Based on a diversity-structured approach which is notably inspired by various natural forms of diversity (biological among others), this book unquestionably offers a framework, on the one hand, to the introduction of non-integer differentiation (otherwise known as fractional differentiation) as a modeling tool and, on the other hand, to the use of such a modeling form to highlight dynamic performances (and notably of damping) unsuspected in an “integer” approach of mechanics and automatic control. The “non-integer” approach indeed enables us to overcome the mass-damping dilemma in mechanics and, consequently, the stability-precision dilemma in automatic control.

This book has been written so that it can be read on two different levels: the first chapter achieves a first level of presentation which goes through the main results while limiting their mathematical development; the five remaining chapters constitute a second level of presentation in which the theoretical passages, deliberately avoided in the first chapter, are then developed at the mathematical level, but with the same goal of simplicity which aspires to make this book an example of pedagogy.

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Diversity and Non-integer Differentiation for System Dynamics
Diversity and Non-integer Differentiation for System Dynamics

Alain Oustaloup

Series Editor
Bernard Dubuisson
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Preface

Non-integer differentiation does not escape to the slogan “different operator, different properties and performances”. This is indeed the concise formula that is likely to explain the “why” of this operator, especially as most of its properties and performances favorably distinguish not only the operator itself but also the models that use it.

It is true that we have established non-integer models that overcome the mass-damping dilemma in mechanics and the stability-precision dilemma in automatic control; the technological achievements associated with these models have been made possible thanks to an adequate synthesis of the non-integer differentiation operator.

It is indeed the idea to synthesize non-integer differentiation (in a medium frequency range) through a recursive distribution of passive components, of transitional frequencies or of zeros and poles, which is at the origin of the non-integer differentiation operator real time use and, therefore, of both analogical and numerical applications that arise from it. As for the corresponding dates, the synthesis as we have led it has been developed by stages and thus proposed and experimented in the 70s for half-integer orders, in the 80s for real non-integer orders and in the 90s for complex non-integer orders.

The first technological applications of this operator (henceforth usable in real time) and notably the first application in 1975 of a “half-integer order controller” to the frequency control of a continuous dye laser, have widely contributed to take the non-integer differentiation out of the mathematician’s drawers and to give rise to new developments likely to enrich the theoretical corpus of circuits and systems.
In this way France has been the first country to experience a interest in non-integer differentiation, this renewal having been well relayed thanks to the dynamism of the foreign scientific communities, at the European level as well as at the international level.

In this context, the French institutions have encouraged research in this field through the acknowledgement of major scientific advances and the support of initiatives or actions aiming to favor the synergies between the different themes and between the academic and industrial components, the university–industry partnership having indeed been nationally rewarded by the Association Française pour la Cybernétique Economique et Technique (AFCET) ’95 Trophy distinguishing the CRONE suspension as the best technological innovation. Concerning the acknowledgements, let us cite the selection of the CRONE control as a “striking fact” of the Centre National de la Recherche Scientifique (CNRS) in 1997 and as “Flagship Innovation” of Alstom in 2000 (Hanover and Baden Baden International Fairs, 2000), a silver medal of the CNRS in 1997 and the Grand Prix Lazare Carnot 2011 of the Science Academy (founded by the Ministry of Defense). Concerning the supports, let us cite the actions financially supported by the CNRS and the Ministry of Research: the edition of “La commande CRONE” (Hermès, 1991) with the exceptional help of the ministry; the International Summer School “Fractal and hyperbolic geometries, fractional and fractal derivatives in engineering, applied physics and economics” (Bordeaux, 1994); the national project of the CNRS, “Non-integer differentiation in vibratory insulation” (1997–1999); the colloquium “Fractional Differential systems” (Paris, 1998); the launching in 1999 of the thematic action of the Ministry of Research “Systems with non-integer derivatives”; the launching in 2004 of the International Federation of Automatic Control (IFAC) Workshop “Fractional Differentiation and its Applications” through the first Workshop FDA ’04 (Bordeaux, 2004) with S. Samko as chairman of the International Programme Committee; the magisterial lecture “From diversity to unexpected dynamic performances” initiated by the French Science Academy (Bordeaux, 5 January 2012).

But this academic support also found a guarantee in the industrial support brought by a strong partnership with major companies such as the Peugeot Société Anonyme (PSA) Peugeot-Citroën, Bosch (Stuttgart) and Alstom, such a partnership indeed led to a high number of patents and technological transfers that have widely proved the industrial interest of non-integer approaches.

Alongside the shared efforts to inscribe these approaches in the realist frame of the university–industry relations, our efforts have never stopped being shared with those of the international scientific community to develop the best relations and collaborations within this community. Without aiming for exhaustiveness, let us cite the involvement of European countries involved in the diffusion, promotion and
animation within the community: the research group “Fracalmo”, which originates from *Fractional calculus* modeling, started with a round table discussion in 1996 during the 2nd International Conference “Transform methods and special functions” held in Bulgaria; the journal “Fractional Calculus and Applied Analysis” (FCAA) started in 1998 with V. Kiryakova as managing editor; the survey on the “Recent history of fractional calculus” (*Communications in Nonlinear Science and Numerical Simulation*, 2010) at the initiative of J.T. Machado who desired to make an inventory of the major documents and events in the area of fractional calculus that had been produced or organized since 1974; the symposium “Fractional Signals and Systems” (FSS) launched by M. Ortigueira in 2009 in Lisbon, then held in 2011 at Coimbra (Portugal) and in 2013 at Ghent (Belgium). Both within and outside of Europe, let us also recall the various events of the Workshop FDA after its launching at Bordeaux in 2004 (under the aegis of IFAC): Porto (Portugal) in 2006; Ankara (Turkey) in 2008; Badajoz (Spain) in 2010; Nanjing (China) in 2012; Grenoble (France) in 2013 and Catania (Italy) in 2014 (under the aegis of IEEE).

As the founder of the CRONE team, which counts today about 10 permanent researchers, I recognize this team to have always escorted me in the federative actions, launched within the national or international scientific community, to energize and harmonize research on both theoretical and applicative aspects of non-integer differentiation.
Introduction

If beyond integer differentiation many people indistinctly talk about fractional, non-integer or generalized differentiation, it seems through database analysis that fractional qualifying is used more in titles, whereas non-integer differentiation is used more in texts, with generalized differentiation appearing to be less used. So, why does non-integer qualifying benefit from our preference in this book?

In the formulation of generalized differentiation, the differentiation order is integer or non-integer, the generalized indeed including the integer and the non-integer. But the specificity of our book is certainly on the non-integer, notably through the link of non-integer differentiation with the recursivity and fractality that diversity covers. Besides, the real or complex non-integer differentiation synthesis, which is major in our contribution, is exclusively a matter for the non-integer, hence our preference. As for the fractional itself that excludes the irrational, it cannot be preferred to the non-integer that does include the irrational and the non-integer rational.

Beyond the originality and the interest of the book’s contents that the thematic nature confers, the originality of the contents’ structuration results from the author’s will to offer readings of the book at several speeds.

To that effect, the book is structured so as to offer to the reader several reading levels corresponding to an increasing level of complexity and/or of specialization. The first three chapters have indeed a more general character whereas the next three chapters are more specialized, notably Chapters 4 and 5 that respectively deal with the CRONE suspension and the CRONE control (CRONE being the French
abbreviation of *Commande Robuste d’Ordre Non Entier*, in English, *non-integer order robust control*).

Thus, the reader who only requires a consciousness-raising to the non-integer approach and to the most striking results on the subject can limit his reading to Chapter 1. The specificity of this chapter is indeed its approach, which is both very general to constitute an overview on the non-integer approach and very targeted to enable a simplified presentation, even an educational example, liable to quickly sensibilize the reader to the thematic interest and thus to give him the desire to go further in his reading.

The reader, who is concerned about the proofs and notably about a deepening of the proofs briefly led in Chapter 1, can partially or fully read the chapters and appendices cited in Chapter 1 with a link enabling then to directly find the expected supplement.

Finally, the reader eager to benefit from the subtleties acquired by the author in 40 years of sustained research in the non-integer field, is invited to browse the whole book, including the Appendices, which constitute mini-chapter, complementary at the physics level as well as the mathematical level.

Chapter 1 largely contributes to the originality of the book. It indeed results from the magisterial conference presented in the framework of the *Grand Prix Lazare Carnot 2011* of the French Academy of Sciences. On the form, given the various disciplines within this academy, our contribution is a simplified presentation at the limit of scientific popularization. On the content, through a *structured approach of diversity*, this chapter unquestionably offers a framework, on the one hand, to the introduction of non-integer derivative as a modeling tool, on the other hand, to the use of such a modeling form to put into light dynamic performances (and notably of damping) unsuspected in an integer approach of mechanics and automatic control. The non-integer approach indeed enables us to overcome the mass-damping dilemma in mechanics and consequently the stability-precision dilemma in automatic control. Furthermore, the metallic and hydropneumatic versions of the CRONE suspension are the subject of a unified presentation through various forms of diversity leading to non-integer differentiation. To illustrate our strategy through a study system, we have studied the relaxation of water on a porous dyke. Such as obtained, the model of the water-dyke interface and the one governing the water relaxation are non-integer models of order between 0 and 1 for the interface and of order between 1 and 2 for the relaxation, these models being valid in a *medium frequency range*.

Chapter 2 proves and validates the *damping robustness* of the water relaxation on a porous dyke, the various study stages going from the object to its performances
and to their experimental verification. The first stage (section 2.2) consists of starting off with a recursive parallel arrangement of serial RC cells to obtain a non-integer differentiation as a model of the water-dyke interface. Obtaining a differentiation non-integer order results from a smoothing of the admittance Bode asymptotic diagrams. The second stage (section 2.3) consists of using the dynamics fundamental principle to change from the non-integer derivative so obtained to a non-integer differential equation as a model governing water relaxation. After the analytical determination of the relaxation, which shows damping robustness in the time domain and constitutes the third stage (section 2.4), the fourth stage (section 2.5) illustrates damping robustness in operational domain and in the frequency domain. Lastly, the fifth stage (section 2.6) consists of experimentally verifying damping robustness through an electronic circuit made of operational amplifiers. This circuit is achieved in such a way that its transmittance respects the non-integer differential equation, which governs the water relaxation on a porous dyke.

Chapter 3 deals with non-integer differentiation, its memory and its synthesis. Section 3.2 presents, in discrete time, non-integer differentiation through the extension, to the non-integer case, of the generic form of integer differentiation. Section 3.3 presents, in continuous time, non-integer differentiation from repeated integer integration via non-integer integration. Thus presented in discrete time then in continuous time, non-integer differentiation is the subject, in section 3.4, of a study of its properties in a sinusoidal steady state. In this operating state and as regards kinematic magnitudes, therefore in terms of position, speed and acceleration, the non-integer derivative of position takes into account “position and speed” or “speed and acceleration” whether the differentiation order $n$ is “between 0 and 1” or “between 1 and 2”. Section 3.5 is devoted to the memory phenomenon associated with non-integer differentiation. The discrete form of the non-integer derivative as presented in section 3.2, directly shows that the function to be differentiated is taken into account through its values at all the past instants. Non-integer differentiation thus introduces a memory notion such that the past attenuation or accentuation is imposed by the differentiation order. As this memory notion is not without evoking a subtle form of memory, an investigation trail is proposed by considering an aspect of the human memory studied in cognitive psychology. Section 3.6 deals with the synthesis of non-integer differentiation through a realistic synthesis turning on a non-integer differentiator bounded in frequency. This synthesis is an approximation of the differentiator ideal version through a (limited) recursive distribution of countable zeros and poles. If the real non-integer differentiator synthesis is based on a recursive distribution of countable real zeros and poles, the complex non-integer differentiator synthesis is based on a recursive distribution of complex zeros and poles, which can also be counted [OUS 00].
Chapter 4 deals with an application of non-integer differentiation in the (automotive) vehicle suspension area, namely the CRONE suspension, CRONE being in this case the French acronym of Comportement Robuste d’Ordre Non Entier. The various synthesis stages, from concept to practical achievement, are developed and performance tests turning on prototypes validate the theoretical expectations. The CRONE suspension is presented through two versions in conformity with the hydropneumatic and metallic technologies used for its achievement: a hydropneumatic version that results from the transposition, in vibratory insulation, of the porous dyke interpretation as defined in Chapter 1 and Appendix 2; a metallic version that is obtained from the usual (or traditional) suspension by replacing the order 1 traditional dashpot with a non-integer order dashpot. As regards stability degree, robustness tests are carried out for different parametric states of the usual and CRONE suspensions. These tests turn successively on the frequency and step responses of the two suspensions and also the roots of their characteristic equations. For the CRONE suspension, they well reveal the robustness of the resonance ratio of the frequency response, the first overshoot of the step response and the damping ratio stemming from the characteristic equation roots.

Chapter 5 shows that the transposition of damping robustness (Chapter 2) in automatic control is at the origin of the initial approach of the CRONE control. Indeed, its first principles stem from the interpretation of the model of water relaxation on a porous dyke in conformity with two legitimate interpretations, which, respectively, define the first generation CRONE control through a controller phase locking and the second generation CRONE control through an open loop phase locking. The change from second to third generation CRONE control results from the generalization of the vertical template in conformity with two generalization levels: in the first generalization level, the vertical template is replaced by a straight line segment of any direction, called a generalized template, the direction of which is given by that of the main axis of the uncertainty domain calculated at the open loop unit gain frequency or the closed loop resonance frequency in tracking; in the second generalization level, the generalized template is replaced by a set of generalized templates, called multi-templates, which leads to a curvilinear template generalizing the rectilinear template formed by the generalized template, the tangent at each point of the curvilinear template being the main axis of the corresponding uncertainty domain.

Chapter 6 studies the close link between recursivity and non-integer differentiation through several recursive electric networks. Thus, section 6.2 presents an exhaustive study of an indefinite recursive parallel arrangement of series RC cells. Section 6.3 is devoted to the study of a recursive arborescent arrangement of gamma RLC cells, the arborescence being obtained according to a bifurcation iterative process in conformity with the lung intern structure seen under the
respiratory angle. Led in such a way, this study presents the advantage reducing the arrangement so obtained to a recursive cascade arrangement of gamma RLC cells. Section 6.4 proposes a unified study of indefinite recursive parallel arrangements of RL, RC and RLC cells. The originality of this study results from methods that are said to be heuristic as they are founded on heuristic assumptions that, in this case, enable us to quickly obtain exact results of remarkable simplicity. Section 6.5 deals with eight recursive arrangements of RC and RL cells obtained from four possible combinations of a resistance and a capacitance and from four possible combinations of a resistance and an inductance. It is about the two arrangements stemming from a recursive parallel arrangement of series RC or RL cells, the two arrangements stemming from a recursive series arrangement of parallel RC or RL cells, the two arrangements stemming from a recursive cascade arrangement of gamma RC or RL cells, and also the two arrangements stemming from a recursive cascade arrangement of gamma CR or LR cells. Section 6.6 shows how to reparameterize the cells to define a unit gain frequency of the indefinite recursive parallel arrangements of series RC and RL cells. Section 6.7 deals with the energy stored by a non-integer differentiator then by a non-integer integrator. Two dual energetic approaches are led through an indefinite recursive parallel arrangement of series RC cells for the non-integer differentiator and an indefinite recursive series arrangement of parallel RL cells for the non-integer integrator.
Chapter 1

From Diversity to Unexpected Dynamic Performances

In the context, this chapter takes inspiration from the magisterial conference presented in the framework of the Grand Prix Lazare Carnot 2011 of the French Academy of Sciences. In its form, given the various disciplines within this academy, our contribution is a simplified presentation, almost in layman’s terms, at the limit of scientific popularization. The content, through a structured approach of diversity, our main contribution is putting the wrong mass-damping dilemma into mechanics and stability-precision dilemma into automatic control.

1.1. Introduction

This chapter offers a framework to the introduction and the use of the non-integer derivative.

On the one hand, this framework allows us to introduce the non-integer derivative as a modeling tool, particularly in the modeling of a porous face and the water relaxation on such a face. On the other hand, from such a non-integer modeling, we are able to solve a physics problem which is without solution so far, which is in this case the mass-damping dilemma in mechanics, to which the stability-precision dilemma in automatic control corresponds.

It turns out that this framework finds its essence and its adequation in a structured approach of diversity. Without a precedent on the matter, this approach that is inspired by various natural forms of diversity (biological among others) leads,
without high mathematical developments in this chapter, to *dynamical performances (and notably damping) unexpected* in an “integer” approach of mechanics and automatic control.

In concrete terms and particularly technologically, there exist as many technological solutions as diversity forms. Therefore, a declension of these forms is proposed here, each of them being the subject of a structural definition, an illustration through a biological example and, eventually, a transposition to vibratory isolation through various technological solutions in terms of *robust suspension* (damping independent of mass):

– A first diversity form is a *multitude of different elements of same action level*. This is the most perceptible elementary form.

It is the case of an element arrangement that is borrowed from *striated muscle fibrous structure or porous face alveolar structure* (whose pores are of different sizes).

It is particularly the case of the *hydropneumatic version* of the CRONE suspension that uses different springs and different dashpots, this version having been implanted on “Citroën” vehicles, the BX (in 1995), the XM (in 2000) and the C5 (in 2010).

– A second diversity form is a *multitude of identical or similar elements of different action level*.

It is the case of an element arrangement that is borrowed from the meshed structure of similar neuron classes (cerebellum or cerebral cortex and motor neurons of the spinal cord). If the neurons are similar, they occupy different positions in the space.

– A third diversity form is a *multitude of different elements of different action level*. This form defines the highest diversity degree.

It is the case of an element arrangement that is borrowed from lung arborescent structure. The deeper the layers, the smaller the branches:

- and the greater their resistance as the restriction effect increases;
- and the smaller their capacitance as the tank effect decreases.

– Last but not least, a fourth diversity form, of different nature, is a *unique element continuously variable* that presents a *multitude of different values* on a time horizon.
It is the case of a variable device that is borrowed from an aorta transverse movement.

It is particularly the case of the controlled dashpot that presents a continuously variable oil laminating section and that equips the metallic version of the CRONE suspension, this version having been implemented on a Peugeot vehicle, the 406 (in 1998).

1.2. An issue raising a technological bottle-neck

A problem that is well known to mechanical engineers and test pilots (car, plane, etc.) consists of a stability degree that decreases when carried mass increases. In other words, the higher the car load, the greater the oscillation through a damping decrease.

This problem, still unsolved by the mechanics integer approach, defines a genuine technological bottleneck, which is called “mass-damping interdependence” or, more precisely, “mass-damping dilemma”, so far an unavoidable dilemma, which well expresses that an increase of mass is accompanied by a damping decrease.

The Citroën 2 CV in civilian domain and the Rafale in military domain are particularly concerned by this phenomenon: the 2 CV, as it is little damped and very light, 570 kg without a load; the Rafale, as its weight varies from 9.5 tons without a load to 24 tons in full load through kerosene and arming.

As an excellent illustration, let us consider a suspension of type mass-spring-dashpot (Figure 1.1) and the bodywork response to a step displacement of the wheel for different masses (Figure 1.2).

![Figure 1.1. Suspension of type mass-spring-dashpot](image)
In terms of performances, the damping ratio, which measures damping by the decrease rate of the successive overshoots, admits an expression of the form (Appendix 1):

\[ \zeta = \frac{K_0}{\sqrt{M}}, \]  

which shows well (with the presence of the mass at the denominator) that the damping ratio decreases when the mass increases.

1.3. An aim liable to answer to the issue

How to answer to such an issue by an unprecedented approach in the matter? It can be answered by seeking a structural damping determined by the structure of a system independently of the system parameters (notably of the mass in our study context).

To better target this aim, we can already assume that a complex structure (like the one of a high-dimensional multi-branch network (Figure 1.3)) is, through its number of components, liable to reduce the effect of each of the components (and then of the associated parameters).

It is true that the higher the number of components, the less the action of each of them on the global behavior. We will indeed see, later, that if multiplicity is not always sufficient, it is nevertheless always necessary.
1.4. A strategy idea liable to reach the aim

*How to reach the aim of a damping exclusively linked to the structure?*

This aim can be reached by seeking a *structure* founded on the *idea of diversity* compatible with the one of a *complex structure*.

1.4.1. *Why diversity?*

The reason for diversity is that this strategy is inspired by the interest of *disorder* taken in the sense of a natural or artificial disorder that borrows from the diversity.

Among the sources on the disorder interest, the following are of importance:

– *the damping nature of highly disturbed media*, observed by the dyke builders in the 17th Century through experimental studies of water relaxation on costal or fluvial dykes;

– *the stabilizing nature of disorder*, noticed by Prigogine, winner of the Nobel Prize in Chemistry in 1977, through his researches on instability and chaos in thermodynamics;

– *the anarchic distribution of regular blocks to artificially recreate disorder*, personally observed on the Socoa jetty in the 1980s, which has enabled us to measure human’s will to recreate, according to his means, disorder.

1.4.2. *What does diversity imply?*

Through multiple different elements, *diversity* associates *multiplicity* and *difference*. 

![High dimension multi-branch network](image)
To avoid any extrapolation, we insist on specifying that the considered elements are (Figure 1.4) geometrical elements (or patterns) or system elements (or components).

1.5. On the strategy itself

1.5.1. The study object

To support the proceedings of our strategy, we have chosen a natural study system, though stylized, as for the representation. This study object (or system) is certainly no stranger to the damping properties of the highly disturbed (or uneven) dykes, notably those forming air pockets compressible by water advance.

This leads us to say that the study object (Figure 1.5) is motion water of mass $M$ which (horizontally) relaxes on a porous dyke whose pore localization is here reduced to the face (Appendix 2).