Advanced Equity Derivatives
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Advanced Equity Derivatives

Volatility and Correlation

SÉBASTIEN BOSSU

WILEY
“As for the expense,” gravely declared the deputy Haffner who never opened his mouth except on great occasions, “our children will pay for it, and nothing will be more just.”

Emile Zola, *La Curée (The Kill)*
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I am pleased to introduce Sébastien Bossu’s new book, *Advanced Equity Derivatives*, which is a great contribution to the literature in our field. Years of practical experience as an exotics structurer, combined with strong theoretical skills, allowed Sébastien to write a condensed yet profound text on a variety of advanced topics: volatility derivatives and volatility trading, correlation modeling, dispersion trading, local and stochastic volatility models, to name just a few.

This book not only reviews the most important concepts and recent developments in option pricing and modeling, but also offers insightful explications of great relevance to researchers as well as traders. For instance, readers will find formulas to overhedge convex payoffs, the derivation of Feller conditions for the Heston model, or an exposition of the latest local correlation models to correctly price basket options.

Perhaps the most exciting aspect of this book is its treatment of the latest generation of equity derivatives, namely volatility and correlation derivatives. Readers will find a wealth of information on these new securities, including original analyses and models to approach their valuation. The chapters on correlation are particularly commendable, as they shed light on an otherwise still obscure area.

The content quality, selection of topics, and level of insight truly set this book apart. I have no doubt that equity derivatives practitioners around the world, be they traders, quants or investors, will find it extremely pertinent, and I wish this book every success.

Peter Carr

*Dr. Peter Carr has over 18 years of experience in the derivatives industry and is currently Global Head of Market Modeling at Morgan Stanley, as well as Executive Director of the Math Finance program at NYU’s Courant Institute. He has over 70 publications in academic and industry-oriented journals and serves as an associate editor for eight journals related to mathematical finance. Dr. Carr is also the Treasurer of the Bachelier Finance Society, a trustee for the Museum of Mathematics in New York, and has received numerous awards, including Quant of the Year by Risk magazine in 2003, the ISA Medal for Science in 2008, and Financial Engineer of the Year in 2010.*
In 2004, while working as an equity derivatives analyst at J.P. Morgan in London, I came upon an esoteric trade: someone was simultaneously selling correlation and buying it back for a (risky) profit using two different methods. I became obsessed with the rationale behind this trade, and, after writing down the math, I discovered with excitement that with some corrections this trade led to a pure dynamic arbitrage strategy—the kind you normally find only in textbooks.

I could see, however, that transaction costs and other market frictions made the strategy very hard to implement in practice, especially for price takers on the buy side. But the fact remained that correlation could be bought and sold at very different prices, and that didn’t make sense to me. So I developed a simple “toy” model to see how this gap might be accounted for, and as I suspected I found that there should be little difference. What this meant is that one of the two correlation instruments involved in the trade, namely the correlation swap, was not priced at “fair value” according to my analysis.

Later on I refined my model, which I introduce in the last chapter of this book among other topics, and reached similar conclusions. I am very pleased that the topic of equity correlation has gained tremendous momentum since 2004, and it is one of this book’s ambitions to introduce the work of others in this highly specialized field. I have no doubt that many new exciting results are yet to be discovered in the coming years.

I also wanted to cover other key advanced concepts in equity derivatives that are relevant to traders, quantitative analysts, and other professionals. Many of these concepts, such as implied distributions and local volatilities, are now well-known and established in the field, while others, such as local and stochastic correlation, lie at the forefront of current research.

To get the most out of this book, readers must already be familiar with the terminology and standard pricing theory of equity derivatives, which can be found in my textbook *Introduction to Equity Derivatives: Theory & Practice*, second edition, also published by John Wiley & Sons.

I relied on a fair amount of advanced mathematics, and therefore a graduate scientific education is a prerequisite here, especially for those readers who want to solve the problems included at the end of each chapter.
The book is made of nine chapters, which are meant to be read sequentially, starting with an exposition of the most widespread exotic derivatives and culminating with cutting-edge concepts on stochastic correlation, which are necessary to correctly price the next generation of equity derivatives such as correlation swaps.

Some simplifications, such as zero interest rates and dividends, were often necessary to avoid convoluted mathematical expressions. I strongly encourage readers to check the particular assumptions used for each formula before transposing it into another context.

I hope this book will prove insightful and useful to its target audience. I am always interested to hear feedback; please do not hesitate to contact me to share your thoughts.
I would like to thank Peter Carr for his foreword, and David Hait and his team at OptionMetrics for providing me with very useful option data. I am grateful to my team at Wiley—Bill Falloon, Meg Freeborn, and associates—for their guidance and professionalism throughout the publication process.

Many thanks also to a group of individuals who, directly or indirectly, made this book possible: Romain Barc, Martin Bertsch, Eynour Boutia, Jose Casino, Mauro Cesa, John Dattorro, Emanuel Derman, Jim Gatheral, Fabrice Rouah, Simone Russo, Roberto Silvotti, and Paul Wilmott.

Last, a special mention to John Lyttle at Ogee Group for his help on many figures and problem solutions.