CASE HISTORIES IN VIBRATION ANALYSIS AND METAL FATIGUE FOR THE PRACTICING ENGINEER
CASE HISTORIES IN VIBRATION ANALYSIS AND METAL FATIGUE FOR THE PRACTICING ENGINEER

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Kingwood, Texas
To The One Who Has Made This All Possible
and
To My Family
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PREFACE

Purpose of the Book

In over 45 years as a practicing engineer, troubleshooting and preventing failures were my primary responsibility, with design, especially of torsional systems, as an additional function. In the area of failure analysis, by far the majority of failures were metal fatigue failures. Since metal fatigue is caused by cycling forces and moments, vibration is introduced.

In a production environment an engineer is burdened with many day-to-day decisions and does not have the luxury of developing elegant mathematical solutions to solve the problem at hand. Trying to understand and utilize differential equations and other concepts and terminology presented in college vibration textbooks is time consuming and may not be cost-effective. Expedient, simple-to-explain solutions are required to get equipment functioning again.

Explaining what caused a failure, along with the proposed solution, to those not well versed in vibration and metal fatigue can be a challenge. This is something engineers must be able to do to generate the necessary funding to implement a solution. Too often we have heard stories about catastrophic failures related to nuclear reactors, space exploration vehicles, and drilling platforms, for example, and that a problem and solution were known by the engineers but ignored by those in control of the budget. A typical comment from those in control might be: “The system has had this problem in the past and worked fine, so there is little risk.” In such cases, time and funding control the decision rather than analysis of the risk involved in not solving the problem. It is the engineer’s responsibility to present risks clearly and concisely in language and, if necessary, in experiments that can be understood.

This book is about helping engineers obtain solutions to difficult vibration problems using techniques that can be easily explained. This is done using personal case histories. The subject of metal fatigue is in the book simply because excessive vibration often results in fatigue failures. Identifying fatigue-based failures can help identify the source of the vibration. It is my hope that the book will help readers understand vibration and metal fatigue and use the contents in a practical manner to solve industrial problems and enhance their careers.
Content and Arrangement

In Chapter 1 we introduce background history on vibration and what we set out to accomplish.

Chapter 2 is a basic introduction to the single-degree-of-freedom problem and an example is used to show how systems can be simplified. Multiple-spring systems are combined into equivalent systems, and some common properties needed in vibration analysis are shown. How to determine the natural frequencies of pipes, beams, and plates, how vibration absorbers function, and how clearance affects the natural frequency of a system are explained.

Chapter 3 addresses methods for measuring and presenting vibration information. The shock pulse method is illustrated, as it has practical use in monitoring vibrations and data trending. A systematic method for identifying the source of vibration is shown in a case history.

Chapter 4 is an important chapter that shows how amplitudes can be calculated using the dynamic magnifier method. The stresses and torques due to vibration can be determined quickly using this method and field data can be used to better define the data. The chapter contains many actual case histories showing use of the method to evaluate several unique and interesting problems.

In Chapter 5 we review problems that vibration can cause and the sources of the problems. Fatigue, wear, bearing failures, why bolts loosen, flow-induced vibrations, and surging of fans are just a few of the topics explained. In addition, the slip-stick phenomenon is introduced and illustrated with actual problems.

In Chapter 6 we discuss imbalance and misalignment. Vibration in pumps, motors, gearboxes, and other equipment, together with their unique vibration problems, are examined in detail. Various types of couplings are also described.

In Chapter 7 we analyze piping and pressure vessel vibration. Here screening charts which show vibration levels that have resulted in failures are presented. Heat-exchanger tube vibration prediction methods and ways to avoid such vibration are explained. Ways to evaluate acoustical vibration problems arising from the amplification of pressure pulses, and fluid water hammer analysis, are introduced using case histories. Also described is crack growth in plates and welds.

Chapter 8 is about torsional vibrations, beginning with what they are and progressing into many case histories on how they were applied. Frequency, amplitude, and excitation calculations are all discussed in detail. Internal combustion engines and electric motors driving geared systems are analyzed. Many are reduced to two-mass systems, but multimass systems are also evaluated. Torque applied suddenly and grid closures that are out of synchronization are evaluated. A Holzer analysis is shown for spreadsheet use and can be used to analyze the frequencies, mode shapes, and relative torques and forces for torsional and linear multimass systems.

In Chapter 9 we examine turbomachinery rotor dynamics, a complex subject, by utilizing simple rotor models to explain the principles and to solve several case histories. The system is modeled as a multidiameter shaft on springs and the fundamental frequency is determined. Various case histories show how the model
is used in troubleshooting problems. Determining the stiffness of hydrodynamic bearings is also reviewed.

In Chapter 10 we look at very low cycle vibrations. These are the types of cyclic loads that can cause metal fatigue failures, which may occur after only a few hundred cycles. Gear face pitting failures and rotary dryer failures are only a few of the types of case histories examined, along with crack growth due to cyclic loads. The chapter ends with examples of the imprinting method to determine the loads causing failures. The use of vibratory and rotational wear equations is also shown.

Chapter 11 contains case histories on some actual failures that I have witnessed, with descriptions of the causes of the failures. Springs, splines, crankshafts, bearings, pistons, and other components are analyzed, and the appearance of the fracture surfaces is discussed. With this information on fatigue failures due to cyclic loads, vibration can be better understood.

Chapter 12 covers the fundamentals of metal fatigue as it applies to investigating vibration problems. What can be expected from a metallurgical examination and how it can be applied to troubleshooting a vibration problem are illustrated. The chapter ends with a brief discussion of risk taking and presentations to management that can benefit an engineer.

In Chapter 13 we present a short history of practical vibration analysis and some of the people responsible for developing much of the theory.

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I also wish to thank Heinz Bloch, a prolific writer, educator, and friend for suggesting that I write the book.

I thank Richard S. Gill, my colleague and friend, for bringing many of these case histories to my attention and for the enjoyable hours of technical discussions on many of them.

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I wrote the book in memory of Dr. J. P. Den Hartog, whose summer seminar and books have made vibration analysis much clearer to me and allowed me to explore new techniques.

Anthony Sofronas
Throughout my career I have been involved in many areas of mechanical engineering and machinery operation, as well as pressure vessel and piping problems. Any analysis was for the purpose of solving an actual problem that was occurring at the time. There usually wasn’t time for a detailed study—an answer was required immediately so that equipment could be restarted safely and reliably with the most probable cause of failure having been determined. High-visibility failures, those drawing top management interest, usually required the attention of many experienced specialists. One discipline that I used to troubleshoot failures was vibration analysis, and over the years sufficient cases with known outcomes were developed that this book could be written. A notable focus is metal fatigue, because where there is excessive vibration, there is usually a fatigue-related problem. For the practicing engineer it is difficult to separate the two when a solution is needed to prevent a repeat failure.

Many books on vibration analysis are available. Some are heavy with theory and others are too simplified for practical everyday use. In this book I fill the void by using actual case histories to discuss the equations presented or the results shown, to heighten their usefulness for practical troubleshooting purposes. I do not consider specific vibration-measuring equipment or computer programs presently in use so that the book will be useful for a long time. The equations don’t change much with time, only the methods used to solve them.

Nearly all machinery, pressure vessels, and piping systems will experience some vibration. In dealing with vibration concerns, the following questions are typically raised:
INTRODUCTION

• What is causing the vibration?
• Is the vibration of sufficient magnitude that it needs to be controlled?
• How can the vibration be controlled?

Vibration-related problems occur less often than static stress-related failures such as bending, torsional, overload, or inadequate material properties. When severe vibration does occur, it can be very costly to remedy, sometimes requiring total system redesign. Vibration is the result of dynamic forces and moments acting on equipment. When severe enough, these can result in fatigue-related failures.

It is therefore important for those who design equipment or for users of the equipment to understand vibration and fatigue.

The book contains cases and information on problems in the petrochemical, component manufacturing, and transportation industries and encompasses over 45 years of personal experience. As such, it should be useful in many sectors of industry and also to those new to industry or new to vibration analysis. Many experienced engineers will also find problems and solutions that they haven’t yet encountered.

The cases used are not based on developing new designs but on investigating the causes of failures or on troubleshooting newly installed, up-rated, or in-service equipment. New machinery, piping systems, and pressure vessel designs are usually based on the manufacturers’ experience with the equipment, and reputable manufacturers use the latest analysis techniques available. It is only when a piece of equipment is the first one ever built, or the biggest ever built, that problems can occur. When large equipment is designed as simply a scale-up of a smaller design, things don’t always scale up as hoped. Manufacturers may also use linear scale-up techniques on nonlinear problems, resulting in fatigue failures.

In a previous book of mine [1] many areas of mechanical engineering and their associated failures were examined using theory and case histories. In this book, only failures due to excessive vibration are considered. Since the book represents primarily my personal experiences, it does not cover all types of vibration. Excellent references are provided to supplement the information. The book does present vibration problems that engineers responsible for many types of equipment will encounter during their careers. Some of the work presented in this book has also been taught to personnel in various companies in seminar format and therefore contains input from many participants.

Most of the examples are simplified so that the reader doesn’t have to have, or purchase, special software to solve many of the vibration problems that occur. The simplified solutions were enough to determine the cause of the vibration problem and implement a solution. I show the development of simplified equations when appropriate. In some cases simplification is not possible, and more complex software must be used. For example, torsional vibration of a multmass system driven by a gasoline or diesel engine can have many harmonics and can require specialized software for frequency and amplitude calculations. Engineers who
send such work to a consulting firm should understand what is being done even if they aren’t proficient in particular areas, as the results will need to be understood and explained to others.

Although exact solutions may not be possible with the simplified equations shown, in most cases they will allow the specialist to better understand the causes of the vibrations and to address them. Sometimes it will just indicate that a vibration consultant should be contacted and provides the specialist with information to discuss the problem intelligently with the consultant.

Many torsional vibration case histories are used in this book. There are two reasons for this. The first is that when a torsional vibration problem occurs in machinery, it is usually an expensive failure and not easily fixed. Calculations need to be expedited and solutions need to be practical. The second reason is that I have an extensive background in analyzing and testing design-related systems for torsional vibration. Thus, there were a considerable number of actual case histories. Most of the failure case histories have interesting stories associated with them, which in some sections are included.

It is difficult to identify a true vibration failure. For example, a shaft failure may be written up as a vibration failure since extreme vibrations were felt. In reality, a rub may have developed that caused bending loads which resulted in a fatigue failure. In this case, rotating bending was the true cause and vibration was a result, not the cause.

The total number of major failures examined during my 45 years in industry and consulting was approximately 400, which would be a total of 100% of the major failures examined. Only 20% were similar to those shown in this book and were defined as vibration problems. This is more typical than atypical for engineers dealing with failures in industry. Not all of an engineer’s time is spent investigating vibration-related failures, but when these types of failures do occur they are usually major investigations. Most vibration alignment or balance problems are just annoying and can be eliminated with a quick realignment or field balance and don’t result in a failure.

Before we begin it will be useful to describe briefly the two types of vibrations that are discussed in this book. The first are linear vibration systems, which in this book have units of in., lb, and sec, and the second are torsional systems, which have units of rad, in.-lb, and sec. Many of the problems are described as single-degree-of-freedom problems, those with one mass and one spring (Figure 1.1). If either is pulled (linear) or twisted (torsional) and released, they will vibrate at their natural frequency, which is \( f_n = 9.55(k/m)^{1/2} \) cycles per minute (cpm) for the linear system and \( f_n = 9.55(C/J)^{1/2} \) cpm for the torsional case. The similarity between the two equations comes in very handy for understanding vibration. Single-degree-of-freedom models are extremely useful, since many complex vibration systems can be reduced to them. They are much easier to understand and analyze and their results are much easier to explain to others, as is shown by the case histories in this book.

Even single-degree-of-freedom (SDF) models can become complex, as in the case of self-excited vibration, transient, or forced damped vibration problems.
Most of the time they are useful because multi-degree-of-freedom problems or several masses with their attaching springs can be “lumped” to form a single mass–spring model. The first example in Chapter 2 shows this.

Also described are the variety of fatigue failures that can occur and that are typical for industry. The types discussed occur many times, and seeing actual photographs should help the specialist understand what may have caused the fatigue. Impacts that occur several times and cause a failure are not truly fatigue failures, which are usually thought of as representing a very high number of cycles. I once worked with an extremely talented and experienced mechanical engineer when I was just starting work in industry. This engineer said: “Everything fails in fatigue.” When I questioned his statement and asked him about a one-cycle impact failure, his remark was: “That’s low-cycle fatigue.” His point was, of course, that most failures on machinery spinning at high speeds are usually fatigue related. With speeds and power increasing over time, these types of fatigue failures are becoming even more common.

REFERENCE

2.1 SPRING–MASS SYSTEMS AND RESONANCE

Vibration problems on compressors, motors, and ship systems can cause extensive damage. The key element of vibration problems is that many can be reduced to a very simple system for troubleshooting calculations. Although exact results cannot be expected, a better understanding of the problem can be.

We discuss vibrations here using case histories and describe the terms therein. There is a good reason for this, as many complex machines or structures can be reduced as described in the examples. This is especially true when modifications of existing equipment are being reviewed. This first case history is based on the vibration of a 2200-hp steam turbine that had a history of startup and in-service vibration problems. Although much vibration testing was done, this analysis looks at understanding the cause and explaining the nomenclature along the way.

This fairly simple multistage steam turbine will be reduced to a simple system so that the fundamental natural frequency can be determined. The rotor is shown out of the turbine in Figure 2.1. Since at this time only the rotating members are of importance, the rotor disks can be combined into one mass, and the bearing supports and bearing oil film into springs, as shown in Figure 2.2.

The shaft can be represented as a simply supported beam and has a $k$ value $P/\delta$ or a load divided by its deflection under the load. It can then be added as a spring in series and the shaft eliminated. This produces the “bouncing” mode.
of the spring–mass system. This is not applicable to rotor dynamics problems, which are discussed in Chapter 9.

The combined springs (Figure 2.3) use the following equations to obtain an equivalent spring, that is, one spring that has the equivalent spring rate. For springs in series,

$$ k = \frac{1}{1/k_1 + 1/k_2} $$
SPRING–MASS SYSTEMS AND RESONANCE

Figure 2.3  Springs in series and parallel.

Figure 2.4  Single-degree-of-freedom system.

or, in general,

\[ \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots \]

For springs in parallel,

\[ k = k_1 + k_2 + \cdots \]

Combining the springs in the case history and calling the rotor and shaft mass \( m \) results in the simplified single-degree-of-freedom system shown in Figure 2.4.

If the mass \( m \) is displaced and then released, or the rotor in the steam turbine is struck, a fundamental motion will occur in an undamped system. The motion of the mass is represented by

\[ x = X \sin \omega t \]

Here \( X \) represents the peak single amplitude and \( 2X \) is the double amplitude, also referred to as the peak-to-peak displacement amplitude, the quantity most used from vibration readings. The value \( x \) is the amplitude at a given time.
For this book we consider peak values only (i.e., the time and phase angle difference have been ignored) and by utilizing differentiation arrive at [1]

Displacement: \( X \) in. peak

Velocity: \( V = 2\pi fX \) in./sec peak

Acceleration: \( a = 4\pi^2 f^2 X \) in./sec^2

The frequency \( f \) represents the number of complete cycles that occur per second. When the frequency is known, the period or time of a cycle is also known:

\[
T = \frac{1}{f} \text{ sec}
\]

This is, of course, only for harmonic motion, as shown in Figure 2.4, and not for other complex waveforms such as those shown in Figure 2.5.

An important fact about this simple single-degree-of-freedom problem is that the frequency at which it will oscillate is simple to calculate:

\[
f_n = 9.55 \left( \frac{K}{m} \right)^{1/2} \text{ cpm}
\]

where \( m = W/g \). In review, \( m \) is simply the concentrated mass of the rotor and shaft. These are vibrating at \( X \) displacement, and \( K \) is the spring constant, that is, how much the load statically displaces the springs.

For a simply supported shaft,

\[
K = \frac{48EI}{L^3} \text{ lb/in.}
\]

If the \( K \) and \( m \) values were determined, it would be a simple task to calculate \( f_n \), the system’s natural frequency. This differs from \( f_f \), which is the forcing frequency. The natural frequency is important since it is the frequency at which the system wants to vibrate. If the forcing frequency (e.g., the rotor speed with unbalance) coincides with the natural frequency, resonance will occur. The speed at which this occurs is also called the critical speed. The resonant frequency is important, as high displacements occur with light damping, and \( \pm 20\% \) from resonance is a good range to design away from.

Figure 2.6 illustrates how the displacement \( X \) increases closer to \( f_f / f_n \) and is called the magnification factor (\( M \)). \( X_0 \) represents the static deflection under the load, and \( X \) is the dynamic peak motion. Also note that damping, represented by \( \zeta \), doesn’t greatly affect the frequency, only the amplitude. The magnification factor is discussed further in Chapter 4 when it is necessary to calculate amplitudes.

This shows the importance of calculating this frequency. If the natural frequency of a device is 4500 rpm, which is the same as saying 4500 cpm, it would not be wise to have its operating speed range within 3600 to 5400 rpm. Somehow the system should be redesigned to be outside this range.
2.2 CASE HISTORY: COMBINING SPRINGS AND MASSES IN A STEAM TURBINE PROBLEM

A speed limitation had been imposed on an old steam turbine, due to a critical speed in the operating range. Due to the turbine’s age, a new high-efficiency turbine was to be purchased. Several manufacturers had bid on the new turbine, and the manufacturer of the original turbine offered the highest efficiency value at the lowest cost. They reported that a design change which stiffened the bearing supports and increased the shaft diameter slightly moved the critical speed to
6800 rpm, a good bit above the original 4800 rpm critical speed. The engineer’s task was to verify that this was the correct direction to go in raising the critical speed.

Returning to the lumped rotor and spring single-degree-of-freedom system shown in Figure 2.2 a simplified system was developed (Figure 2.7). Spring constants were determined from available data and are as follows:

\[
K_{\text{shaft}} = \frac{48EI}{L^3} = \frac{48\pi Ed^4}{64L^3} = 1.7 \times 10^6 \text{ lb/in.}
\]

\[
K_{\text{oil film}} = 1.6 \times 10^6 \text{ lb/in.}
\]

The oil film stiffness is only an approximation, and depending on the bearing type might vary from half this value to twice this value. It is a difficult number to obtain and depends on rotor speeds, designs, viscosities, clearances, load, and other bearing parameter factors. For the bearing type used, this number was realistic.

\[
K_{\text{support}} = 1.6 \times 10^6 \text{ lb/in.}
\]