 10, 11, and 12. Figures 8, 11 and 12 confirm that the arrival of the reflected compression wave to the contact end raises the stress at the contact end, and accordingly the contact force, to its maximum value. Next, contact force starts to decrease and impact is terminated when the stress at the contact end vanishes. The analytical solutions given in [1, 2] predict the dimensionless duration time, displacement of contact end at separation and its maximum value after separation, and maximum contact force. Both the analytical results and the corresponding results of the current finite element simulation are given in Table 1.

It has to be noticed that slight numerical damping is introduced to reduce the oscillations in the numerical solutions.

5 Conclusion

A finite element simulation for the impact of a rigid mass on an elastic rod has been presented in this chapter. The impact model utilizes St.Venant’s classical impact model, and the two cases of free–free and free-fixed elastic rod have been investigated. As contact established, a wave is initiated at the contact end and starts to propagate through the rod. The wave propagation and the contact force differential equations have been obtained and the finite element discretization of the equations of motion has been developed. A numerical solution procedure has been proposed along the lines of Newmark implicit integration method and Newton–Raphson iterative technique. The current simulation calculates the contact force accurately and efficiently which is a significant advantage over other simulations, which just assume the contact force or calculate it inefficiently.

Results show the variation of contact force, displacements, velocities, and stresses with respect to time for both cases of free and fixed far end of the bar. Very good agreement has been found between numerical results and the well-known analytical results. A simulated visualization of the propagation of the stress wave through the bar has been developed. This visualization enhances the understanding of the physical phenomena of impact and wave propagation including the reflection of the wave at free and fixed ends as well as at the contact end. The results demonstrate that the proposed finite element simulation is accurate enough for further investigation in wave analysis and simulation.
References

Hamiltonian Formalisms Applied
to Continuum Mechanics: Potential
Use for Fracture Mechanics

N. Recho

Abstract The first part of this chapter deals with several Hamiltonian formalisms in
elasticity. The formalisms of Zhong ((1995) Dalian Science & Technology
University Press, Liaoning, China) and Bui ((1993) Introduction aux problèmes
inverses en mécaniques des matériaux, Editions Eyrolles, Paris), which resolve
respectively the two-end problem and the Cauchy problem in elasticity, are presented
briefly. Then we propose a new Hamiltonian formalism, which resolves simulta-
neously the two problems mentioned above and shows the link between the two
formalisms. The potential use for fracture mechanics purposes is then mentioned. In
fact, when traditional theories in fracture mechanics are used, asymptotic analyses
are often carried out by using high-order differential equations governing the stress
field near the crack tip. The solution of the high-order differential equations becomes
difficult when one deals with anisotropic or multilayer media etc. The key of our idea
was to introduce the Hamiltonian system, usually studied in rational mechanics, into
continuum mechanics. By this way, one can obtain a system of first-order differential
equations, instead of the high-order differential equation. This method is very
efficient and quite simple to obtain a solution of the governing equations of this class
of problems. It allows dealing with a large range of problems, which may be difficult
to resolve by using traditional methods. Also, recently we developed another new
way to resolve fracture mechanics problems with the use of ordinary differential
equations (ODEs) with respect to the circumferential coordinate $\theta$ around the crack
(or notch) tip. This method presents the opportunity to be coupled with finite element
analysis and then allows resolving more complicated geometries.

Keywords Hamiltonian analysis · Stress-singular fields · V-notch · Boundary
element method

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1 Introduction

Recently, an important effort has been made in the reform of the classical theory of continuum mechanics in the frame of the Hamiltonian system. In these new approaches, the principle of Hamilton is applied in a special manner, i.e., by considering a dimensional parameter as “time”. In this topic, we can distinguish two formalisms: the formalism of Bui [1] and the formalism of Zhong [2]. By seeking the variations of the couple (displacements, traction forces) on an arbitrary front in a solid when this front virtually moves from an initial position to a neighbor one, a first-order differential equation system governing the mechanical fields was explicitly established. That is the Cauchy problem in elasticity resolved by Bui. On the other hand, the formalism of Zhong looks more classical. In simple words, he established an analogy between quantities in rational mechanics and those in continuum mechanics. For example, a dimensional coordinate in continuum mechanics is considered as time in rational mechanics; the displacement vector as the generalized coordinates; the strain energy density as the Lagrange function and so on. This analogy leads to the canonical equations of Hamilton governing the mechanical fields in elastic bodies. The main advantage of these approaches is that the fundamental equations can directly be resolved. The traditional semi-inverse method is then replaced by a direct, systematic and more structural resolution method.

2 Zhong’s Formalism: The Two-End Problem

Let us consider a solid $V$ described by a coordinate system $Z$ in which $z$ is one chosen coordinate. Let us consider now $q$ the displacements in the $Z$ system associated to neighbor displacements, $q + \delta q$. One notes $\dot{q} = \frac{\partial q}{\partial z}$. If we suppose that the displacements are imposed at $z = z_0$ and $z = z_1$, named the two end points, then we have:

$$\delta q(z = z_0) = \delta q(z = z_1) = 0$$  \hspace{1cm} (1)

Let us write the total potential energy $\Pi$ of the solid:

$$\Pi = \int_{S}^{z_1} \int_{z_0}^{S} (U_0 - W) dS dz = \int_{z_0}^{z_1} L dz$$  \hspace{1cm} \text{avec}  \hspace{1cm} L = \int_{S}^{z_1} (U_0 - W) dS$$  \hspace{1cm} (2)

where $U_0$ is the strain energy density and $W$ is the work density of the external forces. We define the Lagrange function as the integral over $S$. If $S$ is constant along $z$ and we neglect the body forces and we just consider a volume element inside the solid, we can write $L = U_0 - W$. In general, $L$ is a function of $q$ and $\dot{q}$. Following the principle of the minimum of total potential energy, $\delta \Pi = 0$ with respect to $\delta q$ and using the conditions (1), one obtains the Euler equation in $L$:
\[
\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial \dot{q}} = 0
\]  
(3)

In rational mechanics, \( L \) is named Lagrange’s function, and (3) Lagrange’s equation. Then we construct the Hamilton function \( H(p, q) \) through the Legendre’s transformation:

\[
p = \frac{\partial L(q, \dot{q})}{\partial \dot{q}}
\]

\[
H(p, q) = p^T \dot{q} - L(q, \dot{q})
\]

(4)

From (3) and (4), one deduces immediately the canonical equations of Hamilton:

\[
\frac{\partial H}{\partial \dot{q}} = -\frac{\partial L}{\partial q} = -p \quad \frac{\partial H}{\partial p} = \dot{q}
\]

(5)

\( q \) and \( p \) are dual conjugate variables. Differently from rational mechanics, these two variables represent respectively the displacement vector and the normalized stress vector.

3 Bui’s Formalism: Cauchy’s Problem in Elasticity

Bui [1] has solved the Cauchy problem in elasticity by seeking the variations of the mechanical quantities (\( q \) as a displacement vector, \( p \) as a traction vector) at an arbitrary front in the solid when it moves from an initial position \( \Gamma_t \) to a neighbour position \( \Gamma_{t+dt} \), where \( t \) defines the movement of the front in the solid. This approach leads to an explicit system of first-order differential equations.

Let us consider a domain divided into two parts \( \Omega \) and \( \Omega_t \) by a contour \( \Gamma \). Suppose that mechanical fields are known at the interior of the contour; consequently \( q \) and \( p \) are known at the contour \( \Gamma_t \). Suppose \( q' \) a virtual compatible displacement. The virtual work principle leads to:

\[
\int_{\Omega_t} \nabla q' \cdot \Lambda \cdot \nabla q' d\Omega = \int_{\Gamma_t} q' \cdot q' d\Gamma
\]

(6)

\( \Lambda \) is the elastic tensor. Let us consider now an evolution of \( \Gamma_t \) to \( \Omega_t \), i.e. at \( t + dt \), the contour \( \Gamma_t \) reaches \( \Gamma_{t+dt} \). It’s suitable to consider that \( \Gamma_{t+dt} \) is deduced from \( \Gamma_t \) following the normal to \( \Gamma_t \) with a quantity \( \psi n dt \) where \( n \) is a unit vector normal to the contour and \( \psi \) is a positive scalar field describing the velocity of the contour evolution. The derivation of (6) with respect to \( dt \) gives:

\[
\frac{d}{dt} \int_{\Omega_t} \nabla q' \cdot \Lambda \cdot \nabla q' d\Omega = \frac{d}{dt} \int_{\Gamma_t} p' \cdot q' d\Gamma
\]

(7)