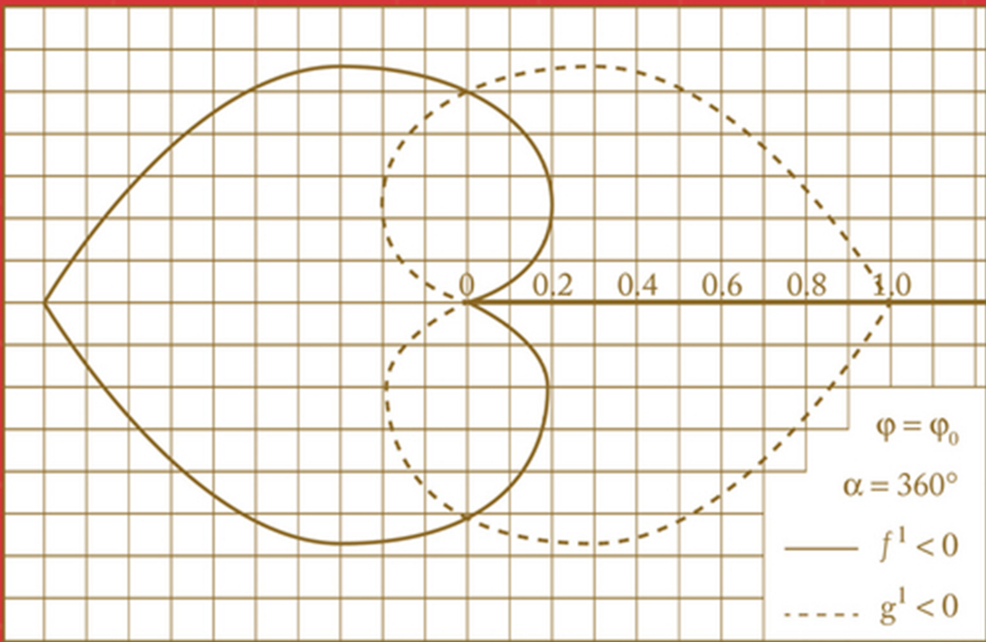
Fundamentals of the Physical Theory of Diffraction

Second Edition



Pyotr Ya. Ufimtsev

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Preface

The physical theory of diffraction (PTD) is a high-frequency asymptotic technique for the investigation of antennas and scattering problems. PTD was announced publically under this name for the first time in a report and a book by Ufimtsev, 1962a,b. An anniversary article (Ufimtsev, 2013a) contains comments on its origination and development. This monograph presents a complete and comprehensive description of modern PTD based on the concept of elementary edge waves. Its basic subject is the diffraction of acoustic and electromagnetic waves by perfectly reflecting objects.

Here are new features of the revised version:

- New Sections 1.3.6 and 1.4.2 establish that the shadow radiation equals zero in the directions of the reflected rays, and the reflected field equals zero in the shadow direction.
- New Sections 1.3.7 and 1.4.3 extend the theory of shadow radiation and reflection to opaque objects.
- New Section 1.5 provides physical interpretations of the shadow radiation via Fresnel diffraction and forward scattering.
- New Section 2.7 develops the "magic zero" procedure to derive fast convergent integrals and uniform asymptotics, which are convenient for numerical and analytic analysis of the canonical wedge diffraction problem.
- Chapter 3 simplifies the physical optics (PO) approximation, introducing new functions $v^{(0)}_{\mathrm{s,h}}(kr, \psi)$.
- New Section 3.3 derives the fast convergent integrals and uniform asymptotics for the PO diffracted field.

- New Section 4.3.2 extends the topic of polarization coupling, which is critical in PTD. It also clarifies the nature of this phenomenon.
- New Section 7.8.2 describes the diffracted field inside a perfectly reflecting wedge and explains its origination as a consequence of the equivalence theorem.
- New Section 7.8.3 presents numerical data for diffraction coefficients of electromagnetic elementary fringe waves.
- New Section 8.3 clarifies the relationships between PTD and GTD.
- New Section 10.4 studies diffraction at a strip to test the asymptotic theory of the secondary diffraction derived in Chapter 10.
- New Section 14.1.5 studies the PO shooting-through rays and their cancellation by fringe rays. It highlights the fundamental role of the nonuniform/fringe components introduced in PTD.
- An essential supplement to the main text is provided by the end-of-chapter problems followed by their solutions. They will be helpful in the study and application of PTD.
- MATLAB codes presented in the appendices allow for quick numerical calculations of fringe waves and axial backscattering at bodies of revolution.
- Compared to the first edition, more attention is given to the theory of electromagnetic waves.
- Additional smaller insertions and corrections are incorporated throughout the book.

The theory developed in the book may find various applications. Among them are problems associated with the design of microwave antennas, estimation of scattering cross-sections, identification of scattering objects, and propagation of waves in an urban environment. The most significant example of practical applications of PTD represents the design of the American F-117 stealth fighter and B-2 stealth bomber. In combination with numerical methods, PTD can be used for the development of efficient hybrid techniques for the investigation of complex diffraction problems.

The book is intended for researchers working on antennas and scattering problems in industry and university laboratories. It can also be useful for teaching a variety of university courses, including topics on high-frequency asymptotic techniques in diffraction theory. University instructors and graduate students will benefit from this book as well.

I am very grateful to Dr. Feray Hacivelioglu and Prof. Levent Sevgi, who produced MATLAB codes for numerical calculation of diffracted waves. Many thanks are also due to the reviewers for their valuable comments.

Pyotr Ya. Ufimtsev

Los Angeles, California
August 2013

Foreword to the First Edition

Ideas have consequences. Great ideas have far-reaching consequences.

The physical theory of diffraction (PTD) that Professor Ufimtsev introduced in the 1950s—a methodology for approximate evaluation at a high enough frequency of the scattering from a body, especially a body of complicated shape—has proven to be a truly great idea.

The first form of PTD developed by Professor Ufimtsev, the vector form applicable to electromagnetic scattering from three-dimensional bodies, has played a key role in the development of modern low-radar-reflectivity weapons systems, such as the Lockheed F-117 stealth fighter and the Northrop B-2 stealth bomber, functioning both as a design tool and as a conceptual framework. These systems in turn have revolutionized the conduct of large-scale government-versus-government warfare and thus have helped to shape history.

Ben Rich, who oversaw the F-117 project as head of Lockheed's fabled Skunk Works, refers to Professor Ufimtsev's work as "the Rosetta Stone breakthrough for stealth technology." At Northrop, where I worked on the B-2 project, we were so enthusiastic about PTD that a co-worker and I sometimes broke into choruses of "Go, Ufimtsev" to the tune of "On, Wisconsin." At both Lockheed and Northrop we referred to PTD as "industrial-strength" diffraction theory, to distinguish it from the approach to diffraction then being favored in the universities, which was not well enough developed to handle the problems of stealth design.

Like many good theories, PTD is much easier to apply than to explain. But let us now nevertheless examine the inner workings of PTD and seek to understand why it is such a useful approach. First of all, PTD is based on two important principles, which it will be convenient to refer to here as the *physical principle* and the *geometrical principle*.

The physical principle shows how the scattered field at a point outside a scattering body can be determined from an integral of appropriate field quantities over the surface of the body. In acoustics these quantities are the pressure at a hard surface, the normal velocity at a soft surface, both at an impedance boundary or the surface of a penetrable body. In electromagnetics they are the tangential magnetic field at the surface of a perfect conductor, the tangential magnetic and electric fields at an impedance boundary or the surface of a penetrable body.

The geometrical principle states that at high enough frequency, when the wavelength is small enough compared to the critical dimensions of the scattering body, the surface integrals can be evaluated asymptotically to yield a description of the total field outside the body in terms of geometrical rays, including diffracted rays. The change in field amplitude along a ray can be calculated geometrically by tracing the divergence and convergence of ray bundles except in the regions surrounding (a) a geometrical shadow boundary, for which ray tracing predicts a field discontinuity across the boundary, and (b) a caustic, that is, a locus where adjacent geometrical rays meet or cross (such as, in the simplest case, a focal point), at which ray tracing predicts an infinite field. The correct value for the field in these regions, which shrink as frequency increases, can be found by using uniform asymptotic techniques to evaluate the surface integrals.

One of the important features of PTD is this ability to calculate the field accurately in shadow boundary and caustic regions. It is especially important in low observables design because we are often interested in far-field scattering of a plane wave from a body with straight or slightly curved edges, a configuration for which parts of the far-field region lie in caustic regions.

The other major advantages of PTD arise from the way the surface fields are handled. There is a *uniform* part that is *defined everywhere on the surface* and a *nonuniform* part that serves as a correction term.

For electromagnetics the uniform part is usually, though not always, given by the physical optics (PO) approximation: namely, that the surface fields at a point are the same as if the point lay on an infinite plane surface tangent to the actual body at the point and with the same boundary conditions as at the point. For acoustics the uniform part is usually given by the analogous approximation. Because this acoustics approximation does not have a firmly established name and because other investigators have set the precedent, Professor Ufimtsev uses the terminology PO in both electromagnetics and acoustics throughout this book. Much of Chapter 1 is devoted to PO and its implications.

The nonuniform fields for a nonpenetrable body—for example, a hard body in acoustics or a perfect conductor in electromagnetics—tend to be strongest near a diffracting feature such as an edge where two faces of a faceted surface meet, and these fields often diminish rapidly with distance from the feature. It should be emphasized here that this desirable behavior is a consequence of the judicious choice of the uniform part.

The nonuniform surface fields are determined using the results of simpler scattering problems, often called *canonical problems*. Consider again, for example, an edge on a faceted surface. Let the body be a perfect conductor and the edge be straight with the wedge angle formed by the two faces constant along its length, let the illuminating field be a plane wave, and let us choose the PO fields as the uniform part. Then the canonical problem is diffraction of an appropriately oriented plane wave from an infinitely long wedge with perfectly conducting flat faces (even if the faces on the body of interest are not flat). This problem reduces to two scalar two-dimensional problems, one for an incident electric field normal to the edge, the other for an incident magnetic field normal to the edge, and exact solutions exist for these problems. The vector surface fields can be constructed from the two scalar solutions, and the nonuniform surface fields associated with the edge are then found by subtracting the physical optics fields of the canonical problem from the full solution.

There now arises the problem of reconciling the uniform part and the nonuniform part, which is defined on a surface that may not exactly match the body surface. Professor Ufimtsev addresses this in Chapter 7, where he reduces the nonuniform part to a continuous array of *elementary edge waves* concentrated along the edge. These elementary edge waves are sources of diffracted rays and have a directivity pattern that is related to the canonical problem. In the parlance of engineering they would be called *diffraction coefficients*.

The nonuniform contribution to the field diffracted from the edge is now given by an integral of the elementary edge waves over the length of the edge. But when we evaluate asymptotically the integral for the physical optics diffraction from a face, we see that it reduces to an integral along the illuminated part of the face perimeter plus possibly other localized terms (such as a specular reflection contribution). Thus, there are edge diffraction contributions from the uniform part of the surface field on both faces that meet at the edge (if both are illuminated) as well as from the nonuniform part, and these three terms give the total edge diffraction. Furthermore, it turns out that each element of the edge produces diffraction in essentially all directions.

We can now, from this investigation of how the surface fields are modeled, extract these additional important features of PTD:

1. PTD can find accurately the reflection and diffraction from a body of complicated shape without having to match the entire body to canonical problems, just the regions that give rise to diffraction.
2. PTD minimizes the difficulty of reconciling the geometries of the body and of the canonical problem.
3. PTD yields diffracted rays in all directions from each element of a linear diffracting feature rather than just in directions on the well-known diffraction cone.

The third point is extremely important in low-observables work, where the off-cone rays can sometimes yield the strongest fields in a region.

This book presents a thorough development of the fundamentals of PTD for both scalar and vector cases as applied to acoustics and electromagnetics, including important aspects of the theory only recently developed by Professor Ufimtsev. For acoustics it is, of course, the scalar theory that is of interest. For electromagnetics both scalar and vector theory should be of interest. Canonical problems are often two-dimensional, and two-dimensional problems can be reduced to scalar form.

Emphasis in the book is on nonpenetrable bodies with "classical" boundary conditions at the surface: the Dirichlet and Neumann problems of applied mathematics, the corresponding soft and hard boundary problems of acoustics, and the perfect conductor problem of electromagnetics.

PTD is, however, in principle readily extended to cases of a body with an impedance boundary condition at its surface and of a penetrable but opaque body, and has in fact been used extensively for such bodies, although much of the work is classified, proprietary, or otherwise restricted. The extension to translucent and transparent bodies is more challenging, not because of any shortcoming of PTD but because it can be necessary to deal with such complicated phenomena as diffracted waves that travel through the body and are then refracted out of the body.

Much has been said and written about the relative merits of the two major modern approaches to diffraction theory: PTD on one hand, and on the other, Professor Joseph Keller's geometrical theory of diffraction (GTD) and its modified versions, the uniform theory of diffraction (UTD) developed at the Ohio State University and the similar uniform asymptotic theory of diffraction (UAT).

Both approaches are valid, each yields a ray description of the field (PTD as an end result, GTD as a starting point), each has its advantages, and the two have now been cross-fertilizing each other for half a century. The work of the next generation, I fervently hope, will be to mold these approaches and other contributions together into a single modern theory of diffraction from bodies.

By his detailed exposition of the fundamentals of PTD in this volume, Professor Ufimtsev has not only produced a work of great contemporary value but also a compendium that can be extremely useful in this reconciliation process.

KENNETH M. MITZNER

November 2006

Preface to the First Edition

The physical theory of diffraction (PTD) is a high-frequency asymptotic technique for the investigation of antennas and scattering problems. This monograph presents the first complete and comprehensive description of the modern PTD based on the concept of elementary edge waves (EEWs). Its subject is the diffraction of acoustic and electromagnetic waves by perfectly reflecting objects located in a homogeneous lossless medium.

The basic idea of PTD is that the diffracted field is considered as the radiation generated by scattering sources (currents) induced on objects. *Uniform* and *nonuniform scattering sources* are introduced in PTD. Uniform sources are defined as sources induced on an infinite plane tangent to the object at a source point. Nonuniform sources are caused by any deviation of the scattering surface from the tangent plane. For large convex objects with sharp edges, the basic contributions to the scattered field are produced by uniform sources and by those nonuniform sources that concentrate near edges (often called *fringe sources*).

The integration of uniform sources leads to the physical optics (PO) approximation for the scattered field. The PTD is the natural extension of the PO approximation by taking into account the additional field created by the nonuniform/fringe sources.

The book provides high-frequency asymptotics for the scattering sources and for the scattered field in the far zone. Scattering characteristics are calculated for a variety of objects, such as strips, polygonal cylinders, cones, bodies of revolution with nonzero Gaussian curvature (including paraboloids and spherical segments), and finite circular cylinders with flat bases.

The title of the book underlines the fact that a great deal of attention is to be given to scattering physics. The analytic expressions derived clearly explain the physical structure of a scattered field and describe, in detail, all of the reflected and diffracted rays and beams, as well as the fields in the vicinity of caustics and foci. Also, a new fundamental component of the field, *shadow radiation,* is introduced. It is shown that this component contains *half* of the total scattered power. The physical manifestations of shadow radiation are the well-known phenomena of Fresnel diffraction and forward scattering.

Plotted numerical results supplement the theory and provide visualizations of the individual contributions of different parts of the scattering objects to the total diffracted field. Detailed comments explain all critical steps in the analytic and numerical calculations to facilitate their examination and utilization by readers. All chapters are followed by problems for independent investigation, which will be helpful in studying PTD, especially for students.

This book is intended for researchers working on antennas and scattering problems in industrial and university laboratories. It can also be useful for teaching a variety of university courses that include topics on high-frequency asymptotic techniques in diffraction theory. University instructors and graduate students will benefit from this book as well.

P. Ya. Ufimtsev

Los Angeles, California
June 2006

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During the preparation of this book I often appealed to my sons Ivan and Vladimir with requests to check and improve my English and to fix computer problems. I am thankful for their assistance.

Thanks are also due to E.V. Jull, K.M. Mitzner, Y. Rahmat-Samii, and A.J. Terzuoli, Jr. for their review of the manuscript and valuable comments.

This book includes, in revised form, materials from certain articles I wrote for the journals *Zhurnal Tekhnicheskoi Fiziki* (Russia), *Journal of the Acoustical Society of America* (United States), *Annals of Telecommunications* (France), and *Electromagnetics* (United States). I thank the editorial boards of the journals for their permission to use these materials.

P. YA. UFIMTSEV

Introduction

The *physical theory of diffraction* (PTD) is an asymptotic high-frequency technique originated in earlier work by this author (Ufimtsev, 1957, 1958a,b,c, 1961). The results of initial journal publications on PTD were summarized in a monograph (Ufimtsev, 1962b), which became a bibliographical rarity a long time ago. To acquaint a new generation of readers with the original form of PTD, some sections of this monograph were updated and included in two books (Ufimtsev, 2003, 2009). Comments on origination and development of PTD were presented recently in an anniversary article (Ufimtsev, 2013a). The selected topics of the modern form of PTD were published in concise form in articles by Butorin and Ufimtsev (1986), Butorin et al. (1987), Ufimtsev (1989, 1991), Ufimtsev and Rahmat-Samii (1995), Ufimtsev (1998, 2006a,b, 2008a,b), and Hacivelioglu et al. (2011).

This book presents the first complete and comprehensive description of modern PTD based on the concept of elementary edge waves (EEW). The theory is developed for acoustic and electromagnetic waves scattered by perfectly reflecting objects.

For acoustic waves, *soft* (Dirichlet) or *hard* (Neumann) boundary conditions are imposed on scattering objects located in a homogeneous nonviscous medium. The absence of viscosity is justified for a fluid (such as air and water) in the linear approximation (Kinsler et al., 1982; Pierce, 1994).

In diffraction problems for electromagnetic waves, the scattering objects are considered as perfectly conducting bodies located in vacuum. The assumption of infinite conductivity is acceptable for metallic objects detected by radar. The boundary condition related to electromagnetic waves states that on the surface of perfectly conducting bodies, the tangential component of the electric vector is equal to zero (Balanis, 1989, 2012).

The diffraction theory of acoustic waves is scalar, and it is simpler than the vector theory of electromagnetic waves. Because of this, we investigate first an acoustic diffraction problem in detail and then present its electromagnetic version, referring to similar elements in acoustic theory. This facilitates the study of electromagnetic problems. Note also that from a mathematical point of view, all two-dimensional diffraction problems have identical solutions for acoustic and electromagnetic waves. These problems are considered in the book for acoustic waves. The relationships between acoustic and electromagnetic diffracted waves are emphasized throughout the book. They are also formulated in the boxes located at the beginning of most chapters and sections.

PTD has found various applications. Some related references are collected at the end of the book in the section "Additional References Related to the PTD Concept: Applications, Modifications, and Developments." In particular, PTD was used successfully in the design of the American F-117 stealth fighter and B-2 stealth bomber (Browne,1991a,b; Rich, 1994; Rich and Janos, 1994; Grant, 2013; see also Mitzner's foreword for three Ufimtsev books (2003, 2007, 2009). The present book contains only original results obtained by the author (some of them in collaboration with colleagues).

The distinctive feature of PTD is that it belongs to the class of source-based theories. The scattered/diffracted field is considered as radiation by surface sources which are induced (due to diffraction) on the scattering objects by incident waves. In the case of electromagnetic waves and metallic scattering objects, these sources are surface electric charges and currents. In the case of acoustic waves, these sources are the surface distributions of the "acoustic pressure" on rigid objects, or the surface distributions of the "fluid velocity" on soft (pressure-release) objects. Compared to ray-based techniques, the advantage of this approach is that it allows calculation of the scattered field everywhere, including diffraction regions, such as foci and caustics, where the diffracted field does not have a ray structure.

The central and original idea of PTD is the separation of surface sources into uniform and nonuniform components. This separation is a flexible procedure, based on an appropriate choice of canonical diffraction problems (Ufimtsev, 1998). In the present book (except Section 7.9), the *uniform* component is defined as the scattering sources induced on the infinite plane tangent to the object at a source point. In the case of incident waves with a ray structure, this component is determined according to the geometrical optics (GO) (geometrical acoustics, GA) for electromagnetic (acoustic) waves. The field found by integration of the uniform component is considered a high-frequency approximation for the scattered field. In acoustic diffraction problems, this approximation is interpreted as the Kirchhoff approximation (KA). In electromagnetic diffraction problems, it is known as the physical optics (PO) approach. In the present book we use the term *physical optics* for both electromagnetic and acoustic waves, just as in the work by Bowman et al. (1987, p. 29).

The PTD is the natural extension of PO and takes into account the additional field generated by the *nonuniform* component, which has a diffraction nature and is caused by *any deviation* of the scattering surface from an infinite tangent plane. Another definition of the uniform and nonuniform scattering sources is introduced in Section 7.9. Here, the uniform component is defined as the field induced on the half-plane tangential to the illuminated face of the scattering edge (and to the edge itself). The nonuniform component is the difference between the exact field on the tangential wedge and this new uniform component. This type of separation of the surface field allows formulation of the advanced version of PTD, which is free of the grazing singularity (Section 7.9).

The localization principle related to the behavior of a high-frequency diffracted fields is used to determine the asymptotic approximations for the nonuniform component. In particular, according to this principle, the nonuniform sources induced in the vicinity of sharp curved edges are asymptotically identical to the nonuniform sources induced on a tangential wedge near the tangency point. Because these sources concentrate in the vicinity of the scattering edge, they are often called *fringe sources*, and the diffracted fields generated by these sources are termed *fringe waves*. The fundamental role of the fringe waves is emphasized and demonstrated throughout the book

Thus, the wedge diffraction is the basic canonical problem for the investigation of edge waves, and it is studied in detail in this book. Exact and asymptotic expressions for two-dimensional edge waves are derived in Chapters 2, 3, and 4. These results are then used in Chapter 5 to construct simple asymptotic expressions for the field diffracted at strips and polygonal cylinders.

Notice that two-dimensional diffraction problems for acoustically soft (hard) scattering objects are equivalent to electromagnetic problems where the electric vector $\vec{E}$ (magnetic vector $\vec{H}$) is parallel to the generatrix of perfectly conducting objects. Due to this equivalence, some results obtained by Ufimtsev (1962b) for two-dimensional electromagnetic problems are transferable for acoustic problems, with proper redefinitions of physical quantities. For the same reason, the asymptotics derived in Chapter 5 for acoustic waves are also valid for electromagnetic waves diffracted at perfectly conducting strips and trilateral cylinders.

A new physical interpretation of classical physical optics is introduced in Chapter 1. The scattered PO field is separated into the *reflected field* and *shadow radiation*. The first part contains ordinary reflected rays and beams and dominates in the geometrical optics region. The shadow radiation is equivalent to the field scattered at a blackbody (of the same shape and size as the actual scattering object) and dominates in the vicinity of the shadow region (Figs. 1.4 and 14.6). Manifestations of the shadow radiation are the well-known phenomena *Fresnel diffraction* and *forward scattering.*

The shadow contour theorem established in Section 1.3.5 states that different objects with identical shadow boundaries on their surfaces generate identical shadow radiation. This theorem significantly facilitates the approximate estimation of scattering at complex objects (Alekseev et al., 2007). It is also shown here that the shadow radiation contains *half* of the total power scattered by perfectly reflecting objects. Thus, the new formulation of the PO field elucidates the scattering physics and explains the nature of the fundamental diffraction law according to which the total scattering cross-section of large (compared to the wavelength) perfectly reflecting objects equals double the transverse area of geometrical optics shadow zone behind the object.

A significant part of this book is devoted to the theory of *elementary edge waves* and to its applications. An elementary edge wave is a wave radiated by surface sources induced in the vicinity of an infinitesimal element of the edge. The high-frequency asymptotic expressions found for elementary edge waves allows one to investigate diffraction at arbitrary curved edges with large radii of curvature (compared to the wavelength).

Elementary edge waves can also be interpreted as *elementary edge-diffracted rays.* The PO field as well can also be understood as the linear superposition of the other type of *elementary rays*. Because of this, PTD can be considered as a ray theory on the level of elementary rays. Even in diffraction regions such as geometrical optics boundaries, foci, and caustics, the wave field can be represented in terms of elementary rays. Ordinary reflected and diffracted rays are found in PTD by the asymptotic evaluation of field integrals and can be interpreted as the beams of elementary rays generated in the vicinity of stationary points. Such a possible interpretation of PTD goes back to the intuitive Huygens' principle, which was rigorously formulated by Helmholtz in terms of elementary spherical waves/rays (Baker and Copson, 1939).

The general theory of elementary waves is utilized in the book to solve a variety of diffraction problems. Backscattering and bistatic scattering at bodies of revolution are considered in Chapter 6. Ray and caustic asymptotics are derived in Chapter 8. Slope and multiple diffraction at large objects are investigated in Chapters 9 and 10. The results of these chapters are utilized in Chapters 11 and 12 to analyze the focusing of multiple edge waves on the symmetry axis of bodies of revolution. An example of the disk diffraction problem (whose exact asymptotic solution is known) establishes that PTD provides correct expressions for the first term in the total asymptotic expansion for each multiple edge-diffracted wave. This result is a matter of principle because it provides validation of PTD. Also notice other examples of theoretical and experimental validation of PTD in diffraction problems for electromagnetic waves (Nefedov and Fialkovsii, 1972; Ufimtsev, 1962b, 2003, 2009).

Chapters 13 and 14 derive the PTD asymptotics for the field scattered at a finite cylinder under oblique incidence of a plane wave. Together with the numerical results illustrated in the figures, they explain the physical structure of the scattered field. New features of the theory are emphasized here. They concern the necessity to calculate high-order terms in the PO field as well as radiation by nonuniform component of the scattering sources caused by smooth bending of a cylindrical surface.

The theory developed in the book can find various applications. Among them are the problems associated with the design of microwave antennas, the estimation of scattering cross-sections, the identification of scattering objects, and the propagation of waves in an urban environment. In combination with numerical methods, it can be used for the development of efficient hybrid techniques for the investigation of complex diffraction problems. The book can also be useful for teaching a variety of university courses, including topics on high-frequency asymptotic techniques in diffraction theory. The problems (together with their solutions) following each chapter will be helpful in studying PTD, especially for students. MATLAB codes presented in the appendices allow for quick numerical calculations of fringe waves and axial backscattering at bodies of revolution.

The International System of Units (SI) and the time dependence of $\exp(-i\omega t)$ for wave fields and sources are used in this book. Readers who prefer the dependence $\exp(j\omega t)$ can easily transform the book equations to this time format by simple replacement of the positive imaginary unit $i = \sqrt{-1}$ by the negative unit $-j = -\sqrt{-1}$.

1

Basic Notions in Acoustic and Electromagnetic Diffraction Problems

1.1 FORMULATION OF THE DIFFRACTION PROBLEM

In this book the physical theory of diffraction (PTD) is developed for both acoustic and electromagnetic waves diffracted at perfectly reflecting objects.

> In two-dimensional problems, this theory is valid for both *electromagnetic* and *acoustic waves*.

First we present the theoretical fundamentals for acoustic waves and then for electromagnetic waves. In the linear approximation, the velocity potential u of harmonic acoustic waves satisfies the Helmholtz wave equation (Kinsler et al., 1982; Pierce, 1994):

$$\nabla^2 u + k^2 u = I. \tag{1.1}$$

Here $k = 2\pi/\lambda = \omega/c$ is the wave number, λ the wavelength, ω the angular frequency, c the speed of sound, and I the source strength characteristic. The time dependence is assumed to be in the form $\exp(-i\omega t)$ and is suppressed below. The acoustic pressure p and the velocity v of fluid particles, caused by sound waves, are determined through the velocity potential (Kinsler et al., 1982; Pierce, 1994),

$$p = -\rho \frac{\partial u}{\partial t} = i\omega\rho\, u, \qquad \vec{v} = \nabla u, \tag{1.2}$$

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where ρ is the mass density of a fluid. The power flux density of sound waves, which is the analog of the Poynting vector for electromagnetic waves, equals

$$\vec{P} = p\vec{v} = p\,\nabla u. \tag{1.3}$$

Its value averaged over the period of oscillations $T = 2\pi/\omega$ equals

$$\vec{P}_{\text{av}} = \frac{1}{2}\text{Re}(p^*\vec{v}). \tag{1.4}$$

Two types of boundary conditions are imposed on the surface of perfectly reflecting objects: the Dirichlet condition,

$$u = 0 \quad \text{or} \quad p = 0 \quad \text{(soft)}, \tag{1.5}$$

for objects with a soft (pressure-release) surface, and the Neumann condition,

$$\frac{\partial u}{\partial n} = \hat{n} \cdot \nabla' u = 0 \quad \text{(hard)}, \tag{1.6}$$

for objects with a hard (rigid) surface. Here u is the total field that is the *sum of incident and scattered waves.* The symbol $\hat{n}$ stands for a unit outward vector, which is normal to the scattering surface S (Fig. 1.1). The gradient operator ∇' is applied to coordinates of the integration/source point Q.

To complete formulation of the diffraction problem and to ensure the uniqueness of its solution, the wave equation and boundary conditions above are supplemented by the Sommerfeld radiation condition for the scattered field:

$$\lim r\left(\frac{\partial u}{\partial r} - iku\right) = 0 \qquad \text{with} \quad r \to \infty, \tag{1.7}$$

where r is the distance from the scattering object to the observation point.

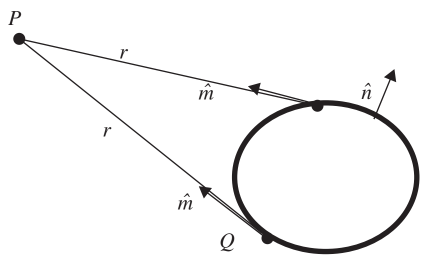

Figure 1.1 Scattering surface S. Here r is the distance between the observation point P (which can be in the far zone) and the integration point Q (on the surface of the scatterer). The unit vector $\hat{m}$ is directed from point Q to point P.