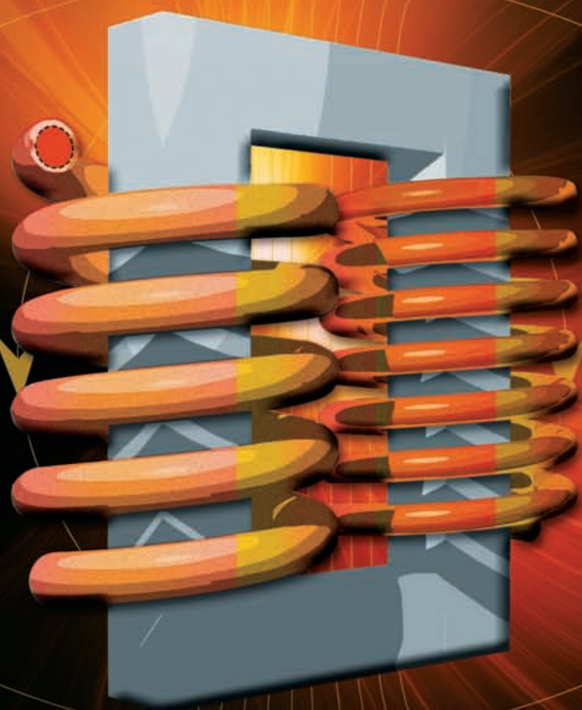
TRANSFORMERS AND INDUCTORS FOR POWER ELECTRONICS

THEORY, DESIGN AND APPLICATIONS



W. G. HURLEY AND W. H. WÖLFLE

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WILEY

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To Our Families

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About the Authors

William Gerard Hurley was born in Cork, Ireland. He received the B.E. degree in Electrical Engineering from the National University of Ireland, Cork in 1974, the M.S. degree in Electrical Engineering from the Massachusetts Institute of Technology, Cambridge MA, in 1976 and the PhD degree at the National University of Ireland, Galway in 1988. He was awarded the D.ENG degree by the National University of Ireland in 2011.

He worked for Honeywell Controls in Canada from 1977–1979, and for Ontario Hydro from 1979–1983. He lectured in electronic engineering at the University of Limerick, Ireland from 1983 to 1991 and is currently Professor of Electrical Engineering at the National University of Ireland, Galway. He is the Director of the Power Electronics Research Centre there. He served on the faculty at the Massachusetts Institute of Technology as a Visiting Professor of Electrical Engineering in 1997–1998. Prof. Hurley has given invited presentations on magnetics in Mexico, Japan, Singapore, Spain, the Czech Republic, Hong Kong, China and USA.

His research interests include high frequency magnetics, power quality, and renewable energy systems. He received a Best Paper Prize for the *IEEE Transactions on Power Electronics* in 2000. Prof. Hurley is a Fellow of the IEEE. He has served as a member of the Administrative Committee of the Power Electronics Society of the IEEE and was General Chair of the Power Electronics Specialists Conference in 2000.

Werner Hugo Wölfle was born in Bad Schussenried, Germany. He graduated from the University of Stuttgart in Germany in 1981 as a Diplom-Ingenieur in Electronics. He completed a PhD degree in Electrical Engineering at the National University of Ireland, Galway in 2003.

He worked for Dornier Systems GmbH from 1982–1985 as a Development Engineer for power converters in space craft applications. From 1986–1988 he worked as a Research and Development Manager for industrial AC and DC power. Since 1989 he has been Managing Director of Convertec Ltd. in Wexford, Ireland, a company of the TRACOPOWER Group. Convertec develops high reliability power converters for industrial applications. He is currently an Adjunct Professor in Electrical Engineering at the National University of Ireland, Galway.

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Foreword

It's too big! It's too hot! It's too expensive! And the litany goes on, recognizable to those of us who have designed inductors and transformers, the bane of power electronics. In writing this book, Professor Hurley and Doctor Wölfle have combined their expertise to produce a resource that, while not guaranteeing freedom from pain, at least provides substantial anaesthesia.

Ger Hurley has been engaged in research, teaching and writing about magnetic analysis and design for almost 40 years, since his time as a graduate student at MIT completing his thesis on induction heating under my supervision. And Werner Wölfle brings to this text, in addition to his extensive industrial experience, the benefit of having been Prof. Hurley's student. So, in some very small way, I take some very small credit for this book.

Today's demands on power electronics are unprecedented and, as their application moves ever further into the commodity marketplace (solar PV converters, EV and hybrid drives, home automation, etc.), the emphases placed on cost and efficiency are driving a sharp focus on the high-cost transformers and inductors in these products. As we venture into design domains, where electroquasistatics no longer obtains, and where the contradictory demands of efficiency and size reduction create an engineering confrontation, we need the guidance that this book provides.

While many books have been written to aid the engineer in the design of magnetics, they almost exclusively present design rules and formulas without exposing the underlying physics that governs their use. Hurley and Wölfle, too, provide formulas and rules, but the emphasis is on understanding the fundamental physical phenomena that lead to them. As we move to higher frequencies, new geometries, new materials and new manufacturing technologies, we can no longer simply find an appropriate formula, go to a catalogue to select a pot core, C-core or E-core, and begin winding. An understanding of electromagnetic fundamentals, modelling and analysis is now critically important to successful design – an understanding that Hurley and Wölfle convey most effectively.

With its comprehensive scope and careful organization of topics, covering fundamentals, high-frequency effects, unusual geometries, loss mechanisms, measurements and application examples, this book is a 'must have' reference for the serious power electronics engineer pursuing designs that are not too big, not too hot and not too expensive. Hurley and Wölfle have produced a text that is destined to be a classic on all our shelves, right next to 'The Colonel's' book[1]. A remarkable achievement.

John G. Kassakian
Professor of Electrical Engineering
The Massachusetts Institute of Technology

[1] McLyman, Colonel W.T. (1978) *Transformer and Inductor Design Handbook*. Marcel Dekker, Inc., New York.

Preface

The design of magnetic components such as transformers and inductors has been of interest to electronic and electrical engineers for many years. Traditionally, treatment of the topic has been empirical, and the 'cook-book' approach has prevailed. In the past, this approach has been adequate when conservative design was acceptable. In recent years, however, space and cost have become premium factors in any design, so that the need for tighter designs is greater. The power supply remains one of the biggest components in portable electronic equipment. Power electronics is an enabling technology for power conversion in energy systems. All power electronic converters have magnetic components in the form of transformers for power transfer and inductors for energy storage.

The momentum towards high-density, high-efficiency power supplies continues unabated. The key to reducing the size of power supplies is high-frequency operation, and the bottleneck is the design of the magnetic components. New approaches are required, and concepts that were hitherto unacceptable to the industry are gaining ground, such as planar magnetics, integrated magnetics and matrix configurations.

The design of magnetic components is a compromise between conflicting demands. Conventional design is based on the premise that the losses are equally divided between the core and the winding. Losses increase with frequency, and high-frequency design must take this into account.

Magnetic components are unique, in that off-the-shelf solutions are not generally available. The inductor is to the magnetic field what the capacitor is to the electric field. In the majority of applications, the capacitor is an off-the-shelf component, but there are several reasons for the lack of standardization in inductors and transformers. In terms of duality, the voltage rating is to the capacitor what the current rating is to the inductor. Dielectric materials used in capacitor manufacture can be chosen so that voltage rating greatly exceeds the design specification without incurring extra cost. In this way, a spectrum of voltage ratings can be covered by a single device.

On the other hand, the current flow in an inductor gives rise to heat loss, which contributes to temperature rise, so that the two specifications are interlinked. This, in turn, determines the size of the conductors, with consequential space implications. Magnetic components are usually the most bulky components in a circuit, so proper sizing is very important.

Returning to the duality analogy, the dielectric material in a capacitor is to the electric field what ferromagnetic material in a magnetic component is to the magnetic field. In general, dielectrics are linear over a very large voltage range and over a very wide frequency range. However, ferromagnetic materials are highly non-linear and can be driven into saturation with small deviations from the design specifications. Furthermore, inductance is a frequency-dependent phenomenon. Dielectric loss does not contribute to temperature rise in a critical way, whereas magnetic core loss is a major source of temperature rise in an inductor.

The totality of the above factors means that magnetic component design is both complex and unique to each application. Failure mechanisms in magnetic components are almost always due to excessive temperature rise, which means that the design must be based on both electrical and thermal criteria. A good designer must have a sound knowledge of circuit analysis, electromagnetism and heat transfer. The purpose of this book is to review the fundamentals in all areas of importance to magnetic component design and to establish sound design rules which are straightforward to implement.

The book is divided into four sections, whose sequence was chosen to guide the reader in a logical manner from the fundamentals of magnetics to advanced topics. It thus covers the full spectrum of material by providing a comprehensive reference for students, researchers and practising engineers in transformer and inductor design.

The Introduction covers the fundamental concepts of magnetic components that serve to underpin the later sections. It reviews the basic laws of electromagnetism, as well as giving a historical context to the book. Self and mutual inductance are introduced and some important coil configurations are analyzed; these configurations form the basis of the practical designs that will be studied later on. The concepts of geometric mean distance and geometric mean radius are introduced to link the formulas for filaments to practical coils with finite wires such as litz wires.

In Section I, the design rules for inductor design are established and examples of different types of inductors are given. The single coil inductor, be it in air or with a ferromagnetic core or substrate, is the energy storage device. A special example is the inductor in a flyback converter, since it has more than one coil. This treatment of the inductor leads on to the transformer in Section II, which has multiple coils and its normal function is to transfer energy from one coil to another.

Section II deals with the general design methodology for transformers, and many examples from rectifiers and switched mode power supplies are given. Particular emphasis is placed on modern circuits, where non-sinusoidal waveforms are encountered and power factor calculations for non-sinusoidal waveforms are covered. In a modern power converter, the transformer provides electrical isolation and reduces component stresses where there is a large input/output conversion ratio. The operation of the transformer at high frequency reduces the overall size of the power supply.

There is an inverse relationship between the size of a transformer and its frequency of operation, but losses increase at high frequency. There is skin effect loss and proximity effect loss in the windings due to the non-uniform distribution of the current in the conductors. The core loss increases due to eddy currents circulating in the magnetic core and also due to hysteresis. General rules are established for optimizing the design of windings under various excitation and operating conditions – in particular, the type of waveforms encountered in switching circuits are treated in detail. A simple, straightforward formula is presented to optimize the thickness of a conducting layer in a transformer winding.

Finally, Section III treats some advanced topics of interest to power supply designers. The authors feel that the book would be incomplete without a section on measurements, a topic that is often overlooked. Advances in instrumentation have given new impetus to accurate measurements. Practitioners are well aware of the pitfalls of incorrect measurement techniques when it comes to inductance, because of the non-linear nature of hysteresis. Planar magnetics have now become mainstream. The incorporation of power supplies into integrated circuits is well established in current practice.

This book is of interest to students of electrical engineering and electrical energy systems – graduate students dealing with specialized inductor and transformer design and practising engineers working with power supplies and energy conversion systems. It aims to provide a clear and concise text based on the fundamentals of electromagnetics. It develops a robust methodology for transformer and inductor design, drawing on historical references. It is also a strong resource of reference material for researchers. The book is underpinned by a rigorous approach to the subject matter, with emphasis on the fundamentals, and it incorporates both depth and breadth in the examples and in setting out up-to-date design techniques.

The accompanying website www.wiley.com/go/hurley_transformers contains a full set of instructors' presentations, solutions to end-of-chapter problems, and digital copies of the book's figures.

Prof. W. G. Hurley and Dr Werner Wölfle
National University of Ireland, Galway, Ireland
March 2013

Nomenclature

The following is a list of symbols used in this book, and their meanings.

A	Average or geometric mean radius
A_c	Cross-sectional area of magnetic core
A_g	Cross-sectional area of the gap
A_L	Inductance per turn
A_m	Effective cross-sectional area of magnetic circuit
A_p	Product of window winding area $\times$ cross-sectional area
A_t	Surface area of wound transformer
A_w	Bare wire conduction area
a	Transformer turns ratio
a_1, a_2	Inside and outside radii of a coil
B_{max}	Maximum flux density
B_o	Optimum flux density
B_{sat}	Saturation flux density
b	Winding dimension: see Figure 6.4
C_{eff}	Effective capacitance of a transformer
D	Duty cycle
d	Thickness of foil or layer
d_1, d_2	Height of filaments or coil centres above ferromagnetic substrate
Φ	Magnetomotive force, mmf
f	Frequency in hertz
G, g	Maximum and minimum air gap lengths
GMD	Geometric mean distance between coils
$g(x)$	Air-gap length at x
h	Winding dimension: see Figure 2.14
h_c	Coefficient of heat transfer by convection
h_1, h_2	Coil heights in axial direction
$\hat{I}$	Peak value of the current waveform
I_{dc}	Average value of current
I_n	RMS value of the nth harmonic of current
$I_n(x)$, $K_n(x)$	Modified Bessel functions of the first and second kind, respectively
I'_{rms}	RMS value of the derivative of the current waveform
I_{rms}	RMS value of the current waveform
J_o	Current density

$J(r)$	Current density at radius r
$J_0(x)$, $J_1(x)$	Bessel functions of the first kind
K_c	Material parameter
$K(f)$, $E(f)$	Complete elliptic integrals of the first and second kind, respectively
K_i	Current waveform factor
K_t	48.2×10^3
K_v	Voltage waveform factor
k	Coupling coefficient
k_a, k_c, k_w	Dimensionless constants (see Equations 3.25, 3.26 and 3.27)
k_f	Core stacking factor A_m/A_c
k_i	Defined in Figure 7.28
k_p	Power factor
k_{pn}	Ratio of the AC resistance to DC resistance at nth harmonic frequency
k_s	Skin-effect factor
k_u	Window utilization factor
L	Self-inductance
L_{eff}	Effective inductance
L_l	Leakage inductance
L_m	Magnetizing inductance
L_s	Additional coil inductance due to ferromagnetic substrate
l_c	Magnetic path length of core
M	Mutual inductance
MLT	Mean length of a turn
m	$\sqrt{(j\omega\mu_0\sigma)}$
N	Number of turns in coil
n	Harmonic number
P_{cu}	Copper or winding loss
P_{fe}	Iron or core loss
P_o	Output power
P_v	Power loss per unit volume
p	Number of layers
R	Average or geometric mean radius
$\mathcal{R}$	Reluctance
R_{ac}	AC resistance of a winding with sinusoidal excitation
R_{dc}	DC resistance of a winding
R_{eff}	Effective AC resistance of a winding, with arbitrary current waveform
R_δ	DC resistance of a winding of thickness δ_0
R_θ	Thermal resistance
r_1, r_2	Inside and outside radii of a coil
r_o	Radius of bare wire
s	Substrate separation in sandwich structure
T	Period of a waveform
T_a	Ambient temperature
$T_{\max}$	Maximum operating temperature
t	Substrate thickness
t_r	Rise time (0–100%)

V_{rms}	RMS value of the voltage waveform
VA	Voltampere rating of winding
V_c	Volume of core
V_o	DC output voltage
V_s	DC input voltage
V_w	Volume of winding
$\langle v \rangle$	Average value of voltage over time τ
W_a	Window winding area of core
W_c	Electrical conduction area
W_m	Stored energy in a magnetic field
w	Winding dimension: see Figure 6.4
Z	Impedance
Z_i	Internal impedance of a conductor
z	Axial separation
α, β	Material constants
α_{20}	Temperature co-efficient of resistivity at 20°C
Δ	Ratio d/δ_0
ΔB	Flux density ripple
ΔT	Temperature rise
ΔV	Output voltage ripple
δ	Skin depth
δ_0	Skin depth at fundamental frequency
δ_n	Skin depth at the *n*th harmonic frequency
ϕ	Flux
$\phi(k)$	Defined in Equation 9.49
ϕ_0	Defined in Equation 9.58
γ	Ratio of iron loss to copper loss
Λ	Defined in Equation 9.36
λ	Flux linkage
μ	Static or absolute permeability
μ_0	Magnetic permeability of free space $4\pi \times 10^{-7}$ H/m
μ_{eff}	Effective relative permeability
μ_i	Initial permeability
μ_{inc}	Incremental permeability
μ_{opt}	Optimum value of effective relative permeability
μ_r	Relative permeability
μ_{rs}	Complex relative permeability
η	Porosity factor
ρ_{20}	Electrical resistivity at 20 °C
ρ_w	Electrical resistivity
σ	Electrical conductivity
τ	Time for flux to go from zero to its maximum value
Ψ	$(5p^2–1)/15$
ω	Angular frequency (rad/s)

1

Introduction

In this chapter, we describe the historical developments that led to the evolution of inductance as a concept in electrical engineering. We introduce the laws of electromagnetism which are used throughout the book. Magnetic materials that are in common use today for inductors and transformers are also discussed.

1.1 Historical Context

In 1820, Oersted discovered that electric current flowing in a conductor produces a magnetic field. Six years later, Ampere quantified the relationship between the current and the magnetic field. In 1831, Faraday discovered that a changing magnetic field causes current to flow in any closed electric circuit linked by the magnetic field, and Lenz showed that there is a relationship between the changing magnetic field and the induced current. Gauss established that magnetic poles cannot exist in isolation. These phenomena established the relationship between electricity and magnetism and became the basis for the science of electromagnetism.

In 1865, Maxwell unified these laws in the celebrated form of Maxwell's equations, which established the basis for modern electrical engineering. He also established the link between phenomena in electromagnetics and electrostatics. Father Nicholas Joseph Callan, who was Professor of Natural Philosophy at the National University of Ireland, Maynooth, in the 1830 s, invented the induction coil. Alexander Anderson was Professor of Natural Philosophy at the National University of Ireland, Galway in the early 1900 s and gave his name to the Anderson Bridge for measuring inductance.

These individuals provide the inspiration for a textbook on magnetic design that focuses on the issues that arise in power electronics. Power electronics is an enabling technology for modern energy conversion systems and inductors and transformers are at the heart of these systems.

Figure 1.1 shows a straight conductor carrying a current, i. The presence of the magnetic field is detected by placing a freely-suspended magnet in the vicinity of the conductor. The direction of the magnetic field (a vector) is given by the direction in which the north pole of the search magnet points. It turns out that the magnitude of the magnetic field is constant on any circle concentric with the conductor, and its direction is tangential to that circle, given by

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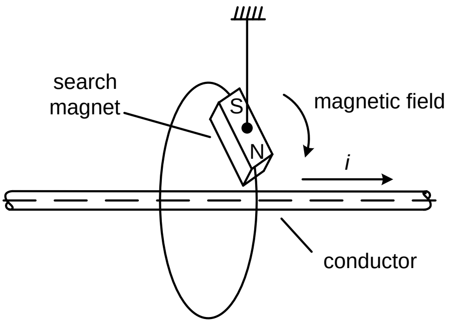

Figure 1.1 Magnetic field created by a current.

the right hand rule – that is, a conventional (right-handed) cork screw will rotate in the direction of the magnetic field if it is driven in the direction of the current flow. It also turns out that the magnitude of the magnetic field is proportional to the current in the conductor and is inversely proportional to the radial distance from the conductor axis.

The magnetic field around a straight conductor is illustrated in Figure 1.2. The direction of the magnetic field as shown complies with the right hand screw rule. An alternative to the right hand screw rule for establishing the direction of the magnetic field created by the current is to point the thumb of your right hand along the conductor in the direction of the current flow, and your fingers will wrap themselves around the conductor in the direction of the magnetic field. The higher density of the lines near the conductor indicates a stronger magnetic field in this area.

The magnetic field around the current carrying conductor is described by two vector quantities: the magnetic flux density **B** and the magnetic field intensity **H**.

The magnetic field intensity **H** is best explained by Ampere's law, which expresses these observations about the current-carrying conductor in their most general form:

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J}_f \cdot \mathbf{n} da \tag{1.1}$$

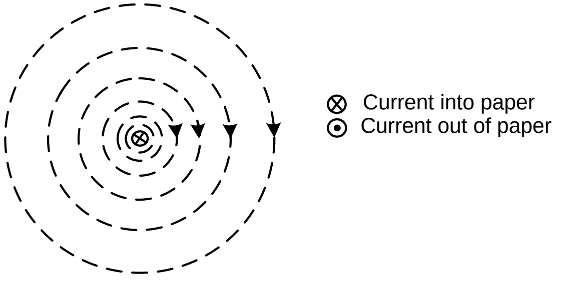

Figure 1.2 Magnetic field around a current-carrying conductor.