WITH MULTIMEDIA CD

WILEY

ANTENNA THEOR

ANALYSIS AND **DESIGN**

INCLUDED

CONSTANTINE A. BALANIS

ANTENNA THEORY

ANTENNA THEORY ANALYSIS AND DESIGN

THIRD EDITION

Constantine A. Balanis

A JOHN WILEY & SONS, INC., PUBLICATION

Copyright $©$ 2005 by John Wiley & Sons, Inc. All rights reserved.

Published by John Wiley & Sons, Inc., Hoboken, New Jersey. Published simultaneously in Canada.

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400, fax 978-646-8600, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services please contact our Customer Care Department within the U.S. at 877-762-2974, outside the U.S. at 317-572-3993 or fax 317-572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print, however, may not be available in electronic format.

Library of Congress Cataloging-in-Publication Data is available.

ISBN: 0-471-66782-X

Printed in the United States of America.

10 9 8 7 6 5 4 3 2 1

To the memory of my parents, uncle and aunt

Στη μνήμη των γονέων, του θείου και της θείας μου

<u>|@|@|@|@|@|@|@|@|@|@|@|@|</u>

Contents

CONTENTS **xi**

Appendix III: Cosine and Sine Integrals 1053 Appendix IV: Fresnel Integrals 1057 **Appendix V: Bessel Functions 1063** Appendix VI: Identities 1075 **Appendix VII: Vector Analysis 1079 Appendix VIII: Method of Stationary Phase 1089 Appendix IX: Television, Radio, Telephone, and Radar Frequency Spectrums 1095**

Preface

The third edition of *Antenna Theory* is designed to meet the needs of electrical engineering and physics students at the senior undergraduate and beginning graduate levels, and those of practicing engineers. The text presumes that the students have knowledge of basic undergraduate electromagnetic theory, including Maxwell's equations and the wave equation, introductory physics, and differential and integral calculus. Mathematical techniques required for understanding some advanced topics in the later chapters are incorporated in the individual chapters or are included as appendices.

The third edition has maintained all of the attractive features of the first two editions, including the three-dimensional graphs to display the radiation characteristics of antennas, especially the amplitude patterns. This feature was hailed as an innovative and first of its kind addition in a textbook on antennas. Additional graphs have been added to illustrate features of the radiation characteristics of some antennas. However, there have been many new features added to this edition. In particular,

- A new chapter on *Smart Antennas* (Chapter 16)
- A section on *Fractal Antennas* (Section 11.6)
- Summary tables of important equations in the respective chapters (Chapters 2, 4, 5, 6, 12–14)
- New figures, photos, and tables
- Additional end-of-the-chapter problems
- ž CD with the following *Multimedia Material:*
	- *Power Point* view graphs of lecture notes for each chapter, in multicolor
	- End-of-the-chapter *Interactive Questionnaires* for review (40–65 for each chapter) based on *Java*
	- ž *Animations* based on *Java*
	- ž *Applets* based on *Java*
	- *MATLAB* programs translated from the *FORTRAN* programs of the second edition
	- A number of new *MATLAB* programs
	- *FORTRAN* programs from the second edition.

The CD is attached to the book, and it will open automatically once inserted in the computer. *It is highly recommended that the reader uses the Internet Explorer (IE) to open the Multimedia Material; other browsers may not perform well.* For additional instructions on how to open and use the material in the CD, there is a HELP file in the CD.

The book's main objective is to introduce, in a unified manner, the fundamental principles of antenna theory and to apply them to the analysis, design, and measurements of antennas. Because there are so many methods of analysis and design and a plethora of antenna structures, applications are made to some of the most basic and practical configurations, such as linear dipoles; loops; arrays; broadband, and frequency-independent antennas; aperture antennas; horn antennas; microstrip antennas; and reflector antennas.

A tutorial chapter on Smart Antennas has been included to introduce the student in a technology that will advance antenna theory and design, and revolutionize wireless communications. It is based on antenna theory, digital signal processing, networks and communications. MATLAB simulation software has also been included, as well as a plethora of references for additional reading.

Introductory material on analytical methods, such as the Moment Method and Fourier transform (spectral) technique, is also included. These techniques, together with the fundamental principles of antenna theory, can be used to analyze and design almost any antenna configuration. A chapter on antenna measurements introduces state-of-theart methods used in the measurements of the most basic antenna characteristics (pattern, gain, directivity, radiation efficiency, impedance, current, and polarization) and updates progress made in antenna instrumentation, antenna range design, and scale modeling. Techniques and systems used in near- to far-field measurements and transformations are also discussed.

A sufficient number of topics have been covered, some for the first time in an undergraduate text, so that the book will serve not only as a text but also as a reference for the practicing and design engineer and even the amateur radio buff. These include design procedures, and associated computer programs, for Yagi–Uda and log-periodic arrays, horns, and microstrip patches; synthesis techniques using the Schelkunoff, Fourier transform, Woodward–Lawson, Tschebyscheff, and Taylor methods; radiation characteristics of corrugated, aperture-matched, and multimode horns; analysis and design of rectangular and circular microstrip patches; and matching techniques such as the binomial, Tschebyscheff, T-, gamma, and omega matches.

The text contains sufficient mathematical detail to enable the average undergraduate electrical engineering and physics students to follow, without too much difficulty, the flow of analysis and design. A certain amount of analytical detail, rigor, and thoroughness allows many of the topics to be traced to their origin. My experiences as a student, engineer, and teacher have shown that a text for this course must not be a book of unrelated formulas, and it must not resemble a "cookbook." This book begins with the most elementary material, develops underlying concepts needed for sequential topics, and progresses to more advanced methods and system configurations. Each chapter is subdivided into sections or subsections whose individual headings clearly identify the antenna characteristic(s) discussed, examined, or illustrated.

A distinguished feature of this book is its three-dimensional graphical illustrations from the first edition, which have been expanded and supplemented in the second and third editions. In the past, antenna texts have displayed the three-dimensional energy radiated by an antenna by a number of separate two-dimensional patterns. With the advent and revolutionary advances in digital computations and graphical displays, an additional dimension has been introduced for the first time in an undergraduate antenna text by displaying the radiated energy of a given radiator by a single threedimensional graphical illustration. Such an image, formed by the graphical capabilities of the computer and available at most computational facilities, gives a clear view of the energy radiated in all space surrounding the antenna. It is hoped that this will lead to a better understanding of the underlying principles of radiation and provide a clearer visualization of the pattern formation in all space.

In addition, there is an abundance of general graphical illustrations, design data, references, and an expanded list of end-of-the chapter problems. Many of the principles are illustrated with examples, graphical illustrations, and physical arguments. Although students are often convinced that they understand the principles, difficulties arise when they attempt to use them. An example, especially a graphical illustration, can often better illuminate those principles. As they say, "a picture is worth a thousand words."

Numerical techniques and computer solutions are illustrated and encouraged. A number of MATLAB computer programs are included in the CD attached to the book. Each program is interactive and prompts the user to enter the data in a sequential manner. Some of these programs are translations of the FORTRAN ones that were included in the first and second editions. However, many new ones have been developed. Every chapter, other than Chapters 3 and 17, have at least one MATLAB computer program; some have as many as four. The outputs of the MATLAB programs include graphical illustrations and tabulated results. For completeness, the FORTRAN computer programs are also included, although there is not as much interest in them. The computer programs can be used for analysis and design. Some of them are more of the design type while some of the others are of the analysis type. Associated with each program there is a READ ME file, which summarizes the respective program.

The purpose of the Lecture Notes is to provide the instructors a copy of the text figures and some of the most important equations of each chapter. They can be used by the instructors in their lectures but need to be supplemented with additional narratives. The students can use them to listen to the instructors' lectures, without having to take detailed notes, but can supplement them in the margins with annotations from the lectures. Each instructor will use the notes in a different way.

The Interactive Questionnaires are intended as reviews of the material in each chapter. The student can use them to review for tests, exams, and so on. For each question, there are three possible answers, but only one is correct. If the reader chooses one of them and it the correct answer, it will so indicate. However, if the chosen answer is the wrong one, the program will automatically indicate the correct answer. An explanation button is provided, which gives a short narrative on the correct answer or indicates where in the book the correct answer can be found.

The Animations can be used to illustrate some of the radiation characteristics, such as amplitude patterns, of some antenna types, like line sources, dipoles, loops, arrays, and horns. The Applets cover more chapters and can be used to examine some of the radiation characteristics (such as amplitude patterns, impedance, bandwidth, etc.) of some of the antennas. This can be accomplished very rapidly without having to resort to the MATLAB programs, which are more detailed.

For course use, the text is intended primarily for a two-semester (or two- or threequarter) sequence in antenna theory. The first course should be given at the senior undergraduate level, and should cover most of the material in Chapters 1 through 7, and Chapters 16 and 17. The material in Chapters 8 through 16 should be covered in a beginning graduate-level course. Selected chapters and sections from the book can be covered in a single semester, without loss of continuity. However, it is almost essential that most of the material in Chapters 2 through 6 be covered in the first course and before proceeding to any more advanced topics. To cover all the material of the text in the proposed time frame would be, in some cases, a very ambitious task. Sufficient topics have been included, however, to make the text complete and to give the teacher the flexibility to emphasize, deemphasize, or omit sections or chapters. Some of the chapters and sections can be omitted without loss of continuity.

In the entire book, an *ejωt* time variation is assumed, and it is suppressed. The International System of Units, which is an expanded form of the rationalized MKS system, is used in the text. In some cases, the units of length are in meters (or centimeters) and in feet (or inches). Numbers in parentheses () refer to equations, whereas those in brackets [] refer to references. For emphasis, the most important equations, once they are derived, are boxed. In some of the basic chapters, the most important equations are summarized in tables.

I would like to acknowledge the invaluable suggestions from all those that contributed to the first and second editions, too numerous to mention here. Their names and contributions are stated in the respective editions. It is a pleasure to acknowledge the invaluable suggestions and constructive criticisms of the reviewers of the third edition: Dr. Stuart A. Long of University of Houston, Dr. Christos Christodoulou of University of New Mexico, Dr. Leo Kempel of Michigan State, and Dr. Sergey N. Makarov of Worcester Polytechnic University. There have been many other contributors to this edition, and their contributions are valued and acknowledged. Many graduate and undergraduate students from Arizona State University who have written many of the MATLAB computer programs. Some of these programs were translated from the FORTRAN ones, which appeared in the first and second editions. However a number of entirely new MATLAB programs have been created, which are included for the first time, and do not have a FORTRAN counterpart. The name(s) of the individual contributors to each program is included in the respective program. The author acknowledges Dr. Sava V. Savov of Technical University of Varna, Bulgaria, for the valuable discussions, contributions and figures related to the integration of equation (5-59) in closed form in terms of Bessel functions; Dr. Yahya Rahmat-Samii and Dr. John P. Gianvittorio of UCLA for the figures on Fractal antennas. I would like to thank Craig R. Birtcher of Arizona State University for proofreading part of the manuscript; Bo Yang of Arizona State University for proofreading part of the manuscript, revising a number of the MATLAB programs, and developing the flow chart for accessing the CD Multimedia material; and Razib S. Shishir of Arizona State University for developing all of the Java-based software, including the Interactive Questionnaires, Applets, and Animations. Special thanks to the many companies (Motorola, Inc., Northrop Grumman Corporation, March Microwave Systems, B.V., Ball Aerospace & Technologies Corporation, Samsung, Midland Radio Corporation, Winegard Company, Antenna Research Associates, Inc., Seavey Engineering Associates, Inc., and TCI, A Dielectric Company) for providing photos, illustrations, and copyright permissions. The author acknowledges the long-term friendship and support from Dennis DeCarlo, George C. Barber, Dr. Karl Moeller, Dr. Brian McCabe, Dr. W. Dev Palmer, Michael C. Miller, Frank A. Cansler, and the entire AHE Program membership, too long to be included here. The friendship and collaborative arrangements with Prof. Thodoros D. Tsiboukis and Prof. John N. Sahalos, both from the Aristotle University of Thessaloniki, Greece, are recognized and appreciated. The loyalty and friendship of my graduate students is acknowledged and valued. To all my teachers, thank you. You have been my role models and inspiration.

I am also grateful to the staff of John Wiley & Sons, Inc., especially George Telecki, Associate Publisher, Wiley-Interscience, for his interest, support, cooperation, and production of the third edition; Danielle Lacourciere, Associate Managing Editor, for the production of the book; and Rachel Witmer, Editorial Assistant, for managing the production of the cover. Finally, I must pay tribute to my family (Helen, Renie, and Stephanie) for their support, patience, sacrifice, and understanding for the many hours of neglect during the completion of the first, second, and third editions of this book. It has been a pleasant but daunting task.

> Constantine A. Balanis Arizona State University Tempe, AZ

1.1 INTRODUCTION

An antenna is defined by Webster's Dictionary as "a usually metallic device (as a rod or wire) for radiating or receiving radio waves." The *IEEE Standard Definitions of Terms for Antennas* (IEEE Std 145–1983)[∗] defines the antenna or aerial as "a means for radiating or receiving radio waves." In other words the antenna is the transitional structure between free-space and a guiding device, as shown in Figure 1.1. The guiding device or transmission line may take the form of a coaxial line or a hollow pipe (waveguide), and it is used to transport electromagnetic energy from the transmitting source to the antenna, or from the antenna to the receiver. In the former case, we have a transmitting antenna and in the latter a receiving antenna.

A transmission-line Thevenin equivalent of the antenna system of Figure 1.1 in the transmitting mode is shown in Figure 1.2 where the source is represented by an ideal generator, the transmission line is represented by a line with characteristic impedance Z_c , and the antenna is represented by a load Z_A [$Z_A = (R_L + R_r) + jX_A$] connected to the transmission line. The Thevenin and Norton circuit equivalents of the antenna are also shown in Figure 2.27. The load resistance *RL* is used to represent the conduction and dielectric losses associated with the antenna structure while R_r , referred to as the *radiation resistance*, is used to represent radiation by the antenna. The reactance *XA* is used to represent the imaginary part of the impedance associated with radiation by the antenna. This is discussed more in detail in Sections 2.13 and 2.14. Under ideal conditions, energy generated by the source should be totally transferred to the radiation resistance R_r , which is used to represent radiation by the antenna. However, in a practical system there are conduction-dielectric losses due to the lossy nature of the transmission line and the antenna, as well as those due to reflections (mismatch) losses at the interface between the line and the antenna. Taking into account the internal impedance of the source and neglecting line and reflection (mismatch) losses, maximum

[∗]*IEEE Transactions on Antennas and Propagation*, vols. AP-17, No. 3, May 1969; AP-22, No. 1, January 1974; and AP-31, No. 6, Part II, November 1983.

Antenna Theory: Analysis Design, Third Edition, by Constantine A. Balanis

ISBN 0-471-66782-X Copyright \odot 2005 John Wiley & Sons, Inc.

Figure 1.1 Antenna as a transition device.

power is delivered to the antenna under *conjugate matching*. This is discussed in Section 2.13.

The reflected waves from the interface create, along with the traveling waves from the source toward the antenna, constructive and destructive interference patterns, referred to as *standing waves*, inside the transmission line which represent pockets of energy concentrations and storage, typical of resonant devices. A typical standing wave pattern is shown dashed in Figure 1.2, while another is exhibited in Figure 1.15. If the antenna system is not properly designed, the transmission line could act to a large degree as an energy storage element instead of as a wave guiding and energy transporting device. If the maximum field intensities of the standing wave are sufficiently large, they can cause arching inside the transmission lines.

The losses due to the line, antenna, and the standing waves are undesirable. The losses due to the line can be minimized by selecting low-loss lines while those of

Figure 1.2 Transmission-line Thevenin equivalent of antenna in transmitting mode.

the antenna can be decreased by reducing the loss resistance represented by *RL* in Figure 1.2. The standing waves can be reduced, and the energy storage capacity of the line minimized, by matching the impedance of the antenna (load) to the characteristic impedance of the line. This is the same as matching loads to transmission lines, where the load here is the antenna, and is discussed more in detail in Section 9.7. An equivalent similar to that of Figure 1.2 is used to represent the antenna system in the receiving mode where the source is replaced by a receiver. All other parts of the transmission-line equivalent remain the same. The radiation resistance R_r is used to represent in the receiving mode the transfer of energy from the free-space wave to the antenna. This is discussed in Section 2.13 and represented by the Thevenin and Norton circuit equivalents of Figure 2.27.

In addition to receiving or transmitting energy, an antenna in an advanced wireless system is usually required to *optimize* or *accentuate* the radiation energy in some directions and suppress it in others. *Thus the antenna must also serve as a directional device in addition to a probing device*. It must then take various forms to meet the particular need at hand, and it may be a piece of conducting wire, an aperture, a patch, an assembly of elements (array), a reflector, a lens, and so forth.

For wireless communication systems, the antenna is one of the most critical components. A good design of the antenna can relax system requirements and improve overall system performance. A typical example is TV for which the overall broadcast reception can be improved by utilizing a high-performance antenna. The antenna serves to a communication system the same purpose that eyes and eyeglasses serve to a human.

The field of antennas is vigorous and dynamic, and over the last 60 years antenna technology has been an indispensable partner of the communications revolution. Many major advances that occurred during this period are in common use today; however, many more issues and challenges are facing us today, especially since the demands for system performances are even greater. Many of the major advances in antenna technology that have been completed in the 1970s through the early 1990s, those that were under way in the early 1990s, and signals of future discoveries and breakthroughs were captured in a special issue of the *Proceedings of the IEEE* (Vol. 80, No. 1, January 1992) devoted to Antennas. The introductory paper of this special issue [1] provides a carefully structured, elegant discussion of the fundamental principles of radiating elements and has been written as an introduction for the nonspecialist and a review for the expert.

4 ANTENNAS

Figure 1.3 Wire antenna configurations.

1.2 TYPES OF ANTENNAS

We will now introduce and briefly discuss some forms of the various antenna types in order to get a glance as to what will be encountered in the remainder of the book.

1.2.1 Wire Antennas

Wire antennas are familiar to the layman because they are seen virtually everywhere—on automobiles, buildings, ships, aircraft, spacecraft, and so on. There are various shapes of wire antennas such as a straight wire (dipole), loop, and helix which are shown in Figure 1.3. Loop antennas need not only be circular. They may take the form of a rectangle, square, ellipse, or any other configuration. The circular loop is the most common because of its simplicity in construction. Dipoles are discussed in more detail in Chapter 4, loops in Chapter 5, and helices in Chapter 10.

1.2.2 Aperture Antennas

Aperture antennas may be more familiar to the layman today than in the past because of the increasing demand for more sophisticated forms of antennas and the utilization of higher frequencies. Some forms of aperture antennas are shown in Figure 1.4. Antennas of this type are very useful for aircraft and spacecraft applications, because they can be very conveniently flush-mounted on the skin of the aircraft or spacecraft. In addition, they can be covered with a dielectric material to protect them from hazardous conditions of the environment. Waveguide apertures are discussed in more detail in Chapter 12 while horns are examined in Chapter 13.

1.2.3 Microstrip Antennas

Microstrip antennas became very popular in the 1970s primarily for spaceborne applications. Today they are used for government and commercial applications. These antennas

Figure 1.4 Aperture antenna configurations.

consist of a metallic patch on a grounded substrate. The metallic patch can take many different configurations, as shown in Figure 14.2. However, the rectangular and circular patches, shown in Figure 1.5, are the most popular because of ease of analysis and fabrication, and their attractive radiation characteristics, especially low cross-polarization radiation. The microstrip antennas are low profile, comformable to planar and nonplanar surfaces, simple and inexpensive to fabricate using modern printed-circuit technology, mechanically robust when mounted on rigid surfaces, compatible with MMIC designs, and very versatile in terms of resonant frequency, polarization, pattern, and impedance. These antennas can be mounted on the surface of high-performance aircraft, spacecraft, satellites, missiles, cars, and even handheld mobile telephones. They are discussed in more detail in Chapter 14.

1.2.4 Array Antennas

Many applications require radiation characteristics that may not be achievable by a single element. It may, however, be possible that an aggregate of radiating elements in an electrical and geometrical arrangement (*an array*) will result in the desired

(b) Circular

Figure 1.5 Rectangular and circular microstrip (patch) antennas.

radiation characteristics. The arrangement of the array may be such that the radiation from the elements adds up to give a radiation maximum in a particular direction or directions, minimum in others, or otherwise as desired. Typical examples of arrays are shown in Figure 1.6. Usually the term *array* is reserved for an arrangement in which the individual radiators are separate as shown in Figures 1.6(a –c). However the same term is also used to describe an assembly of radiators mounted on a continuous structure, shown in Figure 1.6(d).

1.2.5 Reflector Antennas

The success in the exploration of outer space has resulted in the advancement of antenna theory. Because of the need to communicate over great distances, sophisticated forms of antennas had to be used in order to transmit and receive signals that had to travel millions of miles. A very common antenna form for such an application is a parabolic reflector shown in Figures 1.7(a) and (b). Antennas of this type have been built with diameters as large as 305 m. Such large dimensions are needed to achieve the high gain required to transmit or receive signals after millions of miles of travel. Another form of a reflector, although not as common as the parabolic, is the corner reflector, shown in Figure 1.7(c). These antennas are examined in detail in Chapter 15.

Figure 1.6 Typical wire, aperture, and microstrip array configurations.

1.2.6 Lens Antennas

Lenses are primarily used to collimate incident divergent energy to prevent it from spreading in undesired directions. By properly shaping the geometrical configuration and choosing the appropriate material of the lenses, they can transform various forms of divergent energy into plane waves. They can be used in most of the same applications as are the parabolic reflectors, especially at higher frequencies. Their dimensions and weight become exceedingly large at lower frequencies. Lens antennas are classified according to the material from which they are constructed, or according to their geometrical shape. Some forms are shown in Figure 1.8 [2].

In summary, an ideal antenna is one that will radiate all the power delivered to it from the transmitter in a desired direction or directions. In practice, however, such ideal performances cannot be achieved but may be closely approached. Various types of antennas are available and each type can take different forms in order to achieve the desired radiation characteristics for the particular application. Throughout the book, the radiation characteristics of most of these antennas are discussed in detail.

1.3 RADIATION MECHANISM

One of the first questions that may be asked concerning antennas would be "how is radiation accomplished?" In other words, how are the electromagnetic fields generated

(b) Parabolic reflector with Cassegrain feed

(c) Corner reflector

(a) Lens antennas with index of refraction $n > 1$

(b) Lens antennas with index of refraction $n < 1$

Figure 1.8 Typical lens antenna configurations. (SOURCE: L. V. Blake, *Antennas*, Wiley, New York, 1966).

by the source, contained and guided within the transmission line and antenna, and finally "detached" from the antenna to form a free-space wave? The best explanation may be given by an illustration. However, let us first examine some basic sources of radiation.

1.3.1 Single Wire

Conducting wires are material whose prominent characteristic is the motion of electric charges and the creation of current flow. Let us assume that an electric volume charge density, represented by q_v (coulombs/m³), is distributed uniformly in a circular wire of cross-sectional area *A* and volume *V* , as shown in Figure 1.9. The total charge *Q* within volume *V* is moving in the *z* direction with a uniform velocity v_z (meters/sec). It can be shown that the current density J_z (amperes/m²) over the cross section of the wire is given by [3]

$$
J_z = q_v v_z \tag{1-1a}
$$

If the wire is made of an ideal electric conductor, the current density J_s (amperes/m) resides on the surface of the wire and it is given by

$$
J_s = q_s v_z \tag{1-1b}
$$

where q_s (coulombs/m²) is the surface charge density. If the wire is very thin (ideally zero radius), then the current in the wire can be represented by

$$
I_z = q_l v_z \tag{1-1c}
$$

where q_l (coulombs/m) is the charge per unit length.

Instead of examining all three current densities, we will primarily concentrate on the very thin wire. The conclusions apply to all three. If the current is time varying, then the derivative of the current of (1-1c) can be written as

$$
\frac{dI_z}{dt} = q_l \frac{dv_z}{dt} = q_l a_z \tag{1-2}
$$

Figure 1.9 Charge uniformly distributed in a circular cross section cylinder wire.

10 ANTENNAS

where $dv_z/dt = a_z$ (meters/sec²) is the acceleration. If the wire is of length *l*, then (1-2) can be written as

$$
l\frac{dI_z}{dt} = lq_l\frac{dv_z}{dt} = lq_l a_z
$$
\n(1-3)

Equation (1-3) is the basic relation between current and charge, and it also serves as the fundamental relation of electromagnetic radiation [4], [5]. It simply states that *to create radiation, there must be a time-varying current or an acceleration (or deceleration) of charge*. We usually refer to currents in time-harmonic applications while charge is most often mentioned in transients. To create charge acceleration (or deceleration) the wire must be curved, bent, discontinuous, or terminated [1], [4]. Periodic charge acceleration (or deceleration) or time-varying current is also created when charge is oscillating in a time-harmonic motion, as shown in Figure 1.17 for a $\lambda/2$ dipole. Therefore:

- 1. If a charge is not moving, current is not created and there is no radiation.
- 2. If charge is moving with a uniform velocity:
	- a. There is no radiation if the wire is straight, and infinite in extent.
	- b. There is radiation if the wire is curved, bent, discontinuous, terminated, or truncated, as shown in Figure 1.10.
- 3. If charge is oscillating in a time-motion, it radiates even if the wire is straight.

A qualitative understanding of the radiation mechanism may be obtained by considering a pulse source attached to an open-ended conducting wire, which may be connected to the ground through a discrete load at its open end, as shown in Figure 1.10(d). When the wire is initially energized, the charges (free electrons) in the wire are set in motion by the electrical lines of force created by the source. When charges are accelerated in the source-end of the wire and decelerated (negative acceleration with respect to original motion) during reflection from its end, it is suggested that radiated fields are produced at each end and along the remaining part of the wire, [1], [4]. *Stronger radiation with a more broad frequency spectrum occurs if the pulses are of shorter or more compact duration while continuous time-harmonic oscillating charge produces, ideally, radiation of single frequency determined by the frequency of oscillation*. The acceleration of the charges is accomplished by the external source in which forces set the charges in motion and produce the associated field radiated. The deceleration of the charges at the end of the wire is accomplished by the internal (self) forces associated with the induced field due to the buildup of charge concentration at the ends of the wire. The internal forces receive energy from the charge buildup as its velocity is reduced to zero at the ends of the wire. Therefore, charge acceleration due to an exciting electric field and deceleration due to impedance discontinuities or smooth curves of the wire are mechanisms responsible for electromagnetic radiation. While both current density $({\bf J}_c)$ and charge density (q_v) appear as source terms in Maxwell's equation, charge is viewed as a more fundamental quantity, especially for transient fields. Even though this interpretation of radiation is primarily used for transients, it can be used to explain steady state radiation [4].