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From Classical Methods to Modern Approaches
Second, Revised and Enlarged Edition
Muhammad Sahimi
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Flow and Transport in Porous Media and Fractured Rock

From Classical Methods to Modern Approaches

Second, Revised and Enlarged Edition

WILEY-VCH Verlag GmbH & Co. KGaA
Dedicated to the memory of my parents
Habibollah Sahimi (1916–1997) and Fatemeh Fakour Rashid (1928–2006)
and to
Mahnoush, Ali and Niloofar
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Since 1995, when the first edition of this book was published, the science of porous media and the understanding of flow and transport phenomena that occur in them have been advanced greatly. Characterization of porous media – both fractured and unfractured – can now be done in great detail, that is, for samples on the laboratory scale. Reconstruction methods have made it possible to develop realistic models of porous media based on relatively limited experimental data. Many new approaches have been developed that have made it possible to compute various flow and transport properties of porous media with considerable precision. In particular, pore network models on the one hand, and advanced computational techniques, such as the lattice-Boltzmann method, on the other hand, have become invaluable tools for studying flow and transport in porous media and in fractures. New theoretical developments have made it possible to analyze field-scale data, such as, the various types of logs and seismic records, with precisions, hence yielding deeper insights.

The understanding of fractures – particularly the crucial effect of the roughness of their internal surface – and fracture networks – especially the effect of their connectivity on their effective (overall) properties – has deepened. Classical models of fractured porous media, for example, the double-porosity model, though still useful in some special limits, are no longer viewed as the only practicable models for simulating flow and transport in large-scale fractured porous media. Viable alternatives that are much more realistic have been emerging at a rapid pace.

On the experimental side, new instrumentations coupled with advanced theoretical developments have made it possible to measure the various morphological, flow, and transport properties of porous media. Use of such techniques as three-dimensional X-ray computed tomography and nuclear magnetic resonance for measuring the properties have almost become routine.

The new developments motivated the preparation of the second edition of the book. However, the new edition does not merely represent an updated version of the first edition. Almost all the chapters have been completely rewritten. The characterization of unfractured and fractured porous media has been separated, each described and developed in its own chapter. The classical continuum models of fluid flow and transport in porous media and the discrete approaches based on the network models have also been separated, with each subject having its own chapter. On the experimental side, many newer experimental techniques for measuring
the important morphological, flow, and transport properties of porous media have been described. Additionally, instead of describing unconsolidated porous media separately, as in the first edition, they have been merged with consolidated porous media and given equal footing in order to describe both types of porous media in a unified manner.

As the famous song by John Lennon and Paul McCartney goes, *I get by with a little help from my friends*, except that in my case, my students and colleagues have given me a lot of help. Many people have contributed to my understanding of the topics described in this book. First and foremost, I have been blessed by the many outstanding doctoral students that I have worked with throughout my academic career on the problems studied in this book. They include Drs. Sepehr Arbabi, Mitra Dadvar, Fatemeh Ebrahimi, Jaleh Ghassemzadeh, Hossein Hamzehpour, Mehrdad Hashemi, Abdossalam Imdakm, Mahyar Madadi, Ali Reza Mehrabi, Sumit Mukhopadhyay, Mohammad Reza Rasaeei, and Habib Siddiqui. Over the years, I have also been fortunate enough to have fruitful collaborations with many friends and colleagues on research problems related to what is studied in this book, including Professors Joe Goddard, Manouchehr Haghighi, Barry Hughes, Mark Knackstedt, Charles Sammis, Dietrich Stauffer, and Theodore Tsotsis as well as Drs. Adel A. Heiba and Luigi Sartor.

In addition, the preparation of this edition of the book was greatly helped by several people. My former doctoral students, Drs. Faezeh Bagheri-Tar, Fatemeh Ebrahimi, and Mahyar Madadi provided much needed help for the figures. Dr. Mohammad Piri generously gave me the electronic file of his outstanding Ph.D. Thesis (Piri, 2003) that I used as a great source for the discussions of multiphase flows as well as the electronic files of the figures, some of which I have used in Chapter 15.

Throughout my academic life, I have been blessed by great mentors. Dr. Hasan Dabiri, my first academic mentor when I was attending the University of Tehran in Iran, introduced me to the petroleum industry when he worked with me on the project for my B.S. degree, *Evaporation Loss in the Petroleum Industry*. Over 35 years after taking the first of many courses with him, I am still influenced by his outstanding qualities, both as an academic mentor and as a wonderful human being. My advisors for my Ph.D. degree at the University of Minnesota, the late Professors H. Ted Davis and L.E. (Skip) Scriven, introduced me to various porous media problems and percolation theory, and taught me the fundamental concepts.

Michael C. Poulson was the publisher of the first edition of this book (and my first book, *Applications of Percolation Theory*, when he was with Taylor and Francis) as well as a great friend. He passed away on December 31, 1996 at the age of 50. In preparing this edition of the book, I greatly missed his wise advice, humor, and cheerful personality. The staff of Wiley-VCH, particularly Ulrike Werner, were extremely patient with my long delay in delivering the 2nd edition.

I dedicate this book to the special people in my life. My mentors in life, my late parents, Habibollah Sahimi and Fatemeh Fakour Rashid, made me what I am. I will miss them until I see them again. My wife Mahnoush, son Ali, and daughter Niloofar put up with my long absence from family life, and my spending countless
numbers of days, weeks, and months in front of the computer at home. This edition would not have been completed without their love and patience.

Los Angeles, August 2010

Muhammad Sahimi
Preface to the First Edition

Disordered porous media are encountered in many different branches of science and technology, ranging from agricultural, ceramic, chemical, civil and petroleum engineering, to food and soil sciences, and powder technology. For several decades, porous media have been studied both experimentally and theoretically. With the advent of precise instruments and new experimental techniques, it has become possible to measure a wide variety of physical properties of porous media and flow, and transport processes therein. New computational methods and technologies have also allowed us to model and simulate various phenomena in porous media, and thus a deep understanding of them has been gained.

Whether we like it or not, we have to accept the fact that many natural porous systems are fractured, the understanding of which requires new methodologies and ways of thinking. In the past two decades, the understanding of fractured rock has taken on new urgency since, in addition to oil reservoirs, many groundwater resources are also fractured. Thus, flow in fractured rock has attracted the attention of scientists, engineers and politicians as a result of growing concerns regarding pollution and water quality. Highly intense exploitation of groundwater, and the increase in solute concentrations in aquifers due to leaking repositories and use of fertilizers, have made flow in fractured rock a main topic of research.

I have been working on such problems for 15 years, and during this period, I have realized that there are two distinct approaches to modeling flow phenomena in porous media and fractured rock. Some of these approaches belong to a class of models that I call the \textit{continuum models}. Largely based on the classical equations of flow and transport, the continuum models have been very popular with engineers. Although not as widely used as the continuum models, the second approach, which is based on the \textit{discrete models} that represent a porous system by a discrete set of elements and use large-scale Monte Carlo simulations and various statistical methods to analyze flow phenomena in porous media and fractured rock, has also attracted wide attention. Many new ideas and concepts have been developed as the result of using this class of models, and new results have emerged that have helped us gain a much better understanding of porous systems. In addition, such ideas and concepts as percolation processes, universal scaling laws and fractals, the basic tools of the discrete models, have gradually found their rightful positions in the porous systems literature. Currently, such concepts are even taught in graduate
courses on flow through porous media, and in courses on computer simulation of disordered media and statistical mechanical systems.

Realizing these facts, and given that there was no book that discusses and compares both approaches, I decided to write this book. Even a glance at the immense literature on these subjects reveals that it is impossible to discuss every issue and present an in-depth analysis of it in just one book. Percolation theory, fractals, Monte Carlo simulations, and similar topics have been, by themselves, the subjects of several books and monographs. Unless one explains the most important concepts and then provides references where the reader can find more material to read, such a book can easily contain over a thousand pages. Based on this realization and limitation, I selected the topics that are discussed in this book. Largely based on this limitation, I also had to ignore several important topics, for example, flow of non-Newtonian fluids in porous media, filtration, and dissolution of rock by an acid which creates large fractures in the rock. In spite of such limitations, this book represents, in my opinion, a comprehensive review and discussion of the most important experimental and theoretical approaches to flow phenomena in porous media and fractured rock. Therefore, it can be used both as a reference book and a text for graduate courses on the subjects that it discusses. Considering the fact that this book discusses experimental measurement of the most important morphological and transport properties of porous systems, and the fact that many topics, especially those of single-phase flow and transport, are discussed in great detail, I believe that roughly half of the book can also be used in a senior-level undergraduate course on porous media problems that is taught in many chemical, petroleum and civil engineering and geological science departments.

As the famous song by John Lennon and Paul McCartney goes, “I get by with a little help from my friends”, except that in my case, my friends and colleagues have given me a lot of help. Many people have contributed to my understanding of the topics discussed in this book, a complete list of whom would be too long to be given here. However, I would like to mention a few of them who have had greatly influenced my way of thinking. I would like to thank Professors H. Ted Davis and L. E. (Skip) Scriven of the University of Minnesota who introduced me to various porous media problems and percolation theory, and taught me the fundamental concepts when I was their doctoral student. For over a decade, Dietrich Stauffer has greatly influenced my way of thinking about percolation, disordered media, and critical phenomena. I am deeply grateful to him. I would like to thank all of my past collaborators with whom I have published many papers on flow in porous media, especially Adel A. Heiba and Barry D. Hughes. Three other persons helped me write and finish this book. Michael Poulson, my publisher at VCH Publishers, was very patient and helpful. Drs. Sherry Caine and Dalia Goldschmidt helped me to organize my thoughts, focus on writing this book, and have a more positive outlook on life.

My debts of gratitude to them, and to many more who taught and influenced me, thus making this book possible.

Los Angeles, August 1994

Muhammad Sahimi
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Continuum versus Discrete Models

Introduction

Flow and transport phenomena in porous media and fractured rock as well as industrial synthetic porous matrices arise in many diverse fields of science and technology, ranging from agricultural, biomedical, construction, ceramic, chemical, and petroleum engineering, to food and soil sciences, and powder technology. Fifty percent or more of the original oil-in-place is left behind in a typical oil reservoir after the primary and secondary recovery processes end. The unrecovered oil is the main target for the enhanced or tertiary oil recovery methods now being developed. However, oil recovery processes only constitute a small fraction of an enormous and still rapidly growing literature on porous media. In addition to oil recovery processes, the closely related areas of soil science and hydrology are perhaps the best-established topics related to porous media. The study of groundwater flow and the restoration of aquifers that have been contaminated by various pollutants are important current areas of research. The classical research areas of chemical engineers that deal with porous media include filtration, centrifuging, drying, multiphase flow in packed columns, flow and transport in microporous membranes, adsorption and separation, and diffusion and reaction in porous catalysts.

Lesser known, though equally important, phenomena involving porous media are also numerous. For example, for the construction industry, the transmission of water by building materials (bricks or concretes) is an important problem to consider when designing a new building. The same is true for road construction where penetration of water into asphatene damages roads. Various properties of wood, an interesting and unusual porous medium, have been studied for a long time. Some of the phenomena involving wood include drying and impregnation by preservatives. Civil engineers have long studied asphalts as water-resistant binders for aggregates, protection of various types of porous materials from frost heave, and the properties of road beds and dams with respect to water retention. Biological porous media with interesting pore space morphology and wetting behavior include skin, hair, feathers, teeth and lungs. Other types of porous media that are widely used are ceramics, pharmaceuticals, contact lenses, explosives, and printing papers.
In any phenomenon that involves a porous material, one must deal with the complex pore structure of the medium and how it affects the distribution, flow, displacement of one or more fluids, or dispersion (i.e., mixing) of one fluid in another. Each process is, by itself, complex. For example, displacement of one fluid by another can be carried out by many different mechanisms which may involve heat and mass transfer, thermodynamic phase change, and the interplay of various forces such as viscous, buoyancy, and capillary forces. If the solid matrix of a porous medium is deformable, its porous structure may change during flow or some other transport phenomenon. If the fluid is reactive or carries solid particles of various shapes, sizes, and electrical charges, the pore structure of the medium may change due to the reaction of the fluid with the pore surface, or the physicochemical interaction between the particles and the pore surface.

In this book, we describe and study various experimental, theoretical, and computer simulation approaches regarding diffusion, flow, dispersion, and displacement processes in porous media and fractured rock. Most of the discussions regarding porous media are equally applicable to a wide variety of systems, ranging from oil reservoirs to catalysts, woods, and composite materials. We study flow phenomena only in a static medium, that is, one with a morphology that does not change during a given process. Thus, deformable media as well as those that undergo morphological changes due to a chemical reaction, or due to physicochemical interactions between the pore surface and a fluid and its contents, are not studied here. The interested reader is referred to Sahimi et al. (1990) for a comprehensive discussion of transport and reaction in evolving porous media and the resulting changes in their morphology.

1.1 A Hierarchy of Heterogeneities and Length Scales

The outcome of any given phenomenon in a porous medium depends on several length scales over which the medium may or may not be homogeneous. By homogeneous, we mean a porous medium with effective properties that are independent of its linear size. When there are inhomogeneities in the medium that persist at distinct length scales, the overall behavior of the porous medium is dependent on the rate of the transport processes, for example, diffusion, conduction, and convection, the way the fluids distribute themselves in the medium, and the medium’s morphology. Often, the morphology of a porous medium plays a role that is more important than that of other influencing factors.

Consider, as an example, an oil reservoir, perhaps one of the most important heterogeneous porous media. In principle, the reservoir is completely deterministic in that it has potentially measurable properties and features at various length scales. It could have been straightforward to obtain a rather complete description of the reservoir if only we could excavate each and every part of it. In practice, however, this is not possible and, therefore, a description of any reservoir, or any other natural porous medium for that matter, is a combination of the determinis-
tic components – the information that can be measured – and indirect inferences that, by necessity, have stochastic or random elements in them. Over the past four decades, the statistical physics of disordered media has played a fundamental role in developing the stochastic component of description of porous media.

There are several reasons for the development. One is that the information and data regarding the structure and various properties of many porous media are still vastly incomplete. Another reason is that any property that we ascribe to a medium represents an average over some suitably selected volume of the medium. However, the relationship between the property values and the volume of the system over which the averages are taken remains unknown. The issue of a suitably selected volume reminds us that any proper description of a porous medium or fractured rock must have a length scale associated with it. In general, the heterogeneities of a natural porous medium are described at mainly four distinct length scales that are as follows (Haldorsen and Lake, 1984).

1. The microscopic heterogeneities are at the level of the pores or grains, and are discernible only through scanning electron microscopy or thin sections.
2. The macroscopic heterogeneities are at the level of core plugs, and are routinely collected in fields and analyzed. Such heterogeneities are found in every well with property values varying widely from core to core. In most theoretical studies, however, cores are assumed homogeneous and the average effective properties are assigned to them, notwithstanding their microscopic heterogeneities.
3. The megascopic or field-scale heterogeneities are at the level of the entire reservoir that may have large fractures and faults. They can be modeled as a collection of thousands, perhaps millions, of cores, oriented and organized in some fashion.
4. The gigascopic heterogeneities are encountered in landscapes that may contain many such reservoirs as described in the third example, along with mountains, rivers, and so on.

Not all of the aforementioned heterogeneities are important to all porous media. For example, porous catalysts usually only contain microscopic heterogeneities, and packed beds may be heterogeneous at both the microscopic and macroscopic levels. In this book, we consider the first three classes of heterogeneities and their associated length scales.

1.2 Long-Range Correlations and Connectivity

In the early years of studying flow phenomena in porous media and fractured rock, most researchers almost invariably assumed that the heterogeneities in one region or segment of the system were random and uncorrelated with those in other regions. Moreover, it was routinely assumed that such heterogeneities occur at length scales much smaller than the overall linear size of the system. Such assumptions were partly due to the fact that it was very difficult to model the system in a more
realistic way due to the computational limitations and lack of precise experimental techniques for collecting the required information. At the same time, the simple conceptual models, such as random heterogeneities, did help us gain a better understanding of some of the issues. However, increasing evidence suggests that rock and soils do not conform to such simplistic assumptions. They exhibit *correlations* in their properties, and such correlations are often present at *all* the length scales. The existence of such correlations has necessitated the introduction of *fractal distributions* that tell us how property values of various regions of a porous medium depend on the length (or even time) scale of the observations, how they are correlated with one another, and how one can model such correlations realistically. Such concepts and modeling techniques are described in this book.

Once we accept that natural porous media and fractured rock are heterogeneous at many length scales, we also have to live with its consequences. As a simple, yet very important, example, consider the permeability of a porous medium, which is a measure of how easily a fluid can flow through it. In a natural porous medium and at large length scales (of the order of a few hundred meters or more), the permeabilities of various regions of the medium follow a broad distribution. That is, while parts of the medium may be highly permeable, other parts can be practically impermeable. If we consider a natural porous medium, then, the low permeability regions can be construed as the impermeable zones as they contribute little or nothing to the overall permeability, while the permeable zones provide the paths through which a fluid flows. Thus, the impermeable zones divide the porous medium into compartments according to their permeabilities. This implies that the permeable regions may or may not be connected to one another, and that there is disorder in the connectivity of various regions of the porous medium. Thus, if we are to develop a realistic description of a porous medium, the connectivity of its permeable regions must be taken into account.

The language and the tools for taking into account the effect of the connectivity of the permeable regions of a pore space are provided by *percolation theory*. Similar to fractal distributions, percolation has its roots in the mathematics and physics literature, although it was first used by chemists for describing polymerization and gelation phenomena. Percolation theory teaches us how the connectivity of the permeable regions of a porous medium affects its overall properties. Most importantly, percolation theory predicts that if the volume fraction of the permeable regions is below some critical value, the pore space is not permeable and its overall permeability is zero.

In the classical percolation that was studied over 50 years ago, it was assumed that the permeable and impermeable regions are distributed randomly and independently of each other throughout the pore space. Since then, more refined and realistic percolation models have been developed for taking into account the effect of correlations and many other influencing factors. Such ideas and concepts are developed and used throughout this book for describing various flow phenomena in porous media and fractured rock.