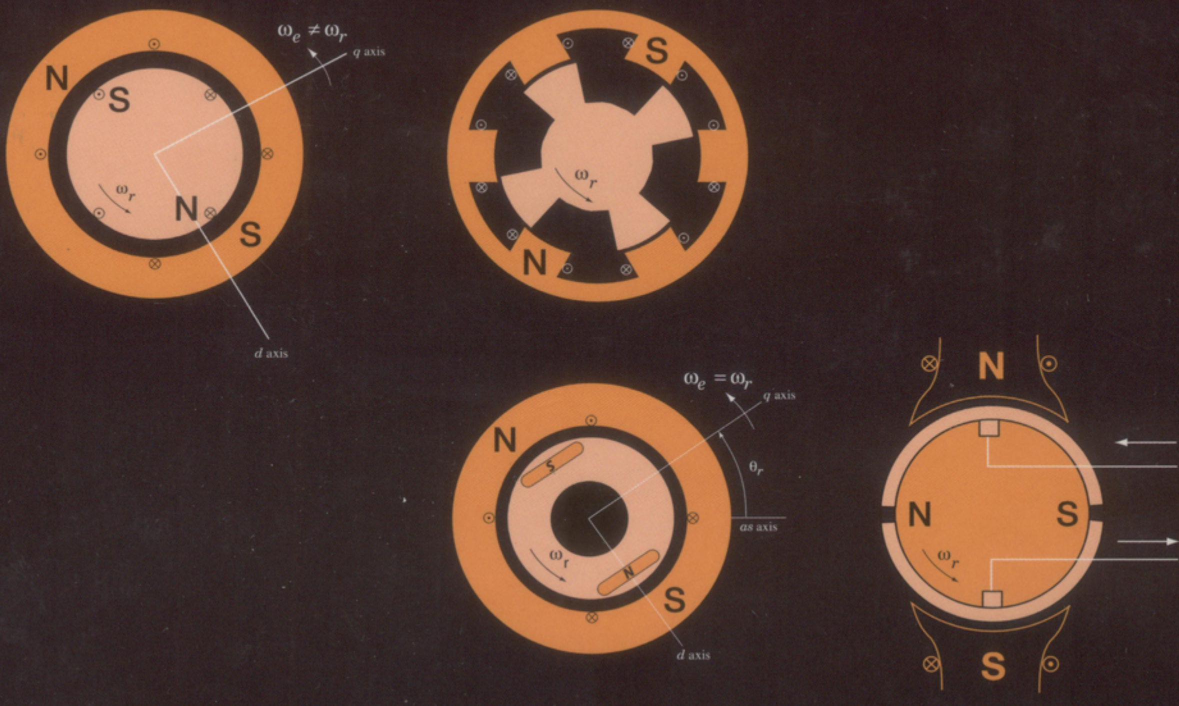
Electromechanical Motion Devices

Second Edition



PAUL KRAUSE • OLEG WASYNCZUK • STEVEN PEKAREK

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PREFACE

Performance control of electric machines began in earnest with the advent of electronic switching devices in the mid 20th century and has since grown into a major industry. This growth has been accelerated in the last 25 years by the ever-increasing sophistication of switching devices and the emergence of electric drives, and now, the recent push to develop economically competitive hybrid and electric vehicles and a more efficient and cleaner power grid. These device improvements have enabled major breakthroughs in the performance control of ac machines. For example, the permanent-magnet ac machine and the induction machine can be controlled so that the resulting performance characteristics are unrecognizable from the traditional steady-state, torque-speed characteristics. However, it has been found that in the design of these controls, it is convenient if not necessary to incorporate a transformation for the ac variables so that the substitute variables resemble those of a dc machine and that this transformation must be embedded within the control. In addition, detailed computer simulations, which include the electric and mechanical transients, have become a design necessity. Reference-frame theory is the key player in all of this and it would be highly beneficial if it were at least introduced in undergraduate study of electric machines. The present-day academic maturity of the third-year electrical engineering student is more than sufficient to follow the concept of reference-frame theory if it is introduced in a straightforward and concise manner. This second edition is an attempt to accomplish this modernization goal.

The analysis of magnetically coupled windings, a direct approach to energy conversion that minimizes the traditional array of summations, distributed windings, and dc machines are covered in the first four chapters. Therein, the advantages and the performance features of the dc machine, which are the emulation goals of controlled ac machines, are established. Controlled converter switching for a dc drive is covered briefly; however, this is presented without the need for a background in automatic control or in semiconductor physics.

Reference-frame theory is introduced in Chapter 5. This is not a lengthy, involved three-phase dissertation; instead, it is a concise two-phase approach that, if studied carefully, makes the analysis of the electric machines covered in later chapters a straightforward and less-time consuming task. It has been the authors' experience that the concepts and advantages of reference-frame theory is often lost in the maze of the trigonometry involved in a three-phase analysis. Since most, if not all, of the concepts are contained in the two-phase approach, the student is able to focus on the basic principles and advantages of reference-frame theory with minimum trigonometric distraction. In fact, once familiar with the material in Chapter 5, the student is able to foresee the change of variables needed for the machines considered in the later chapters and the form of resulting transformed voltage equations without going through any additional derivation. Therefore, the instructor will find that the time spent on the material in Chapter 5 is paid back with handsome dividends in later chapters. Moreover, the analysis and the transient and steady-state performance characteristics of the two- and three-phase machines are essentially identical. The minor differences are addressed briefly at the end of the two-phase treatment of each machine, making the extension to the three-phase machine direct and easily presented.

Field-oriented control of induction machines, constant-torque and constant-power regions of permanent-magnet ac machines, brushless dc machines, and control of doubly fed induction machines for wind turbines are all applications that have become common in the last 25 years. Several of these applications are introduced in this text, not in extensive detail, but in detail sufficient to give the reader a clear first-look at these modern machine applications.

Topics from Chapters 1 through 5 form the basis for the subsequent chapters and the text is purposely written so that once these topics are covered, Chapters 6 through 9 can be covered in any order. Although topics from Chapter 6 should be covered before Chapter 10, the ordering of Chapters 6 through 9 is not based on requisites nor should the ordering in the text be taken as recommended. Although the ordering and depth of coverage are optional, there are perhaps two scenarios that bracket the possible classroom use of this text. To emphasize the electric-power area of study, parts of Chapter 3 could be omitted and topics from Chapters 6, 7, and 10 added. For the electric-drives area, topics from Chapters 6, 8, 9, and part of Chapter 10 could be added. Actually, this book can play various roles depending upon the background of the students and the goals of the instructor. Certainly, it is not intended for all of the material to be taught in one undergraduate course. The instructor can select the topics and depth of coverage so that the student is prepared for advanced study and to provide a modern background and a ready reference for the practicing engineer. Moreover, this text could be used in a two-course series in which the second course is at the senior or introductory graduate level. The text is purposely organized with material being repeated for convenient use as a reference. Once the instructor has become familiar with this feature, it will be found that topics can be covered thoroughly without presenting material previously covered.

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Chapter 1

MAGNETIC AND MAGNETICALLY COUPLED CIRCUITS

1.1 INTRODUCTION

Before diving into the analysis of electromechanical motion devices, it is helpful to review briefly some of our previous work in physics and in basic electric circuit analysis. In particular, the analysis of magnetic circuits, the basic properties of magnetic materials, and the derivation of equivalent circuits of stationary, magnetically coupled devices are topics presented in this chapter. Much of this material will be a review for most, since it is covered either in a sophomore physics course for engineers or in introductory electrical engineering courses in circuit theory. Nevertheless, reviewing this material and establishing concepts and terms for later use sets the appropriate stage for our study of electromechanical motion devices.

Perhaps the most important new concept presented in this chapter is the fact that in all electromechanical devices, mechanical motion must occur, either translational or rotational, and this motion is reflected into the electric system either as a change of flux linkages in the case of an electromagnetic system or as a change of charge in the case of an electrostatic system. We will deal primarily with electromagnetic systems. If the magnetic system is linear, then the change in flux linkages results, owing to a change in the inductance. In other words, we will find that the inductances of the electric circuits associated with electromechanical motion devices are functions of the mechanical motion. In this chapter, we shall learn to express the self- and mutual inductances for simple translational and rotational electromechanical devices, and to handle these changing inductances in the voltage equations describing the electric circuits associated with the electromechanical system.

Throughout this text, we will give short problems (SPs) with answers following most sections. If we have done our job, each short problem should take less than ten minutes to solve. Also, it may be appropriate to skip or deemphasize some material in this chapter depending upon the background of the students. For example, those familiar with the concept of phasors may opt to skip all or most of the following section. At the close of each chapter, we shall take a moment to look back over some of the important aspects of the material that we have just covered and mention what is coming next and how we plan to fit things together as we go along.

1.2 PHASOR ANALYSIS

Phasors are used to analyze steady-state performance of ac circuits and devices. This concept can be readily established by expressing a steady-state sinusoidal variable as

$$F_a = F_p \cos \theta_{ef} \tag{1.2-1}$$

where capital letters are used to denote steady-state quantities and F_p is the peak value of the sinusoidal variation, which is generally voltage or current but could be any electrical or mechanical sinusoidal variable. For steady-state conditions, θ_{ef} may be written as

$$\theta_{ef} = \omega_e t + \theta_{ef}(0) \tag{1.2-2}$$

where ω_e is the electrical angular velocity and $\theta_{ef}(0)$ is the time-zero position of the electrical variable. Substituting (1.2-2) into (1.2-1) yields

$$F_a = F_p \cos[\omega_e t + \theta_{ef}(0)] \tag{1.2-3}$$

Since

$$e^{j\alpha} = \cos \alpha + j \sin \alpha \tag{1.2-4}$$

equation (1.2-3) may also be written as

$$F_a = Re\left\{F_p e^{j[\omega_e t + \theta_{ef}(0)]}\right\} \tag{1.2-5}$$

where Re is shorthand for the "real part of." Equations (1.2-3) and (1.2-5) are equivalent. Let us rewrite (1.2-5) as

$$F_a = Re\left\{F_p e^{j\theta_{ef}(0)} e^{j\omega_e t}\right\} \tag{1.2-6}$$

We need to take a moment to define what is referred to as the root-mean-square (rms) of a sinusoidal variation. In particular, the rms value is defined as

$$F = \left(\frac{1}{T}\int_0^T F_a^2(t)dt\right)^{\frac{1}{2}} \tag{1.2-7}$$

where F is the rms value of $F_a(t)$ and T is the period of the sinusoidal variation. It is left to the reader to show that the rms value of (1.2-3) is $F_p/\sqrt{2}$. Therefore, we can express (1.2-6) as

$$F_a = Re\left[\sqrt{2}\, F e^{j\theta_{ef}(0)} e^{j\omega_e t}\right] \tag{1.2-8}$$

By definition, the phasor representing F_a, which is denoted with a raised tilde, is

$$\tilde{F}_a = F e^{j\theta_{ef}(0)} \tag{1.2-9}$$

which is a complex number. The reason for using the rms value as the magnitude of the phasor will be addressed later in this section. Equation (1.2-6) may now be written as

$$F_a = Re\left[\sqrt{2}\tilde{F}_a e^{j\omega_e t}\right] \tag{1.2-10}$$

A shorthand notation for (1.2-9) is

$$\tilde{F}_a = F\underline{/\theta_{ef}(0)} \tag{1.2-11}$$

Equation (1.2-11) is commonly referred to as the *polar form* of the phasor. The *cartesian form* is

$$\tilde{F}_a = F\cos\theta_{ef}(0) + jF\sin\theta_{ef}(0) \tag{1.2-12}$$

When using phasors to calculate steady-state voltages and currents, we think of the phasors as being stationary at $t = 0$. On the other hand, a phasor is related to the instantaneous value of the sinusoidal quantity it represents. Let us take a moment to consider this aspect of the phasor and, thereby, give some physical meaning to it. From (1.2-4), we realize that $e^{j\omega_e t}$ is a constant-amplitude line of unity length rotating counterclockwise at an angular velocity of ω_e. Therefore,

$$\sqrt{2}\tilde{F}_a e^{j\omega_e t} = \sqrt{2}F\left\{\cos[\omega_e t + \theta_{ef}(0)] + j\,\sin\left[\omega_e t + \theta_{ef}(0)\right]\right\} \tag{1.2-13}$$

is a constant-amplitude line $\sqrt{2}F$ in length rotating counterclockwise at an angular velocity of ω_e with a time-zero displacement from the positive real axis of $\theta_{ef}(0)$. Since $\sqrt{2}F$ is the peak value of the sinusoidal variation, the instantaneous value of F_a is the real part of (1.2-13). In other words, the real projection of the phasor $\tilde{F}_a$ is the instantaneous value of $F_a/\sqrt{2}$ at time zero. As time progresses, $\tilde{F}_a e^{j\omega_e t}$ rotates at ω_e in the counterclockwise direction, and its real projection, in accordance with (1.2-10), is the instantaneous value of $F_a/\sqrt{2}$. Thus, for

$$F_a = \sqrt{2}F\,\cos\,\omega_e t \tag{1.2-14}$$

the phasor representing F_a is

$$\tilde{F}_a = Fe^{j0} = F\underline{/0^\circ} = F + j0 \tag{1.2-15}$$

For

$$F_a = \sqrt{2}F\,\sin\,\omega_e t \tag{1.2-16}$$

the phasor is

$$\tilde{F}_a = Fe^{-j\pi/2} = F\underline{/-90^\circ} = 0 - jF \tag{1.2-17}$$

Although there are several ways to arrive at (1.2-17) from (1.2-16), it is helpful to ask yourself where must the rotating phasor be positioned at time zero so that, when it rotates counterclockwise at ω_e, its real projection is $(1/\sqrt{2})F_p\,\sin\,\omega_e t$? Is it clear that a phasor of amplitude F positioned at $\frac{\pi}{2}$ represents $-\sqrt{2}F\,\sin\,\omega_e t$?

In order to show the facility of the phasor in the analysis of steady-state performance of ac circuits and devices, it is useful to consider a series circuit consisting of a resistance, an inductance, and a capacitance. Thus,

$$v_a = Ri_a + L\frac{di_a}{dt} + \frac{1}{C}\int i_a dt \tag{1.2-18}$$

For steady-state operation, let

$$V_a = \sqrt{2}\,V\,\cos\left[\omega_e t + \theta_{ev}(0)\right] \tag{1.2-19}$$

$$I_a = \sqrt{2}I\,\cos\left[\omega_e t + \theta_{ei}(0)\right] \tag{1.2-20}$$

where the subscript a is used to distinguish the instantaneous value from the rms value of the steady-state variable. The steady-state voltage equation may be obtained by substituting (1.2-19) and (1.2-20) into (1.2-18), whereupon we can write

$$\sqrt{2}V\cos[\omega_e t + \theta_{ev}(0)] = R\sqrt{2}I\cos[\omega_e t + \theta_{ei}(0)] + \omega_e L\sqrt{2}I\cos[\omega_e t + \tfrac{1}{2}\pi + \theta_{ei}(0)] + \frac{1}{\omega_e C}\sqrt{2}I\cos[\omega_e t - \tfrac{1}{2}\pi + \theta_{ei}(0)] \quad (1.2\text{-}21)$$

The second term in the right-hand side of (1.2-21), which is $L\frac{dI_a}{dt}$, can be written

$$\omega_e L\sqrt{2}I\ \cos\left[\omega_e t + \tfrac{1}{2}\pi + \theta_{ei}(0)\right] = \omega_e L Re[\sqrt{2}Ie^{j\frac{1}{2}\pi}e^{j\theta_{ei}(0)}e^{j\omega_e t}] \quad (1.2\text{-}22)$$

Since $\tilde{I}_a = Ie^{j\theta_{ei}(0)}$, we can write

$$L\widetilde{\frac{dI_a}{dt}} = \omega_e L e^{j\frac{1}{2}\pi}\tilde{I}_a \quad (1.2\text{-}23)$$

Since $e^{j\frac{1}{2}\pi} = j$, (1.2-23) may be written

$$L\widetilde{\frac{dI_a}{dt}} = j\omega_e L\tilde{I}_a \quad (1.2\text{-}24)$$

If we follow a similar procedure, we can show that

$$\frac{1}{C}\widetilde{\int I_a dt} = -j\frac{1}{\omega_e C}\tilde{I}_a \quad (1.2\text{-}25)$$

It is interesting that differentiation of a steady-state sinusoidal variable rotates the phasor counterclockwise by $\frac{1}{2}\pi$, whereas integration rotates the phasor clockwise by $\frac{1}{2}\pi$.

The steady-state voltage equation given by (1.2-21) can be written in phasor form as

$$\widetilde{V}_a = \left[R + j(\omega_e L - \frac{1}{\omega_e C})\right]\ \widetilde{I}_a \quad (1.2\text{-}26)$$

We can express (1.2-26) compactly as

$$\tilde{V}_a = Z\,\widetilde{I}_a \quad (1.2\text{-}27)$$

where Z, the impedance, is a complex number; it is not a phasor. It is often expressed as

$$Z = R + j(X_L - X_C) \tag{1.2-28}$$

where $X_L = \omega_e L$ is the inductive reactance and $X_C = \dfrac{1}{\omega_e C}$ is the capacitive reactance.

The instantaneous power is

$$\begin{aligned} P &= V_a I_a \\ &= \sqrt{2}V \cos[\omega_e t + \theta_{ev}(0)]\sqrt{2}I \cos[\omega_e t + \theta_{ei}(0)] \end{aligned} \tag{1.2-29}$$

After some manipulation, we can write (1.2-29) as

$$P = VI \cos[\theta_{ev}(0) - \theta_{ei}(0)] + VI \cos[2\omega_e t + \theta_{ev}(0) + \theta_{ei}(0)] \tag{1.2-30}$$

Therefore, the average power P_{ave} may be written

$$P_{\text{ave}} = |\tilde{V}_a|\,|\tilde{I}_a|\,\cos[\theta_{ev}(0) - \theta_{ei}(0)] \tag{1.2-31}$$

where $|\tilde{V}|$ and $|\tilde{I}|$ are the magnitude of the phasors (rms value), $\theta_{ev}(0) - \theta_{ei}(0)$ is the power factor angle φ_{pf}, and $\cos[\theta_{ev}(0) - \theta_{ei}(0)]$ is referred to as the power factor. If current is positive in the direction of voltage drop then (1.2-31) is positive if power is consumed and negative if power is generated. It is interesting to point out that in going from (1.2-29) to (1.2-30), the coefficient of the two right-hand terms is $\frac{1}{2}(\sqrt{2}V\sqrt{2}I)$ or one-half the product of the peak values of the sinusoidal variables. Therefore, it was considered more convenient to use the rms values for the phasors, whereupon average power could be calculated by the product of the magnitude of the voltage and current phasors as given by (1.2-31).

We see from (1.2-30) that the instantaneous power of a single-phase ac circuit oscillates at $2\omega_e t$ about an average value. Let us take a moment to calculate the steady-state power of a two-phase ac system. Balanced, steady-state, two-phase variables (a and b phase) may be expressed as

$$V_a = \sqrt{2}V \cos[\omega_e t + \theta_{ev}(0)] \tag{1.2-32}$$

$$I_a = \sqrt{2}I \cos[\omega_e t + \theta_{ei}(0)] \tag{1.2-33}$$

$$V_b = \sqrt{2}V \cos[\omega_e t - \tfrac{1}{2}\pi + \theta_{ev}(0)] \tag{1.2-34}$$

$$I_b = \sqrt{2}I \cos[\omega_e t - \tfrac{1}{2}\pi + \theta_{ei}(0)] \tag{1.2-35}$$

The total instantaneous power is

$$P = V_a I_a + V_b I_b \tag{1.2-36}$$

Substituting (1.2-32) through (1.2-35) into (1.2-36) and after some trigonometric manipulation, the total power for a balanced two-phase system becomes

$$P = 2\,|\tilde{V}_a|\,|\tilde{I}_a| \cos \varphi_{\text{pf}} \tag{1.2-37}$$

It is important to note that the $2\omega_e t$ oscillation is not present. In other words, the total instantaneous steady-state power is constant. In the case of a three-phase balanced system, the phasors of the three voltages or currents are displaced 120° and the instantaneous steady-state power is also constant and three times the average power of one phase. In other words the 2 in (1.2-37) becomes 3 when considering a three-phase system.

Example 1A. It is often instructive to construct a phasor diagram. For example, let us consider a voltage equation of the form

$$\tilde{V} = Z\tilde{I} + \tilde{E} \tag{1A-1}$$

where Z is given by (1.2-28). Let us assume that $\tilde{V}$and $\tilde{I}$ are known and that we are to calculate $\tilde{E}$. The phasor diagram may be used as a rough check on these calculations. Let us construct this phasor diagram by assuming that $|X_L| > |X_C|$ and $\tilde{V}$and $\tilde{I}$ are known as shown in Fig. 1A-1. Solving (1A-1) for $\tilde{E}$ yields

$$\tilde{E} = \tilde{V} - [R + j(X_L - X_C)]\tilde{I} \tag{1A-2}$$

To perform this graphically, start at the origin in Fig. 1A-1 and walk to the terminus of $\tilde{V}$. Now, we want to subtract $R\tilde{I}$. To achieve the proper orientation to do this, stand at the terminus of $\tilde{V}$, turn, and look in the $\tilde{I}$ direction which is at the angle $\theta_{ei}(0)$. But we must subtract $R\tilde{I}$; hence, $-\tilde{I}$ is 180° from $\tilde{I}$, so do an about-face and now we are headed in the $-\tilde{I}$ direction, which is $\theta_{ei}(0) - 180°$. Start walking in the direction of $-\tilde{I}$ for the distance $R|\tilde{I}|$ and then stop. While still facing in the $-\tilde{I}$ direction, let us consider the next term. Now since we have assumed that $|X_L| > |X_C|$, we must subtract $j(X_L - X_C)\tilde{I}$, so let us face in the direction of $-j\tilde{I}$. We are still looking in the $-\tilde{I}$ direction, so we need only to j ourselves. Thus, we must rotate 90° in the counterclockwise direction, whereupon we are standing at the end of $\tilde{V} - R\tilde{I}$ looking in the direction of $\theta_{ei}(0) - 180° + 90°$. Start walking in this direction for the distance of $(X_L - X_C)|\tilde{I}|$, whereupon we are at

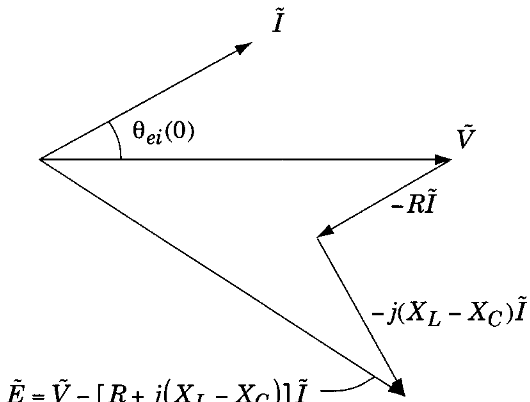

Figure 1A-1: Phasor diagram for (1A-2).

the terminus of $\tilde{V} - [R + j(X_L - X_C)]\tilde{I}$. According to (1A-2), $\tilde{E}$ is the phasor drawn from the origin of the phasor diagram to where we are.

The average steady-state power for a single-phase circuit may be calculated using (1.2-31). We will mention in passing that the reactive power is defined as

$$Q = |\tilde{V}|\,|\tilde{I}|\sin[\theta_{ev}(0) - \theta_{ei}(0)] \tag{1A-3}$$

The units of Q are var (voltampere reactive). An inductance is said to absorb reactive power and thus, by definition, Q is positive for an inductor and negative for a capacitor. Actually, Q is a measure of the interchange of energy stored in the electric (capacitor) and magnetic (inductance) fields.

SP1.2-1 If $\tilde{V} = 1\underline{/0^\circ}$ and $\tilde{I} = 1\underline{/180^\circ}$ in the direction of the voltage drop, calculate Z and P_{ave}. Is power generated or consumed? [$(-1 + j0)$ ohms, 1 watt, generated]

SP1.2-2 For SP1.2-1, express instantaneous voltage, current, and power if the frequency is 60 Hz. [$V = \sqrt{2}\cos 377t$, $I = \sqrt{2}\cos(377t + \pi)$, $P = -1 + 1\cos(754t + \pi)$]

SP1.2-3 $A = \sqrt{2}\,\underline{/0^\circ}$, $B = \sqrt{2}\,\underline{/90^\circ}$. Calculate $A + B$ and $A \times B$. [$2\underline{/45^\circ}$, $2\underline{/90^\circ}$]

SP1.2-4 In Example 1A, $X_L > X_C$ and yet $\tilde{I}$ was given as leading $\tilde{V}$. How can this be? [$\tilde{E}$]

1.3 MAGNETIC CIRCUITS

An elementary magnetic circuit is shown in Fig. 1.3-1. This system consists of an electric conductor wound N times about the magnetic member, which is generally some type of ferromagnetic material. In this example system, the magnetic member contains an air gap of uniform length between points a and b. We will assume that the magnetic system (circuit) consists only of the magnetic member and the air gap. Recall that Ampere's law states that the line integral of the field intensity $\mathbf{H}$ about a closed path is equal to the net current enclosed within this closed path of integration. That is,

$$\oint \mathbf{H} \cdot d\mathbf{L} = i_n \tag{1.3-1}$$

where i_n is the net current enclosed. Let us apply Ampere's law to the closed path depicted as a dashed line in Fig. 1.3-1. In particular,

$$\int_a^b H_i dL + \int_b^a H_g dL = Ni \tag{1.3-2}$$

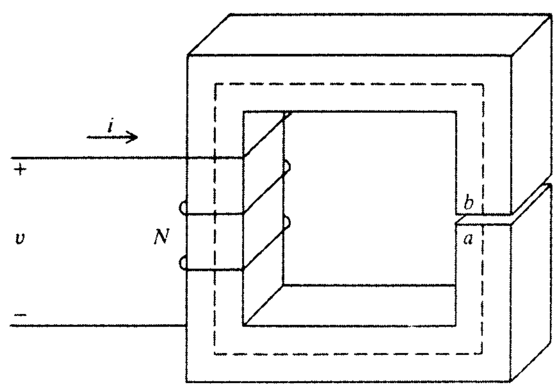

Figure 1.3-1: Elementary magnetic circuit.

where the path of integration is assumed to be in the clockwise direction. This equation requires some explanation. First, we are assuming that the field intensity exists only in the direction of the given path, hence we have dropped the vector notation. The subscript i denotes the field intensity (H_i) in the ferromagnetic material (iron or steel) and g denotes the field intensity (H_g) in the air gap. The path of integration is taken as the mean length about the magnetic member, for purposes we shall explain later. The right-hand side of (1.3-2) represents the net current enclosed. In particular, we have enclosed the current i, N times. This has the units of amperes but is commonly referred to as ampere-turns (At) or magnetomotive force (mmf). We will find that the mmf in magnetic circuits is analogous to the electromotive force (emf) in electric circuits. Note that the current enclosed is positive in (1.3-2) if the current i is positive. The sign of the right-hand side of (1.3-2) may be determined by the so-called "corkscrew" rule. That is, the current enclosed is positive if its assumed positive direction is in the same direction as the advance of a right-hand screw if it were turned in the direction of the path of integration, which in Fig. 1.3-1 is clockwise. Before continuing, it should be mentioned that we refer to **H** as the field intensity; however, some authors prefer to call **H** the field strength.

If we carry out the line integration, (1.3-2) can be written

$$H_i l_i + H_g l_g = Ni \tag{1.3-3}$$

where l_i is the mean length of the magnetic material and l_g is the length across the air gap. Now, we have some explaining to do. We have assumed that the magnetic circuit consists only of the ferromagnetic material and the air gap, and that the magnetic field intensity is always in the direction of the path of integration or, in other words, perpendicular to a cross section of the magnetic material taken in the same sense as the air gap is cut through the material. The assumed direction of the magnetic field intensity is valid except in the vicinity of the corners. The direction of the field intensity changes gradually rather than abruptly at the corners. Nevertheless, the "mean length approximation" is widely used as an adequate means of analyzing this type of magnetic circuit.

Let us now take a cross section of the magnetic material as shown in Fig. 1.3-2. From our study of physics, we know that for linear, isotropic magnetic materials the flux density **B** is related to the field intensity as

$$\mathbf{B} = \mu \mathbf{H} \tag{1.3-4}$$

Where μ is the permeability of the medium. Hence, we can write (1.3-3) in terms of flux density as

$$\frac{B_i}{\mu_i} l_i + \frac{B_g}{\mu_g} l_g = Ni \tag{1.3-5}$$

The surface integral of the flux density is equal to the flux Φ, thus

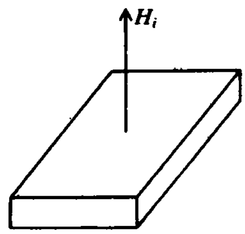

Figure 1.3-2: Cross section of magnetic material.

$$\Phi = \int_A \mathbf{B} \cdot d\mathbf{S} \tag{1.3-6}$$

If we assume that the flux density is uniform over the cross-sectional area, then

$$\Phi_i = B_i A_i \tag{1.3-7}$$

where Φ_i is the total flux in the magnetic material and A_i is the associated cross-sectional area. In the air gap,

$$\Phi_g = B_g A_g \tag{1.3-8}$$

where A_g is the cross-sectional area of the gap. From physics, it is known that the streamlines of flux density $\mathbf{B}$ are closed; hence, the flux in the air gap is equal to the flux in the core. That is, $\Phi_i = \Phi_g$, and, if the air gap is small, $A_i \cong A_g$, and, therefore, $B_i \cong B_g$. However, the effective area of the air gap is larger than that of the magnetic material, since the flux will tend to balloon or spread out (fringing effect), covering a maximum area midway across the air gap. Generally, this is taken into account by assuming that $A_g = kA_i$, where k, which is greater than unity, is determined primarily by the length of the air gap. Although we shall keep this in mind, it is sufficient for our purposes to assume $A_g = A_i$. If we let $\Phi_i = \Phi_g = \Phi$ and substitute (1.3-7) and (1.3-8) into (1.3-5), we obtain

$$\frac{l_i}{\mu_i A_i}\Phi + \frac{l_g}{\mu_g A_g}\Phi = Ni \tag{1.3-9}$$

The analogy to Ohm's law is at hand. Ni (mmf) is analogous to the voltage (emf), and the flux Φ is analogous to the current. We can complete this analogy if we recall that the resistance of a conductor is proportional to its length and inversely proportional to its conductivity and cross-sectional area. Similarly, $l_i/\mu_i A_i$ and $l_g/\mu_g A_g$ are the *reluctances* of the magnetic material and air gap, respectively. Generally, the permeability is expressed in terms of relative permeability as

$$\mu_i = \mu_{ri}\mu_0 \tag{1.3-10}$$

$$\mu_g = \mu_{rg}\mu_0 \tag{1.3-11}$$

where μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ Wb/A·m or $4\pi \times 10^{-7}$ H/m, since Wb/A is a henry) and μ_{ri} and μ_{rg} are the relative permeability of the magnetic material and the air gap, respectively. For all practical purposes, $\mu_{rg} = 1$; however, μ_{ri} may be as large as 500 to 4000 depending upon the type of ferromagnetic material. We will use $\Re$ to denote reluctance so as to distinguish reluctance from resistance, which will be denoted by r or R. We can now write (1.3-9) as

$$(\Re_i + \Re_g)\Phi = Ni \tag{1.3-12}$$

where $\Re_i$ and $\Re_g$ are the reluctance of the iron and air gap, respectively.

Example 1B. A magnetic system is shown in Fig. 1B-1. The total number of turns is 100, the relative permeability of the iron is 1000, and the current is 10 A. Calculate the total flux in the center leg.

Let us draw the electric circuit analog of this magnetic system for which we will need to calculate the reluctance of the various paths:

$$\begin{aligned}\Re_{ab} &= \frac{l_{ab}}{\mu_{ri}\mu_0 A_i} \\ &= \frac{0.22}{1000(4\pi \times 10^{-7})(0.04)^2} = 109,419\ \mathrm{H}^{-1}\end{aligned} \tag{1B-1}$$

Similarly,

$$\Re_{bcda} = \frac{0.25 + 0.22 + 0.25}{(1000)(4\pi \times 10^{-7})(0.04)^2} = 358,099\ \mathrm{H}^{-1} \tag{1B-2}$$

Neglecting the air gap length,

$$\Re_{bef} = \Re_{gha} = \tfrac{1}{2}\Re_{bcda} = 179,049\ \mathrm{H}^{-1} \tag{1B-3}$$

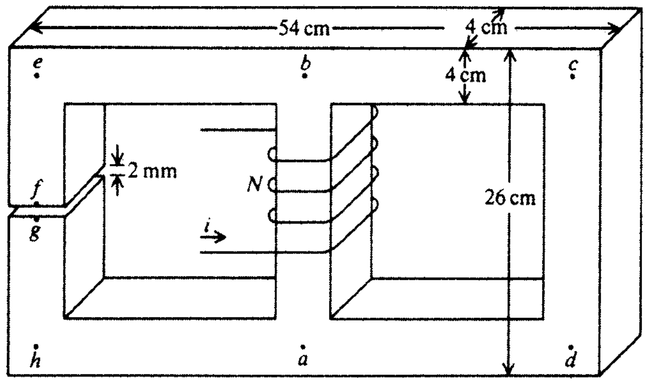

Figure 1B-1: Single-winding magnetic system.

The reluctance of the air gap is

$$\Re_{fg} = \frac{0.002}{(4\pi \times 10^{-7})(0.04)^2} = 994,718 \text{ H}^{-1} \tag{1B-4}$$

The electric circuit analog is given in Fig. 1B-2. The polarity of the mmf is determined by the right-hand rule. That is, if we grasp one of the turns of the winding with our right hand with the thumb pointed in the direction of positive current, then our fingers will point in the direction of positive flux which flows in the direction of an mmf rise. Or if we grasp the winding (center leg) with the fingers of our right hand in the direction of positive current, then our thumb will be in the direction of positive flux and in the direction of a rise in mmf.

We can now apply dc circuit theory to solve for the total flux, $\Phi_1 + \Phi_2$, flowing in the center leg. For example, we can use loop equations or, as we will do here, reduce the series-parallel circuit to an equivalent reluctance. The equivalent reluctance of the parallel combination is

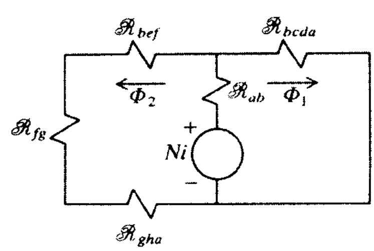

Figure 1B-2: Electric-circuit analog of Fig. 1B-1.

$$\begin{aligned}
\Re_{eq} &= \frac{(\Re_{bcda})(\Re_{bef} + \Re_{fg} + \Re_{gha})}{\Re_{bcda} + \Re_{bef} + \Re_{fg} + \Re_{gha}} \\
&= \frac{(358,099)(179,049 + 994,718 + 179,049)}{358,099 + 179,049 + 994,718 + 179,049} \\
&= \frac{(358,099)(1,352,816)}{1,710,915} = 283,148 \text{ H}^{-1} \qquad \text{(1B-5)}
\end{aligned}$$

$$\begin{aligned}
\Phi_1 + \Phi_2 &= \frac{Ni}{\Re_{ab} + \Re_{eq}} \\
&= \frac{(100)(10)}{109,419 + 283,148} = 2.547 \times 10^{-3} \text{ Wb} \qquad \text{(1B-6)}
\end{aligned}$$

Example 1C. Consider the magnetic system shown in Fig. 1C-1. The windings are supplied from ac sources and, in the steady state, $I_1 = \sqrt{2}\cos\omega_e t$ and $I_2 = \sqrt{2}\,0.3\cos(\omega_e t + 45^\circ)$, where capital letters are used to denote steady-state conditions. $N_1 = 150$ turns, $N_2 = 90$ turns, and $\mu_r = 3000$. Calculate the flux in the center leg.

The electric circuit analog is given in Fig. 1C-2. The reluctance $\Re_x$ is the reluctance of the center leg and $\Re_y$ is the reluctance of one of the two parallel paths from the top of the center leg through an outside leg to the bottom of the center leg. In particular,

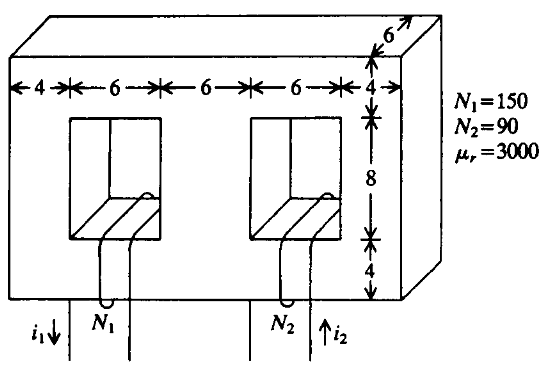

Figure 1C-1: A two-winding magnetic system with dimensions in centimeters.