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RELIABILITY ENGINEERING

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RELIABILITY ENGINEERING

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RELIABILITY ENGINEERING

Second Edition

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PREFACE

Reliability is one of the most important quality characteristics of components, products, and large and complex systems. Reliability is important to each one of us, every day, when we start a vehicle, attempt to place a phone call, or use a copier, a computer, or a fax machine. In all instances, the user expects the machine or the system to provide the designed functions when requested. As you probably have experienced, machines do not always function or deliver the desired quality of service when needed. Machines also experience failures and interruption, if not termination of service.

Engineers spend a significant amount of time and resources during the design, product (or service) development, and production phases of the product life cycle to ensure that the product or system will provide the desired service level. In doing so, engineers start with a concept design, select its components, test its functionality, and estimate its reliability. Modifications and design changes are usually made and these steps are repeated until the product (or service) satisfies its requirements. The prelude of this book presents these steps in the design of the "One-Hoss-Shay."

Designing the product may require redundancy of components (or subsystems), introduction of newly developed components or materials, or changes in design configuration. These will have a major impact on the product reliability. Once the product is launched and used in the field, data are collected so improvements can be made in the newer versions of the product. Moreover, these data become important in identifying potential safety issues or hazards for the users so recalls can be quickly made to resolve these issues. In other words, reliability is a major concern during the entire life of the product and is subject to continual improvements.

This book is an *engineering* reliability book. It is organized according to the same sequence followed when designing a product or service. The book consists of three parts. Part I focuses on system reliability estimation for time-independent and time-dependent models. Chapter 1 focuses on the basic definitions of reliability, its metrics, and methods for its calculations. Extensive coverage of different hazard functions is given. Chapter 2 describes, in greater detail, methods for estimating reliabilities of a variety of engineering systems configurations starting with series systems, parallel systems, series-parallel, parallel-series, consecutive *k*-out-of-*n* : *F*, *k*-out-of-*n*, and complex network systems. It also addresses systems with multistate devices and concludes by estimating reliabilities of redundant systems and the optimal allocation of components in a redundant system. The next step in product design is to study the effect of time on system reliability. Therefore, Chapter 3 discusses, in detail, time- and failure-dependent reliability and the calculation of mean time to failure of a variety of system configurations. It also introduces availability as a measure of system reliability.

PREFACE

Once the design is "firm," the engineer assembles the components and configures them to achieve the desired reliability objectives. This may require conducting reliability tests on components or using field data from similar components. Therefore, Part II of the book, starting with Chapter 4, presents the concept of constructing the likelihood function and its use in estimating the parameters of a failure time distribution. Chapter 5 provides a comprehensive coverage of parametric and nonparametric reliability models for failure data. The extensive examples and methodologies presented in this chapter will aid the engineer in appropriately modeling the test data. Confidence intervals for the parameters of the models are also discussed. More important, the book devotes all of Chapter 6 to accelerated life testing and degradation testing. The main objective of this chapter is to provide varieties of statistical based models, physics-statistics based models, and physics-experimental based models to relate the failure time and data at accelerated conditions to the normal operating conditions at which the product is expected to operate.

Finally, once a product is produced and sold, the manufacturer must ensure its reliability objectives by providing preventive and scheduled maintenance and warranty policies. Part III of the book focuses on these topics. It begins with Chapter 7, which presents different methods (exact and approximate) for estimating the expected number of system failures during a specified time interval. These estimates are used in Chapter 8 in order to determine optimal preventive maintenance schedules and optimum inspection policies. Methods for estimating the inventory levels of spares required to ensure predetermined reliability and availability values are also presented. Finally, Chapter 9 presents different warranty policies and approaches for determining the product price, including warranty cost as well as the estimation of the warranty reserve fund.

Chapter 10 concludes the book. It presents actual case studies that demonstrate the use of the approaches and methodologies discussed throughout the book in solving real cases. The role of reliability during the design phase of a product or a system is particularly emphasized.

Every theoretical development in this book is followed by an engineering example to illustrate its application. Moreover, many problems are included at the end of each chapter. These two features increase the usefulness of this book as a comprehensive reference for practitioners and professionals in the quality and reliability engineering area. In addition, this book may be used for either a one- or two-semester course in reliability engineering geared toward senior undergraduates or graduate students in industrial and systems, mechanical, and electrical engineering programs. It can also be adapted for use in a life data analysis course in a graduate program in statistics. The book presumes a background in statistics and probability theory and differential calculus.

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This book represents the work of not just the author, but also many others whose work is referenced throughout the book. I have tried to give adequate credit to all whose work has influenced this book. Particular acknowledgment is made to the Institute of Electrical and Electronic Engineers, CRC Press, Institute of Mathematical Statistics, American Society of Mechanical Engineers, Seimens AG, Electronic Products, and Elsevier Applied Science Publishers for the use of figures, tables in the appendices, and permission to include material in this book.

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E.A. Elsayed Piscataway, New Jersey

PRELUDE

DESIGN FOR RELIABILITY: A LOGICAL STORY

"The Deacon's Masterpiece, or The Wonderful One-Hoss-Shay" is a perfectly logical story that demonstrates the concept of designing a product for reliability. It starts by defining the objective of the product or service to be provided. The reliability structure of the system is then developed and its components and subsystem are selected. A prototype is constructed and tested. The failure data of the components are collected and analyzed. The system is then redesigned and retested until its reliability objectives are achieved. This is indeed what is considered today as "reliability growth." These logical steps are elegantly described below.

THE DEACON'S MASTERPIECE,

or The Wonderful One-Hoss-Shay¹

I. System's Objective and Structural Design

Have you heard of the wonderful one-hoss-shay, It ran a hundred years to a day, And then, of a sudden, it—ah, but stay, I'll tell you what happened without delay, Scaring the parson into fits, Frightening people out of their wits,— Have you ever heard of that, I say?

Seventeen hundred and fifty-five. Georgius Secundus was then alive,— Snuffy old drone from the German hive. That was the year when Lisbon-town Saw the earth open and gulp her down, And Braddock's army was done so brown, Left without a scalp to its crown. It was on the terrible Earthquake-day That the Deacon finished the one-hoss-shay.

¹ Oliver Wendell Holmes, "The Deacon's Masterpiece," in *The Complete Poetical Works of Oliver Wendell Holmes*, Fourth Printing, Houghton Mifflin, 1908.

II. System Prototyping and Analysis of Failure Observations

Holmes' preface to the poem:

Observation shows us in what point any particular mechanism is most likely to give way. In a wagon, for instance, the weak point is where the axle enters the hub or nave. When the wagon breaks down, three times out of four, I think, it is at this point that the accident occurs. The workman should see to it that this part should never give way, then find the next vulnerable place, and so on, until he arrives logically at the perfect result attained by the deacon.

This is a continuation of reliability growth methodology.

Now in building of chaises, I tell you what, There is always <u>somewhere</u> a weakest spot,— In hub, tire, felloe, in spring or thill, In panel, or crossbar, or floor, or sill, In screw, bolt, thoroughbrace,—lurking still, Find it somewhere you must and will,— Above or below, or within or without,— And that's the reason, beyond a doubt, That a chaise <u>breaks down</u>, but doesn't <u>wear out</u>.

But the Deacon swore (as Deacons do, With an "I dew vum," or an "I tell <u>yeou</u>") He would build one shay to beat the taown 'N' the keounty 'n' all the kentry raoun'; It should be so built that it <u>couldn</u>' break daown: "Fur," said the Deacon, "t's mighty plain Thut the weakes' place mus' stan' the strain; 'N' the way t' fix it, uz I maintain, Is only jest T' make that place uz strong uz the rest."

III. Design Changes and System Improvement

So the Deacon inquired of the village folk Where he could find the strongest oak, That couldn't be split nor bent nor broke,— That was for spokes and floor and sills; He sent for lancewood to make the thills; The crossbars were ash, from the straightest trees, The panels of white-wood, that cuts like cheese, But last like iron for things like these; The hubs of logs from the "Settler's ellum,"—

Last of its timber,—they couldn't sell 'em, Never an axe had seen their chips,

PRELUDE

And the wedges flew from between their lips, Their blunt ends frizzled like celery-tips; Step and prop-iron, bolt and screw, Spring, tire, axle, and linchpin too, Steel of the finest, bright and blue; Thoroughbrace bison-skin, thick and wide; Boot, top, dasher, from tough old hide Found in the pit when the tanner died. That was the way he "put her through." "There!" said the Deacon, "naow she'll dew!"

Do! I tell you, I rather guess She was a wonder, and nothing less! Colts grew horses, beards turned gray, Deacon and deaconess dropped away, Children and grandchildren—where were they? But there stood the stout old one-hoss-shay As fresh as on Lisbon-earthquake-day!

IV. System Monitoring During Operation

EIGHTEEN HUNDRED;—it came and found The Deacon's masterpiece strong and sound. Eighteen hundred increased by ten;— "Hahnsum kerridge" they called it then. Eighteen hundred and twenty came;— Running as usual; much the same. Thirty and forty as last arrive, And then come fifty, and FIFTY-FIVE.

Little of all we value here Wakes on the morn of its hundredth year Without both feeling and looking queer. In fact, there's nothing that keeps its youth, So far as I know, but a tree and truth. (This is a moral that runs at large; Take it. —You're welcome. —No extra charge.)

V. System Aging, Wear Out, and Replacement

FIRST OF NOVEMBER,—the Earthquake-day,— There are traces of age in the one-hoss-shay, A general flavor of mild decay, But nothing local, as one may say. There couldn't be,—for the Deacon's art Had made it so like in every part That there wasn't a chance for one to start.

PRELUDE

For the wheels were just as strong as the thills, And the floor was just as strong as the sills, And the panels just as strong as the floor, And the whipple-tree neither less nor more, And the back crossbar as strong as the fore, And spring and axle and hub <u>encore</u>. And yet, <u>as a whole</u>, it is past a doubt In another hour it will be <u>worn out</u>!

VI. System Reaches Its Expected Life

First of November, 'Fifty-five! This morning the parson takes a drive. Now, small boys, get out of the way! Here comes the wonderful one-hoss-shay, Drawn by a rat-tailed, ewe-necked bay. "Huddup!" said the parson.—Off went they. The parson was working his Sunday's text,-Had got to fifthly, and stopped perplexed At what the—Moses—was coming next. All at once the horse stood still, Close by the meet'n'-house on the hill. First a shiver, and then a thrill, Then something decidedly like a spill,-And the parson was sitting upon a rock, At half past nine by the meet'n'-house clock,— Just the hour of the Earthquake shock! What do you think the parson found, When he got up and stared around? The poor old chaise in a heap or mound, As if it had been to the mill and ground! You see, of course, if you're not a dunce, How it went to pieces all at once,-All at once, and nothing first,— Just as bubbles do when they burst.

End of the wonderful one-hoss-shay. Logic is logic. That's all I say.

CHAPTER

RELIABILITY AND HAZARD FUNCTIONS

1.1 INTRODUCTION

One of the quality characteristics that consumers require from the manufacturer of products is reliability. Unfortunately, when consumers are asked what reliability means, the response is usually unclear. Some consumers may respond by stating that the product should always work properly without failure or by stating that the product will always function properly when required for use, while others will completely fail to explain what reliability means to them.

What is reliability from your viewpoint? Take, for instance, the example of starting your car. Would you consider your car reliable if it starts immediately? Would you still consider your car reliable if it takes you two times to turn on the ignition key for the car to start? How about three times? As you can see, without quantification, it becomes more difficult to define or measure reliability. We define reliability later in this chapter, but for now, to further illustrate the importance of reliability as a field of study and research, we present the following cases.

On April 9, 1963, the USS *Thresher*, a nuclear submarine, slipped beneath the surface of the Atlantic and began a run for deep waters (1000 feet below surface). *Thresher* exceeded its maximum test depth and imploded. Its hull collapsed, causing the death of 129 crewmembers and civilians. It should be noted that the *Thresher* had been the most advanced submarine of its day, with a destructive power beyond that of the Navy's entire submarine force in World War II. Though it was designed to sustain stresses at this depth, it failed catastrophically.

In 1979, a DC-10 commercial aircraft crashed, killing all passengers aboard. The cause of failure was poor maintenance procedure. The engineers specified that the engine should have been taken off before the engine mounting assembly, because of the excessive weight of the engines. Apparently, those guidelines were not followed when maintenance was conducted, causing excessive stresses and forces that cracked the engine mounts.

On December 2, 1982, a team of doctors and engineers at Salt Lake City, Utah, performed an operation to replace a human heart by a mechanical one—the Jarvik heart. Two days later, the patient underwent further operations due to a malfunction of the valve of the mechanical heart. Here, a failure of the system may directly affect one human life at a time. In January 1990, the Food and Drug Administration stunned the medical community by recalling the world's first artificial heart because of deficiencies in manufacturing quality, training, and other areas. This heart affected the lives of 157 patients over an eight-year period. Now, consider the following case, where the failures of the systems have a much greater effect.

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CHAPTER 1 RELIABILITY AND HAZARD FUNCTIONS

On April 26, 1986, two explosions occurred at the newest of the four operating nuclear reactors at the Chernobyl site in the former USSR. It was the worst commercial disaster in the history of the nuclear industry. A total of 31 site workers and members of the emergency crew died as a result of the accident. About 200 people were treated for symptoms of acute radiation syndrome. Economic losses were estimated at \$3 billion, and the full extent of the long-term damage has yet to be determined.

More recently, on July 25, 2000, a Concorde aircraft while taking off at a speed of 175 knots ran over a strip of metal from a DC-10 airplane, which had taken off a few minutes before. This strip cut the tire on wheel No. 2 of the left landing gear resulting in one or more pieces of the tire, which were thrown against the underside wing fuel tank. This led to the rupture of the tank causing fuel leakage and consequently resulting in a fire in the landing gear system. Fire spread to both engines of the aircraft causing loss of power and crash of the aircraft. Clearly, such field condition was not considered in the design process. This type of failure has ended the operation of the Concorde fleet indefinitely.

The explosions of the space shuttle *Challenger* in 1986 and the space shuttle *Columbia* in 2003, as well as the loss of the two external fuel tanks of the space shuttle *Columbia* in an earlier flight (at a cost of \$25 million each), are other examples of the importance of reliability in the design, operation, and maintenance of critical and complex systems. Indeed, field conditions similar to those of the Concorde aircraft have lead to the failure of the *Columbia*. The physical cause of the loss of *Columbia* and its crew was a breach in the Thermal Protection System of the leading edge of the left wing. The breach was initiated by a piece of insulating foam that separated from the left bipod ramp of the External Tank and struck the wing in the vicinity of the lower half of Reinforced Carbon-Carbon panel 8 at 81.9 seconds after launch. During the reentry, reheated air penetrated the leading-edge insulation and progressively melted the aluminum structure until increasing aerodynamic forces caused loss of control, failure of the wing, and breakup of the Orbiter (Walker and Grosch, 2004).

Reliability plays an important role in the service industry. For example, to provide virtually uninterrupted communications for its customers, American Telephone and Telegraph Company (AT&T) installed the first transatlantic cable with a reliability goal of a maximum of one failure in 20 years of service. The cable surpassed the reliability goal and was replaced by new fiber optic cables for economic reasons. The reliability goal of the new cables is one failure in 80 years of service!

Another example of the reliability role in structural design is illustrated by the Point Pleasant Bridge (West Virginia/Ohio border), which collapsed on December 15, 1967, causing the death of 46 persons and the injuries of several dozen persons. The failure was attributed to the metal fatigue of a crucial eyebar, which started a chain reaction of one structural member falling after another. The bridge failed before its designed life.

The failure of a system can have a widespread effect and a far reaching impact on many users and on the society as a whole. On August 14, 2003, the largest power blackout in North American history affected eight U.S. states and the Province of Ontario, leaving up to 50 million people with no electricity. Controllers in Ohio, where the blackout started, were overextended, lacked vital data, and failed to act appropriately on outages that occurred more than an hour before the blackout. When energy shifted from one transmission line to another, overheating caused lines to sag into a tree. The snowballing cascade of shunted power that rippled across the Northeast in seconds would not have happened had the grid not been operating so near to

RELIABILITY DEFINITION AND ESTIMATION

its transmission capacity and assessment of the entire power network reliability when operating at its peak capacity were carefully estimated (The Industrial Physicist, 2003; U.S.-Canada Power System Outage Task Force, 2004).

Most of the above examples might imply that failures and their consequences are due to hardware. However, many systems' failures are due to human errors and software failures. For example, the Therac-25, a computerized radiation therapy machine, massively overdosed patients at least six times between June 1985 and January 1987. Each overdose was several times the normal therapeutic dose and resulted in the patient's severe injury or even death (Leveson and Turner, 1993). Overdoses, although they sometimes involved operator error, occurred primarily because of errors in the Therac-25's software and because the manufacturer did not follow proper software engineering practices. Other software errors might result from lack of validation of the input parameters. For example, in 1998, a crew member of the guided-missile cruiser USS Yorktown mistakenly entered a zero for a data value, which resulted in a division by zero. The error cascaded and eventually shut down the ship's propulsion system. The ship was dead in the water for several hours because a program did not check for valid input.

Another recent example of software reliability includes the Mars Polar Lander which was launched in January 1999 and was intended to land on Mars in December of that year. Legs were designed to deploy prior to landing. Sensors would detect touchdown and turn off the rocket motor. It was known and understood that the deployment of the landing legs generated spurious signals of the touchdown sensors. The software requirements, however, did not specifically describe this behavior and the software designers therefore did not account for it. The motor turned off at too high an altitude and the probe crashed into the planet at 50 mi/h and was destroyed. Mission costs exceeded \$120 million (Gruhn, 2004). Reliability also has a great effect on the consumers' perception of a manufacturer. For example, consumers' experiences with car recalls, repairs, and warranties will determine the future sales and survivability of that manufacturer. Most manufactures have experienced car recalls and extensive warranties that range from as low as 1.2% to 6% of the revenue. Some car recalls are extensive and costly such as the recall of 8.6 million cars due to the ignition causing small engine fires. In 2010, an extensive recall of several car models due to sudden acceleration resulted in the shutdown of the entire production system and hundreds of lawsuits. One of the causes of the recall is lack of thoroughness in testing new cars and car parts under varying weather conditions; the gas-pedal mechanism tended to stick more as humidity increased. Clearly, the number and magnitude of the recalls are indicative of the reliability performance of the car and potential survivability of the manufacturer.

1.2 RELIABILITY DEFINITION AND ESTIMATION

A formal definition of reliability is given as follows:

1.2.1 Reliability

Reliability is the probability that a product will operate or a service will be provided properly for a specified period of time (design life) under the design operating conditions (such as temperature, load, volt ...) without failure.

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In other words, reliability may be used as a measure of the system's success in providing its function properly during its design life. Consider the following.

Suppose n_o identical components are subjected to a design operating conditions test. During the interval of time $(t - \Delta t, t)$, we observed $n_f(t)$ failed components, and $n_s(t)$ surviving components $[n_f(t) + n_s(t) = n_o]$. Since reliability is defined as the cumulative probability function of success, then at time *t*, the reliability R(t) is

$$R(t) = \frac{n_s(t)}{n_s(t) + n_f(t)} = \frac{n_s(t)}{n_o}.$$
(1.1)

In other words, if T is a random variable denoting the time to failure, then the reliability function at time t can be expressed as

$$R(t) = P(T > t). \tag{1.2}$$

The cumulative distribution function (CDF) of failure F(t) is the complement of R(t), that is,

$$R(t) + F(t) = 1. \tag{1.3}$$

If the time to failure, *T*, has a probability density function (p.d.f.) f(t), then Equation 1.3 can be rewritten as

$$R(t) = 1 - F(t) = 1 - \int_0^t f(\zeta) d\zeta.$$
(1.4)

Taking the derivative of Equation 1.4 with respect to t, we obtain

$$\frac{dR(t)}{dt} = -f(t). \tag{1.5}$$

For example, if the time to failure distribution is exponential with parameter λ , then

$$f(t) = \lambda e^{-\lambda t},\tag{1.6}$$

and the reliability function is

$$R(t) = 1 - \int_0^t \lambda e^{-\lambda \zeta} d\zeta = e^{-\lambda t}.$$
(1.7)

From Equation 1.7, we express the probability of failure of a component in a given interval of time $[t_1, t_2]$ in terms of its reliability function as

$$\int_{t_1}^{t_2} f(t)dt = R(t_1) - R(t_2).$$
(1.8)

We define the failure rate in a time interval $[t_1, t_2]$ as the probability that a failure per unit time occurs in the interval given that no failure has occurred prior to t_1 , the beginning of the interval. Thus, the failure rate is expressed as

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$$\frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)}.$$
(1.9)

If we replace t_1 by t and t_2 by $t + \Delta t$, then we rewrite Equation 1.9 as

$$\frac{R(t) - R(t + \Delta t)}{\Delta t R(t)}.$$
(1.10)

The hazard function is defined as the limit of the failure rate as Δt approaches zero. In other words, the hazard function or the instantaneous failure rate is obtained from Equation 1.10 as

$$h(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{\Delta t R(t)} = \frac{1}{R(t)} \left[-\frac{d}{dt} R(t) \right]$$

or

 $h(t) = \frac{f(t)}{R(t)}.$ (1.11)

From Equations 1.5 and 1.11, we obtain

$$R(t) = e^{\left[-\int_0^t h(\zeta) d\zeta\right]},$$
(1.12)

$$R(t) = 1 - \int_0^t f(\zeta) d\zeta,$$
 (1.13)

and

$$h(t) = \frac{f(t)}{R(t)}.$$
 (1.14)

Equations 1.5, 1.12–1.14 are the key equations that relate f(t), F(t), R(t), and h(t).

The following example illustrates how the hazard rate, h(t), and reliability are estimated from failure data.

EXAMPLE 1.1

A manufacturer of light bulbs is interested in estimating the mean life of the bulbs. Two hundred bulbs are subjected to a reliability test. The bulbs are observed, and failures in 1000-h intervals are recorded as shown in Table 1.1.

Plot the failure density function estimated from data $f_e(t)$, the hazard-rate function estimated from data $h_e(t)$, the cumulative probability function estimated from data $F_e(t)$, and the reliability function estimated from data $R_e(t)$. The subscript *e* refers to *estimated*. Comment on the hazard-rate function.

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Time interval (hours)	Failures in the interval
0–1000	100
1001–2000	40
2001–3000	20
3001–4000	15
4001–5000	10
5001-6000	8
6001–7000	7
Total	200

TABLE 1.1 Number of Failures in the Time Intervals

SOLUTION

We estimate $f_e(t)$, $h_e(t)$, $R_e(t)$, and $F_e(t)$ using the following equations:

$$f_e(t) = \frac{n_f(t)}{n_o \Delta t},\tag{1.15}$$

$$h_e(t) = \frac{n_f(t)}{n_s(t)\Delta t},\tag{1.16}$$

$$R_{e}(t) = \frac{f_{e}(t)}{h_{e}(t)} = \frac{n_{s}(t)}{n_{o}},$$
(1.17)

and

$$F_e(t) = 1 - R_e(t). \tag{1.18}$$

Note that $n_s(t)$ is the number of surviving units at the beginning of the period Δt . Summaries of the calculations are shown in Tables 1.2 and 1.3. The plots are shown in Figures 1.1 and 1.2.

Time interval (h)	Failure density $f_e(t) \times 10^{-4}$	Hazard rate $h_e(t) \times 10^{-4}$
0–1000	$\frac{100}{200 \times 10^3} = 5.0$	$\frac{100}{200 \times 10^3} = 5.0$
1001–2000	$\frac{40}{200 \times 10^3} = 2.0$	$\frac{40}{100 \times 10^3} = 4.0$
2001–3000	$\frac{20}{200 \times 10^3} = 1.0$	$\frac{20}{60 \times 10^3} = 3.33$
3001-4000	$\frac{15}{200 \times 10^3} = 0.75$	$\frac{15}{40 \times 10^3} = 3.75$
4001–5000	$\frac{10}{200 \times 10^3} = 0.5$	$\frac{10}{25 \times 10^3} = 4.0$
5001-6000	$\frac{8}{200 \times 10^3} = 0.4$	$\frac{8}{15 \times 10^3} = 5.3$
6001–7000	$\frac{7}{200 \times 10^3} = 0.35$	$\frac{7}{7 \times 10^3} = 10.0$

TABLE 1.2 Calculations of $f_{e}(t)$ and $h_{e}(t)$

TABLE 1.3	Calculations	of	$R_{\rm e}(t)$	and	$F_{e}(t)$	
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Time interval	Reliability $R_e(t) = f_e(t)/h_e(t)$	Unreliability $F_e(t) = 1 - R_e(t)$
0–1000	$\frac{5.0}{5.0} = 1.000$	0.000
1001–2000	$\frac{2.0}{4.0} = 0.500$	0.500
2001–3000	$\frac{1.0}{3.33} = 0.300$	0.700
3001-4000	$\frac{0.75}{3.75} = 0.200$	0.800
4001–5000	$\frac{0.5}{4.0} = 0.125$	0.875
5001–6000	$\frac{0.4}{5.3} = 0.075$	0.925
6001–7000	$\frac{0.35}{10.0} = 0.035$	0.965

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As shown in Figure 1.1, the hazard rate is constant until time of 5000 h and then increases linearly with *t*. Thus, $h_e(t)$ can be expressed as

$$h_e(t) = \begin{cases} \lambda_0 & 0 \le t \le 6,000\\ \lambda_1 t & t > 6,000 \end{cases},$$

where λ_0 and λ_1 are constants.

The above example shows the hazard-rate function is constant for a period of time and then linearly increases with time. In other situations, the hazard-rate function may be decreasing, constant, or increasing, and the rate at which the function decreases or increases may be constant, linear, polynomial, or exponential with time. The following example is an illustration of an exponentially increasing hazard-rate function.

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EXAMPLE 1.2

Facsimile (fax) machines are designed to transmit documents, figures, and drawings between locations via telephone lines. The principle of a fax machine is shown in Figure 1.3. The document on the sending unit drum is scanned in both the horizontal and rotating directions. The document is divided into graphic elements, which are converted into electrical signals by a photoelectric reading head. The signals are transmitted via telephone lines to the receiving end where they are demodulated and reproduced by a recording head.

The quality of the received document is affected by the reliability of the photoelectric reading head in converting the graphic elements of the document being sent into proper electrical signals. A manufacturer of fax machines performs a reliability test to estimate the mean life of the reading head by subjecting 180 heads to repeated cycles of readings. The threshold times, at which the quality of the received document is unacceptable, are recorded in Table 1.4.

Estimate the hazard rate and reliability function of the machines.



FIGURE 1.3 The principle of a fax machine.

TABLE 1.4	Failure Data of	the Facsimile	Machines
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Time interval (hours)	0–150	151–300	301–450	451–600	601–750	751–900
Number of failures	20	28	27	32	33	40

SOLUTION

Using Equations 1.15–1.17, we calculate $f_e(t)$, $h_e(t)$, and $R_e(t)$ as shown in Table 1.5. Plots of the hazard rate and the reliability function are shown in Figures 1.4 and 1.5, respectively.