CURRENT INTERRUPTION TRANSIENTS CALCULATION

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Current Interruption Transients Calculation is a comprehensive resource for the understanding, calculation and analysis of the transient recovery voltages (TRVs) and related re-ignition or re-striking transients associated with fault current interruption and the switching of inductive and capacitive load currents in circuits.

This book provides an original, detailed and practical description of current interruption transients, origins and the circuits involved, and how they can be calculated and then analyzed using only a hand calculator and a spreadsheet program.

Key features:

- Detailed theory of all the current interruption cases that can occur in a power system.
- Clear explanation of how to calculate transients, showing how four basic circuits can represent almost all transients and can be solved in general without any use of the “classical” Laplace transform method.
- Series and parallel RLC circuit theory, followed by the calculation of pole factors using the symmetrical component method to derive the base power frequency components, with results applied to all the relevant transient cases.
- Fault current interruption and inductive and capacitive load current switching cases illustrated using real application examples.

With its practical approach, this book offers engineers the tools they need when analyzing circuit breaker applications. It will be of great interest to practicing engineers and electric utility staff involved in circuit breaker design, specification, testing, switchgear applications, system operation and planning, and, in particular, for engineers using simulation programs for transient calculations. It will also be useful for senior undergraduate and postgraduate electrical power engineering students looking to work in switchgear manufacturing units and testing laboratories.
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Preface

After a fortunate and rewarding career that started at ASEA in Ludvika, Sweden, and was followed by 28 years at BC Hydro in Vancouver, Canada, I took early retirement in May 2001. Not long afterward, I was asked by the Association of Professional Engineers and Geoscientists of British Columbia (B.C.) if I would be interested in presenting continuing professional development courses on circuit breaker application, and this started a second career in teaching.

The first course was 4 hours long and eventually grew into much more detailed courses, some up to 5 days’ duration. Experience with the courses showed that the part that generated the most questions from participants related to all types of current interruption transients, and I started to consider developing a course on transients alone. At about the same time, the engineering manager at one of my consulting clients lamented the fact that engineers today, particularly the younger generation of engineers, are much too dependent on software and have lost sight of theory and practical reality. He asked if a course could be developed to provide a fundamental understanding of transients and enable estimations using only a hand calculator and a spreadsheet program.

The approach taken (after a number of false starts) was to draw the circuit diagrams for all possible making, breaking, reignition and restriking cases. Comparison showed that practically all cases are covered by four basic circuits (Tables 2.1 and 2.2). Some exceptions, of course, occur but are variations on a common theme. Three of the circuits involve second-order linear homogeneous differential equations that, instead of individually resorting to Laplace transformation-based solutions, have a common solution of the form

$$y = A e^{r_1 x} + B e^{r_2 x},$$

where the roots $r_1$ and $r_2$ are derived from the circuit RLC components and the constants $A$ and $B$ from the initial or boundary conditions. The equation in turn has three possible variations: the roots are real, equal or complex corresponding to overdamping, critical damping and under-damping, respectively. After being derived, the three equations enable a generic approach to RLC oscillatory circuit calculations (Table A.1).

The fourth case involves a second-order non-homogeneous differential equation that is more difficult to solve than the homogeneous case. However, mathematicians have long resolved the difficulty by providing lookup tables, basically making a guess at the solution, and then using the method of undetermined coefficients to solve the equation (Appendix A).
At this stage, we now have three equations for each of the four circuits incorporating the \( r_1 \) and \( r_2 \) roots. The next step is to apply the boundary conditions, and the equations for current or voltage in real time are derived. The final step is to convert the equations to a generic format by expressing the circuit damping and time in relative terms, that is, damping relative to critical damping and time relative to the period of the frequency of the transient oscillation (Tables 2.1 and 2.2). General curves can then be drawn and are easily convertible to current or voltage in real time for any switching case.

For multiphase faults, sequential interruption of the fault current in the individual circuit breaker poles leads to AC recovery voltages higher than rated voltage. The AC recovery voltages are related to rated voltage—actually prefault voltage at the point of the fault—by pole factors calculated using the method of symmetrical components. A number of approaches are considered, including a generic approach to first-pole-to-clear pole factor calculation.

As readers will learn, there is a certain symmetry to current interruption transients. For any switching event, taking first the status before the switching operation and then the status after the operation, the transient is the transition from “before” to “after.” On this basis, all transients have a starting point, an aiming point or axis of oscillation and a maximum point. Take, for example, the transient recovery voltage (TRV) for a terminal fault on an effectively earthed system: the starting point is zero, the axis of oscillation is around the AC recovery voltage and the maximum value is dependent on the damping and nature of the involved circuit. Understanding this overall concept enables a graphical approach to transient calculation in many cases (see Figures 6.7 and 6.8).

This is not a book about circuit breaker application, and readers are referred in this regard to the Bibliography in Chapter 1. Also, it is not a book about how to use Excel for equation-based calculations; guidance is readily available in instruction manuals and online. Using the generic approach to transient calculation is well suited to Excel because generic time is always in radians, a prerequisite for plotting sinusoidal and hyperbolic functions. A note of caution with respect to plotting in Excel is that, in contrast to software that permits the plotting of functions, Excel plots points. This means, for example, if no point is calculated at a maximum value, then the maximum value will not appear in the plot. A further note is in combining plots with different frequencies, such as the case of adding series reactors, all plots have to be referred to common real-time coordinates before attempting addition or subtraction.

The book is intended to be inclusive. The switching cases are covered in detail in the main text, and supporting calculations and information can be found in Appendices A–G. The evolution of TRVs and their understanding is interesting and is reviewed in Appendix H. The first circuit breakers became commercially available around 1910, and technical papers started to appear within a few years in AIEE publications. The notion of a TRV was first recognized in 1927 by J.D. Hilliard of GE, who used the descriptive term “voltage kick” for the concept. The first standards for fault current TRVs were developed in the 1950s and evolved further into the standards of today.

I would not have been able to write this book without the support that made my career possible. I am grateful to BC Hydro for supporting my participation in learned societies, principally CIGRE and the IEEE, and in the development of circuit breaker standards in IEC; to my colleagues past and present at BC Hydro and in the IEEE Switchgear Committee, CIGRE Study Committee A3 and IEC Technical Committee 17A; to those who have attended the course and asked the great questions that contributed to the book content; and, most of all, to
Sandra Giasson for her patient and diligent word processing of the text through several drafts to the final version.

Writing this book took 10 months, but really it has been 30 years in the making. Now it’s done, and I hope that you will find it to be useful and of value.

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1

Introduction

1.1 Background

The intent of this textbook is to explain the origin and nature of the transients associated with fault and inductive and capacitive load current interruption. The transients in general have a power frequency and an oscillatory component. The oscillatory components have an RLC circuit basis with such a degree of commonality between the above current interruption cases that a generic calculation approach is possible. The power frequency component is either a balanced or momentarily unbalanced quantity and in some cases is the axis of oscillation for the oscillatory component. In overview, the following transients will be analyzed and the resulting equations applied to real current interruption cases:

- **Fault current interruption**: The transient of interest is the transient recovery voltage (TRV) that appears across the circuit breaker after current interruption. For terminal faults, that is, a fault at the circuit breaker, the power frequency component is dependent on the system earthing and the type of fault. The oscillatory component can be either overdamped or underdamped with travelling waves contributing to the former oscillation. The TRV may be on one side of the circuit breaker only, for example, a three-phase-to-earth fault on an effectively earthed system, or on both sides of the circuit breaker as for the out-of-phase switching and short-line fault cases.

- **Inductive current interruption**: The transients for consideration in this case are the TRV, which is the difference between the source power frequency and the load circuit oscillation, as well as the transients due to re-ignitions. The load circuit and re-ignition transient oscillations are underdamped.

- **Capacitive current interruption**: The transients in this case are related to both current and voltage. The transient currents to be considered are those due to inrush on switching in a single shunt capacitor bank or in back-to-back switching and outrush current when a bank discharges into a fault. At current interruption, the TRV is the difference between the source power frequency voltage and the trapped DC voltage on the capacitive load at current interruption. The voltage transient of issue is that due to re-striking.
The structure of the textbook is the following:

- Chapter 1: The short-circuit rating basis for high-voltage circuit breakers is described with reference to the IEC circuit breaker standard IEC 62271-100 followed by a review of current interruption terminology.
- Chapter 2: Oscillatory RLC circuits are treated using a generic solution approach without any recourse to the traditional Laplace transform method. An examination of all the various circuits involved in current interruption, re-ignitions or re-striking and making showed that by treating four basic circuit configurations, almost all switching cases can be covered. Some exceptions, of course, occur but, as readers will appreciate later, these are actually variations on a common theme. A basic knowledge of travelling waves is required later in the text, and an overview of basic considerations is included in this chapter.
- Chapter 3: Symmetrical component theory is applied to calculate the unbalanced power frequency voltage values, expressed as per-unit pole factors, that occur during fault and inductive and capacitive load current interruption.
- Chapter 4: The basis for the TRVs for terminal faults, that is, faults located at the terminals of the circuit breaker, is derived. This basis is then applied to the test duties required by IEC 62271-100 and further to show the effects of added capacitance, opening resistors and series reactors. The special cases of out-of-phase switching and double earth faults are then treated. This is followed by the derivation of asymmetrical current requirements and the relationship to time constants and so-called $X/R$ values.
- Chapter 5: The short-line fault is a special case with the circuit breaker being stressed by the difference between TRVs on the source and line sides. The derivation of the line-side transient, which is not oscillatory in the usual sense but rather is travelling wave based, is described and related to standard requirements.
- Chapter 6: Inductive load current switching includes the switching out of unloaded transformers and shunt reactors. The former switching case is not onerous for circuit breakers but the same cannot be said for the latter case. The multiple variations of shunt reactor switching configurations are treated using the generic approach.
- Chapter 7: Capacitive load current switching involves both the switching in and switching out of shunt capacitor banks and unloaded cables and lines. The derivation of inrush and outrush currents, TRVs and re-striking events are treated in detail.
- Chapter 8: Circuit breaker type testing requirements for fault current interruption and load current switching are reviewed.

No chapter is stand-alone as such, and readers should note that Chapters 2 and 3 provide the basic theory for the calculations of Chapters 4–7.

Supporting calculations and information relating to the main text are provided in Appendices A–G. Finally, a brief history of how the understanding and appreciation of TRVs evolved and became standards is provided in Appendix H.

### 1.2 Short-Circuit Rating Basis for High-Voltage Circuit Breakers

High-voltage circuit breakers are rated on the basis of clearing three-phase faults. The most onerous case with respect to TRVs is for the first-pole-to-clear (FPTC or fptc). This is because
after the first circuit breaker pole clears, the system becomes unbalanced, causing the AC recovery voltage across the pole to exceed its normal phase-to-earth value. Two cases can be distinguished based on the earthing of the power system:

Case 1: Power system effectively earthed.
An effectively earthed power system is one in which the ratio of the zero-sequence reactance to the positive-sequence reactance is positive and equal to 3 or less (neutrals solidly or low impedance earthed). Circuit breakers applied on such systems are rated on the basis of clearing a three-phase-to-earth fault. After the most onerous first pole clearing, this leaves a double-phase-to-earth fault and, in turn, after second pole clearing, leaves a single-phase-to-earth fault to be cleared by the third pole. This sequence is shown in Figure 3.6.

Case 2: Power system non-effectively earthed.
A non-effectively earthed power system is not defined by sequence reactances but rather as one where the neutral is isolated, high impedance or resonant earthed. Circuit breakers applied on such systems are rated on the basis of clearing a three-phase unearthed fault. First pole clearing leaves a phase-to-phase fault to be cleared simultaneously by the second and third poles in series. Before the second and third pole clearing, the fault-side neutral will shift by 0.5 pu and the AC recovery voltage for the first-pole-to-clear is 1.5 pu. This sequence is shown in Figure 3.7.

The standard TRV requirements for a 245 kV circuit breaker on an effectively earthed system and a 72.5 kV circuit breaker on a non-effectively earthed system are given in Tables 1.1 and 1.2, respectively.

Without going into detail at this point, the TRVs are based on the two components briefly discussed earlier: a power frequency component given by the first-pole-to-clear factor $k_{pp}$ (Chapter 3) and an oscillatory component, which may actually be aperiodic, given by the amplitude factor $k_{af}$ (Chapter 2).

The short-line fault and out-of-phase switching requirements are also shown in Tables 1.1 and 1.2.

In general, circuit breakers are designed to withstand voltage, carry load current and clear faults. However, circuit breakers are also required to interrupt load currents. Load currents at or around unity power factor present no difficulty, but at zero power factor leading or lagging,

### Table 1.1 Standard transient recovery voltage values for 245 kV rated circuit breaker on an effectively earthed system.

<table>
<thead>
<tr>
<th>Rated voltage, $U_r$ (kV)</th>
<th>Test duty</th>
<th>First-pole-to-clear factor, $k_{pp}$ (pu)</th>
<th>Amplitude factor, $k_{af}$ (pu)</th>
<th>First reference voltage, $u_1$ (kV)</th>
<th>Time, $t_1$ (μs)</th>
<th>TRV peak value, $u_c$ (kV)</th>
<th>Time, $t_2$ (μs)</th>
<th>Time delay, $t_d$ (μs)</th>
<th>Voltage, $u'$ (kV)</th>
<th>Time, $t'$ (μs)</th>
<th>RRRV, $u_1/t_1$ (kV/μs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>245 Terminal fault</td>
<td>1.3</td>
<td>1.40</td>
<td>195</td>
<td>98</td>
<td>364</td>
<td>392</td>
<td>2</td>
<td>98</td>
<td>51</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Short-line fault</td>
<td>1</td>
<td>1.40</td>
<td>150</td>
<td>75</td>
<td>280</td>
<td>300</td>
<td>2</td>
<td>75</td>
<td>40</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Out-of-phase</td>
<td>2</td>
<td>1.25</td>
<td>300</td>
<td>196</td>
<td>500</td>
<td>392–784</td>
<td>2–20</td>
<td>150</td>
<td>117</td>
<td>1.54</td>
<td></td>
</tr>
</tbody>
</table>

RRRV: rate of rise of recovery voltage.
current interruption is an onerous duty. No rated interrupting current values are stated in the standards because in practice they are application dependent. Preferred capacitive current switching ratings are stated in the expectation that type testing to these values will cover a majority of actual applications.

### 1.3 Current Interruption Terminology

Current interruption terminology can be understood by considering an actual event. Figures 1.1–1.3 show the trace of a close-open (CO) three-phase unearthed fault current test on a vacuum circuit breaker. Taking each figure in turn, the terminology is as follows:

**Table 1.2  Standard transient recovery voltage values for 72.5 kV rated circuit breaker on a non-effectively earthed system.**

<table>
<thead>
<tr>
<th>Rated voltage, $U_r$ (kV)</th>
<th>Type of test</th>
<th>First-pole-to-clear factor, $k_{pp}$ (pu)</th>
<th>Amplitude factor, $k_{af}$ (pu)</th>
<th>TRV peak value, $u_c$ (kV)</th>
<th>Time, $t_3$ ($\mu$s)</th>
<th>Time delay, $t_d$ ($\mu$s)</th>
<th>Voltage, $u'$ (kV)</th>
<th>Time, $t'$ ($\mu$s)</th>
<th>RRRV, $u_c/t_3$ (kV/\mu s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>72.5</td>
<td>Terminal fault</td>
<td>1.5</td>
<td>1.54</td>
<td>137</td>
<td>93</td>
<td>5</td>
<td>45.6</td>
<td>36</td>
<td>1.47</td>
</tr>
<tr>
<td></td>
<td>Short-line fault</td>
<td>1</td>
<td>1.54</td>
<td>91.2</td>
<td>93</td>
<td>5</td>
<td>30.4</td>
<td>36</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Out-of-phase</td>
<td>2.5</td>
<td>1.25</td>
<td>185</td>
<td>186</td>
<td>28</td>
<td>61.7</td>
<td>90</td>
<td>0.99</td>
</tr>
</tbody>
</table>

**Figure 1.1** Current interruption terminology: timing-related quantities (trace courtesy of KEMA).
The circuit breaker is initially open, and a close signal is applied to the close coil to initiate closing.

After a short electrical delay time, the moving contact starts in motion (travel curve at the bottom of the trace) and makes contact with the circuit breaker fixed contact. This instant is referred to as contact touch or contact make. In practice, actual electrical making of the circuit may precede mechanical contact because of a prestrike between the contacts. The time between application of the close signal and contact touch is the mechanical closing time of the circuit breaker.

The circuit breaker is now closed and carrying fault current. A trip signal is applied to the trip coil initiating opening, also referred to as tripping, of the circuit breaker. After a short electrical time delay, the moving contact is set in motion and mechanical separation of the fixed and moving contacts occurs. This instant is referred to as contact part, contact parting or contact separation. The time between application of the trip signal and contact part is the mechanical opening time.

An arc is drawn between the contacts, and current interruption attempts are made as the zero crossings occur, first on b-phase, then on a-phase and successfully on c-phase. c-phase is thus the first-pole-to-clear with an arcing time—time between contact part and current interruption—of about one half-cycle. The interrupting time, also referred to as the break time, on c-phase is the mechanical opening time plus the arcing time.
At current interruption in c-phase, the currents in a-phase and b-phase become equal in magnitude and opposite in polarity by means of a 30° shift, a shortened half-cycle in the former phase and a longer half-cycle in the latter. The total break time is the mechanical opening time plus the maximum arcing occurring in these two phases.

For a fault initiated at a voltage peak, the current will be symmetrical. Symmetrical means that the each half-cycle of the current, also referred to as a loop of current, will be identical to the preceding half-cycle of current. The current in a-phase is near symmetrical as a result of fault initiation just before the voltage peak.

The currents in b-phase and c-phase are asymmetrical and consist of long and short loops of current referred to as major loops and minor loops, respectively. Maximum asymmetry occurs when the fault is initiated at a voltage zero crossing. Asymmetrical currents are discussed in detail in Section 4.7.

Current zeros occur every 60°, and the pole closest to a zero after contact part will make the first attempt to interrupt the current. The b-phase pole that is the closest to the first zero makes the attempt to interrupt the current but reignites because the contacts are too close to
withstand the TRV. The a-phase pole in turn also makes an attempt but reignites followed by successful interruption on c-phase, that is, recovering against the TRV and AC recovery voltage.

- The TRV is a transient oscillation as the voltage on the source side of the circuit breaker recovers to the prefault system voltage. The TRV oscillates around the AC recovery voltage, its aiming point or axis of oscillation, reaching a peak value depending on the damping in the circuit. As the trace shows, the TRV rings down within a power frequency quarter cycle. The first-pole-to-clear is exposed to the highest TRV. The theory behind TRVs is discussed in Chapters 2 and 3 and applied in the later chapters.

- a-phase and b-phase poles clear 90° later, each with its own TRV of lower magnitude than for c-phase and of opposite polarity. The AC recovery voltage is the line voltage and is shared by both poles.

Bibliography

The following references are for the textbooks covering the broad range of circuit breaker types and related switching transients in high-voltage networks (see also Bibliography in Appendix H).

2.1 General
The transients associated with transient recovery voltage (TRV), re-ignition or re-striking events can in general be related to either a series or parallel RLC circuit, each with specific boundary or initial conditions. The transients of most interest with respect to current interruption can, in fact, be represented by four basic RLC circuits, as shown in Figure 2.1. Each of these circuits will be treated in detail, and the following considerations apply to all cases:

1. All transients have a starting point, which may be zero or a finite value.
2. All transients have an axis of oscillation—or aiming point—that then becomes the ultimate steady-state value after the transient has died out.
3. All transients have a maximum value dependent on the degree of damping in the circuit.
4. All transients have a certain frequency determined by the values of $L$ and $C$ in the circuit; however, note that not all transients are oscillatory, the exceptions being the cases where the oscillations are aperiodic.

The calculation of the transient oscillations in each will be based on the general solutions of second-order linear homogeneous or non-homogeneous differential equations discussed in Appendix A. The boundary conditions to be used in each case are those of the value or rate of change of the value of the applicable current or voltage transient at time zero.

2.2 Series RLC Circuit with Step Voltage Injection
The circuit for this case is as shown in Figure 2.1a. The solutions for the transient current can be used to calculate inrush currents associated with single and back-to-back capacitor bank switching and currents associated with re-ignition and re-striking events.

Applying Kirchoff’s voltage law to the circuit in Figure 2.1a, we can write

$$V = Ri(t) + L \frac{di(t)}{dt} + \frac{q(t)}{C}. \quad (2.1)$$
Differentiating Eq. (2.1) across then gives

\[ L \frac{d^2i(t)}{dt^2} + R \frac{di(t)}{dt} + \frac{1}{C} i(t) = 0 \]

or

\[ \frac{d^2i(t)}{dt^2} + \frac{R}{L} \frac{di(t)}{dt} + \frac{1}{LC} i(t) = 0. \] (2.2)
Equation (2.2) has the \( abc \) format discussed in Appendix A, and we can write

\[
a = 1, \quad b = \frac{R}{L}, \quad c = \frac{1}{LC}
\]

and further that

\[
\alpha = \frac{R}{2L} \quad \text{and} \quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = \sqrt{\alpha^2 - \omega^2},
\]

where

\[
\omega = \frac{1}{\sqrt{LC}}.
\]

Case 1: The circuit is overdamped \((\alpha^2 > \omega^2)\).

From Table A.1, the solution for \( i(t) \) is given by

\[
i(t) = e^{-\alpha t}(k_1 \cosh \beta t + k_2 \sinh \beta t).
\] (2.3)

To determine the values of \( k_1 \) and \( k_2 \), we must apply the boundary conditions given by \( i(0) \) and \( di(0)/dt \).

At \( t = 0 \), \( i(0) = 0 \) and \( q = 0 \) and Eq. (2.3) becomes \((\sinh 0 = 0 \text{ and } \cosh 0 = 1)\)

\[
0 = e^{-0}(k_1(1) + k_2(0))
\]

and therefore \( k_1 = 0 \), giving

\[
i(t) = k_2 e^{-\alpha t} \sinh \beta t.
\]

For the second boundary condition,

\[
\frac{di(t)}{dt} = k_2 e^{-\alpha t}(\beta \cosh \beta t - \alpha \sinh \beta t)
\]

\[
= k_2 \beta \quad \text{at} \ t = 0.
\]

From Eq. (2.1), the current at time zero plus is determined by the inductance \( L \), and we can write

\[
V = R(0) + L \frac{di(0 +)}{dt} + \frac{0}{C}
\]

or

\[
\frac{di(0 +)}{dt} = \frac{V}{L}
\]
and

\[ k_2 = \frac{V}{L\beta}. \]

The solution for \( i(t) \) is

\[ i(t) = \frac{V}{L\sqrt{\alpha^2 - \omega^2}} e^{-\alpha t} \sinh \sqrt{\alpha^2 - \omega^2} t. \quad (2.4) \]

Case 2: The circuit is critically damped (\( \alpha^2 = \omega^2 \)).

From Table A.1, the solution for \( i(t) \) is

\[ i(t) = (k_1 + k_2 t)e^{-\alpha t}. \]

At \( t = 0, i(0) = 0 \):

\[ 0 = (k_1 + 0)e^{-0}, \]

giving \( k_1 = 0 \) and

\[ i(t) = k_2 t e^{-\alpha t}, \]

\[ \frac{di(t)}{dt} = k_2 e^{-\alpha t}(1 - \alpha t), \]

\[ \frac{di(0 +)}{dt} = k_2 \]

\[ = \frac{V}{L}. \]

The solution for \( i(t) \) is

\[ i(t) = \frac{V}{L} t e^{-\alpha t}. \quad (2.5) \]

Case 3: The circuit is underdamped (\( \omega^2 > \alpha^2 \)).

From Table A.1, the solution for \( i(t) \) is

\[ i(t) = e^{-\alpha t}(k_1 \cos \beta t + k_2 \sin \beta t). \]

Applying the boundary conditions gives the same results as for Case 1:

\[ k_1 = 0, \]
\[ k_2 = \frac{V}{L\beta}, \]

and the solution for \( i(t) \) is