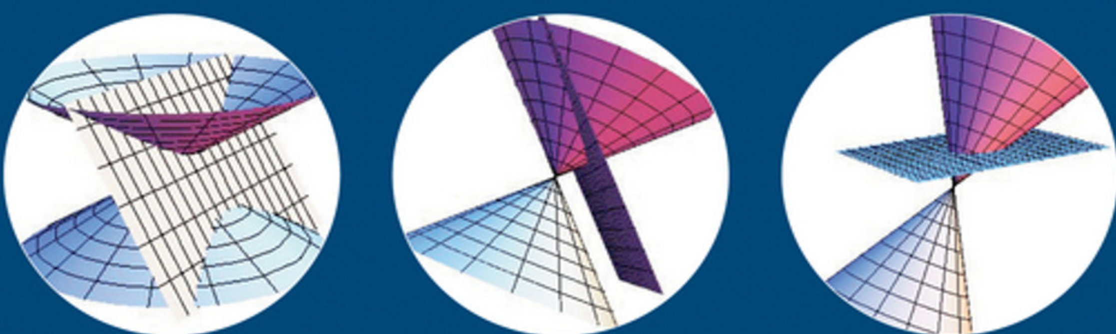
THIRD EDITION

THE HISTORY OF MATHEMATICS

A BRIEF COURSE



ROGER L. COOKE

WILEY

THE HISTORY OF MATHEMATICS

THE HISTORY OF MATHEMATICS
A BRIEF COURSE

THIRD EDITION

Roger L. Cooke
Department of Mathematics and Statistics
University of Vermont
Burlington, VT

WILEY
A JOHN WILEY & SONS, INC., PUBLICATION

Published by John Wiley & Sons, Inc., Hoboken, New Jersey.
Published simultaneously in Canada.

Library of Congress Cataloging-in-Publication Data:

Cooke, Roger, 1942-
The history of mathematics : a brief course / Roger L. Cooke. – 3rd ed.
p. cm.
Includes bibliographical references and index.
ISBN 978-1-118-21756-6 (cloth)
1. Mathematics–History. I. Title.
QA21.C649 2013
510'.9–dc23
2012020963

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

CONTENTS

PREFACE

Like its immediate predecessor, this third edition of *The History of Mathematics: A Brief Course* must begin with a few words of explanation to users of the earlier editions. The present volume, although it retains most of the material from the second edition, has been reorganized once again. In the first edition each chapter was devoted to a single culture or period within a single culture and subdivided by mathematical topics. In the second edition, after a general survey of mathematics and mathematical practice in Part 1, the primary division was by subject matter: numbers, geometry, algebra, analysis, mathematical inference. After long consideration, I found this organization less desirable than a chronological ordering. As I said in the preface to the second edition,

> For reasons that mathematics can illustrate very well, writing the history of mathematics is a nearly impossible task. To get a proper orientation for any particular event in mathematical history, it is necessary to take account of three independent "coordinates": the time, the mathematical subject, and the culture. To thread a narrative that is to be read linearly through this three-dimensional array of events is like drawing one of Peano's space-filling curves. Some points on the curve are infinitely distant from one another, and the curve must pass through some points many times. From the point of view of a reader whose time is valuable, these features constitute a glaring defect. The problem is an old one, well expressed eighty years ago by Felix Klein, in Chapter 6 of his *Lectures on the Development of Mathematics in the Nineteenth Century*:
>
> > I have now mentioned a large number of more or less famous names, all closely connected with Riemann. They can become more than a mere list only if we look into the literature associated with the names, or rather, with those who bear the names. One must learn how to grasp the main lines of the many connections in our science out of the enormous available mass of printed matter without getting lost in the time-consuming discussion of every detail, but also without falling into superficiality and dilettantism.

I have decided that in the lexicographic ordering of the three-dimensional coordinate system mentioned above, culture is the first coordinate, chronology the second, and mathematical content the third. That is the principle on which the first six parts of the present edition are organized. In the seventh and final part, which covers the period from 1800 on, the first coordinate becomes irrelevant, as mathematics acquires a worldwide scope. Because so much new mathematics was being invented, it also becomes impossible to give any coherent description of its whole over even a single decade, and so the chronological ordering has to become the second coordinate, as mathematical content becomes the first.

Changes from the Second Edition

Besides the general reorganization of material mentioned above, I have also had a feeling that in the previous edition I succumbed in too many places to the mathematician's impulse to go into mathematical detail at the expense of the history of the subject and to discuss some questions of historical minutiae that are best omitted in a first course. I have therefore condensed the book somewhat. The main difference with earlier editions is that I have tried to adapt the text better to the needs of instructors. To that end, I have made the chapters more nearly uniform in length, usually ten to twelve pages each, putting into each chapter an amount of material that I consider reasonable for a typical 50-minute class. In addition, I have scrutinized the problems to be sure that they are reasonable as homework problems. They are of three types: (1) those that develop a mathematical technique, such as the Chinese method of solving polynomial equations numerically, the *kuttaka*, computation by the Egyptian method, *prosthaphæresis*, and the like; (2) those that ask the student to recall a specific set of historic facts (these generally have brief answers of a sentence or two and should be answerable directly from the narrative); and (3) those that ask the student to speculate and synthesize the history into a plausible narrative, including possible motives for certain investigations undertaken by mathematicians. In survey chapters at the beginning of some parts, only the last two types occur.

The book is divided into seven parts. The first six, comprising the first 34 chapters, contain as systematic a discussion as I can manage of the general history of mathematics up to the nineteenth century. Because it is aimed at a general audience, I have given extra attention to topics that continue to be in the school curriculum, while at the same time trying to discuss each topic within the context of its own time. At the end of each chapter are a few questions to provide a basis for classroom discussions. More such questions can be found in the accompanying teacher's manual. I believe that these 34 chapters, totaling about 400 pages, constitute a one-semester course and that any extra class meetings (I assume 42 such meetings) will be devoted to quizzes, midterms, and perhaps one or two of the specialized chapters in Part VII.

The seventh and last part of the book consists of more narrowly focused discussions. Except for Chapter 35, which discusses a small portion of the history of women in mathematics, these are updates, arranged by subject matter and carrying the history of the topics they treat into the twentieth century. Since this material involves modern mathematics, it is technically much more difficult than the first six parts of the book, and the mathematical homework problems reflect this greater difficulty, making much higher demands on the reader's mathematical preparation. Instructors will of course use their own judgment as to the mathematical level of their students. In some of these chapters, I have exceeded the self-imposed limit of 12 pages that I tried to adhere to in the first six parts of the book, assuming that instructors who wish to discuss one of these chapters will be willing to devote more than one class meeting to it.

Elementary Texts on the History of Mathematics

A textbook on the history of mathematics aimed at a first course in the subject, whose audience consists of teachers, mathematicians, and interested students from other specialties, cannot be as complete or as focused as an encyclopedia of the subject. Connections with other areas of science deserve attention quite as much as historical issues of transmission and innovation. In addition, there are many mathematical skills that the reader cannot be presumed to have, and these need to be explained as simply as possible, even when the explanation does not faithfully reproduce the historical text in which the subject arose. Thus, I have hybridized and simplified certain mathematical techniques in order to provide a usable model of what was actually done while stripping away complications that make the original texts obscure. This much sacrifice of historical accuracy is necessary, I believe, in order to get to the point within the confines of a single semester. At the same time, I think the exposition of these and other topics gives a reasonable approximation to the essence of the original texts.

This concept of a reasonable approximation to the original presents a problem that requires some judgment to solve: How "authentic" should we be when discussing works written long ago and far away, using concepts that have either disappeared or evolved into something very different? Historians have worked out ways of giving some idea of what original documents looked like. We can simply write numbers, for example, in our own notation. But when those numbers are part of a system with operational connections, it is necessary to invent something that is isomorphic to the original system, so that, for example, numbers written in sexagesimal notation still have a sexagesimal appearance, and computations done in the Egyptian manner are not simply run through a calculator and the output used. This problem is particularly acute in Euclidean geometry, which makes no reference to any units of length, area, or volume. The "Euclidean" geometry that is taught to students in high school nowadays freely introduces such units and makes use of algebraic notation to give formulas for the areas and volumes of circles, spheres, cones, and the like. This modernization conceals the essence of Euclid's method, especially his theory of proportion. He did not speak of the area of a circle, for example, only of the ratio of one circle to another, proving that it was the same as the ratio of the squares having their diameters as sides (Book 12, Proposition 2). How much of that authentic Euclidean geometry, which I call *metric-free*, should the student be subjected to? Without it, many of the most important theorems proved by Euclid, Archimedes, and Apollonius look very different from their original forms. On the other hand, it *is* cumbersome to expound, and one is constantly tempted to capitulate and "modernize" the discussion. I have made the decision in this book to draw the line at conic sections, using symbolic notation to describe them, though I do so with a very bad conscience. But I would never dream of presenting, in an introductory text, the actual definition of the latus rectum given by Apollonius. I try to hold the use of symbolic algebra to a minimum, but compromises are necessary in the real world.

When it comes to algebra, symbolic notation is a very late arrival. Algorithms for solving cubic and quartic equations preceded it, and those algorithms are very cumbersome to explain without symbols. Once again, I surrender to necessity and try to present the essence of the method without getting bogged down in the technical details of the original works. There is a further difficulty that most students have learned algebra by rote and can carry out certain operations, but have no insight into the essence of the problems they have been taught to solve. They may know what American students call the FOIL method of solving quadratic equations with integer coefficients, and some of them may even remember the quadratic formula, but I have yet to encounter a student who has grasped the simple fact that solving a quadratic equation is a way of finding two numbers if one knows their sum and product. Nor have I found a student who has the more general insight that classical algebra is the search for ways of rendering explicit numbers that are determined only implicitly, even though this insight is crucial for recognizing algebra when it occurs in early treatises, where there is no symbolic notation.

Besides the enormous amount of mathematics that the human race has created, so enormous that no one can be really expert except over a tiny region of it, the historian has the additional handicap of trying to fit that mathematics into the context of a wide range of cultures, most of which will not be his or her area of expertise. I feel these limitations with particular keenness when it comes to languages. Despite a lifetime spent trying to acquire new languages in what spare time I have had, I really feel comfortable (outside of English, of course) only when working in Russian, French, German, Latin, and ancient Greek. (I have acquired only a modest ability to read a bit of Japanese, which I constantly seek to expand.) Of course, having a language from each of the Romance, Germanic, and Slavic groups makes it feasible to attempt reading texts in perhaps two dozen languages, but one needs to be on guard and never rely on one's own translations in such cases. I am most sharply aware of my total dependence on translations of works written in Chinese, Sanskrit, and Arabic. Even though I report what others have said about certain features of these languages for the reader's information, let it be noted here and now that anything I say about any of these languages is pure hearsay.

Just to reiterate: One can really glimpse only a small portion of the history of mathematics in an introductory course. Some idea of how much is being omitted can be seen by a glance at the website at the University of St Andrews.

http://www-history.mcs.st-and.ac.uk/

That site provides biographies of thousands of mathematicians. Under the letter G alone there are 125 names, fewer than 40 of which appear in this book. While many of the "small fry" have made important contributions to mathematics, they do not loom large enough to appear on a map the size and scale of the present work. Thus, it needs to be kept in mind that the picture is being painted in very broad brush strokes, and many important details are simply not being shown. Every omission is regrettable, but omissions are necessary if the book is to be kept within 600 pages.

And, finally, a word about the cover. When I was asked what kind of design I wished, I thought of a collage of images encompassing the whole history of the subject: formulas and figures. In the end, I decided to keep it simple and let one part stand for the whole. The part I chose was the conic sections, because of the length and breadth of their influence on the history of the subject. Arising originally as tools to solve the problems of trisecting the angle and doubling the cube, they were the subject of one of the profoundest treatises of ancient times, that of Apollonius. Later, they turned out to be the key to solving cubic and quartic equations in the work of Omar Khayyam, and they became a laboratory for the pioneers of analytic geometry and calculus to use in illustrating their theories. Still later, they were a central topic in the study of projective geometry, and remained so in algebraic geometry far into the nineteenth century. It is no accident that non-Euclidean geometries are classified as elliptic and hyperbolic, or that linear partial differential equations are classified as elliptic, parabolic, and hyperbolic. The structure revealed by this trichotomy of cases for the intersection of a plane with a cone has been enormous. If any one part deserves to stand for the whole, it is the conic sections.

PART I

WHAT IS MATHEMATICS?

This first part of our history is concerned with the "front end" of mathematics (to use an image from computer algebra)—its relation to the physical world and human society. It contains some general considerations about mathematics, what it consists of, and how it may have arisen. This material is intended as an orientation for the main part of the book, where we discuss how mathematics has developed in various cultures around the world. Because of the large number of cultures that exist, a considerable paring down of the available material is necessary. We are forced to choose a few sample cultures to represent the whole, and we choose those that have the best-recorded mathematical history. The general topics studied in this part involve philosophical and social questions, which are themselves specialized subjects of study, to which a large amount of scholarly literature has been devoted. Our approach here is the naive commonsense approach of an author who is not a specialist in either philosophy or sociology. Since present-day governments have to formulate *policies* relating to mathematics and science, it is important that such questions not be left to specialists. The rest of us, as citizens of a republic, should read as much as time permits of what the specialists have to say and make up our own minds when it comes time to judge the effects of a policy.

Contents of Part I

1. Chapter 1 (Mathematics and Its History) considers the general nature of mathematics and gives an example of the way it can help to understand the physical world. We also outline a series of questions to be kept in mind as the rest of the book is studied, questions to help the reader flesh out the bare bones in the historical documents.
2. Chapter 2 (Proto-mathematics) studies the mathematical reasoning invented by people in the course of solving the immediate and relatively simple practical problems of administering a government or managing a construction site. In this area we are dependent on archaeologists and anthropologists for the historical information available.

The History of Mathematics: A Brief Course, Third Edition. Roger L. Cooke.