Chaos Analysis and Chaotic EMI Suppression of DC-DC Converters

Bo Zhang | Xuemei Wang
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AND CHAOTIC EMI
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Preface

This book focuses on the analysis and application of chaos to reduce harmful EMI (electromagnetic interference) of DC–DC converters. DC–DC converters are typical switching systems which have plenty of nonlinear behaviors, such as bifurcation and chaos. The nonlinear behaviors of DC–DC converters have been studied in depth over the last 30 years; in contrast, people are still puzzled by the practical applications of bifurcations and chaos in switching converters. The EMI, due to the high rates of changes of voltage and current resulting from the switching process of power semiconductors, has been a major design constraint of DC–DC converters for a long time. How to reduce the annoying, harmful EMI has consistently attracted much research interest. The conventional filtering and shielding approaches have disadvantages mainly in cost and weight. Is there any method of suppressing EMI which is simple, ingenious, and may solve the EMI problem fundamentally?

Pulse width modulation (PWM) control is the most common and important control method for switching converters. Over the last 10 years, researchers have concerned themselves with applying frequency modulation technology to the suppression EMI of switching converters. The basic principle of frequency modulation makes the EMI energy distribution uniform and reduces the peak values of spectrum, thus suppressing EMI. Some commonly used frequency modulation techniques are jitter frequency, periodic frequency modulation, random frequency modulation, and so on. Comparing these frequency modulation techniques, it is recognized that chaotic frequency modulation may reduce the harmonic greatly and improve the spectral distribution because the chaos signal has a prominent characteristic of continuous power spectral density. Combining the chaos and PWM control technique can distribute the harmonics of the DC–DC converters continuously and evenly over a wide frequency range, consequently the EMI may be reduced.

Although chaotic frequency modulation has significant advantages, this technology has not been applied in industrial products because there are two barriers between the theory research and applications. One is that the current research results of chaos are almost all abstract mathematical presentations baffling to power electronics engineers. Another is that the spectral feature of the chaotic signal is inner-harmonics and the non-integral multiplies harmonics which are difficult to estimate and quantify in the
traditional Fast Fourier Transform (FFT) method. Toward the first objective, analysis approaches associated with chaos phenomena in power electronics, which are easy to understand and employ, are needed. For the second objective, it is necessary to adapt some available method beyond FFT to express the chaos power spectral density correctly. So, there are two parts in this book. The first part, including Chapters 2–4, is concerned with a series of the new chaos quantified method of DC–DC converters; the second part, including Chapters 5–10, is about chaotic power spectral density estimating and chaotic modulating technologies.

A brief outline of the 10 core chapters is as follows. After a review of the fundamentals of chaos behaviors of DC–DC converters, the authors present some recent findings such as symbolic entropy, complexity, and invariant probability distribution, to analyze the characteristics of chaotic DC–DC converters in the next three chapters. Using these methods, the statistic characteristics of chaotic DC–DC converters are extracted and the foundations for the following researches of chaotic EMI suppression are reinforced. The focus then is transferred to estimating the power spectral density of chaotic PWM converters after the introduction of EMI and Electromagnetic Compatibility (EMC) of switching converters. After an introduction to the basic principles of spectrum analysis, chaos point process, Prony, and wavelet analysis methods are suggested for estimating the power spectral density of chaotic PWM converters. Finally, some design-oriented applications which provide a good example of applying chaos theory in engineering practice illustrate the effectiveness in suppressing EMI of the proposed chaotic PWM.

The book highlights the advanced research works in the fields of statistic characteristics of nonlinear behavior and chaotic PWM modulation technology to suppress the EMI of switching converters. The proposed analysis method will directly contribute to the studies of EMI suppression and be available to practical design problems. We think, in the near future, the method and technology proposed in this book will benefit power electronics engineers and will be widely used in power electronics.

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1 Nonlinear Models and Behaviors of DC–DC Converters

1.1 Introduction

DC–DC converters are widely used in industrial and commercial applications with the requirement of regulating DC power, such as computers, spacecrafts, medical instruments, communication devices, electrical vehicles, and so on [1–6]. The technology of DC–DC conversion is a major subject area in the field of power engineering and drives, and has been under development for seven decades [5]. It is well known that DC–DC converters are typical nonlinear systems because of their switching processes. Some irregular behavior, such as subharmonics and intermittent instability, has been observed constantly in practice. Over nearly three decades, lots of nonlinear behavior, such as period-doubling bifurcation, Hopf bifurcation, border collision bifurcation, torus bifurcation, coexistence attractor, intermittent chaos and chaos, has been studied by power electronics researchers one after the other [7–17].

Although the nonlinear behavior of DC–DC converters has been intensively studied, people are still puzzled by the practical application of bifurcations and chaos in switching converters [18]. Miniaturization is the main approach to reduce the size of DC–DC converters, and a higher switching frequency must be utilized with the requirement of the smaller size. Electromagnetic interference (EMI), resulting from bursting changes of voltage or current (dv/dt or di/dt), has been a major design constraint for a long time due to the high switching frequency. The question of how to reduce annoying and harmful EMI has attracted much research interest.

Fortunately, chaos research opens up the possibility of the application of nonlinear characteristics in power electronic systems, and might be a new method to improve the performance of power electronic systems in the future. For example, the switching speed between unstable periodic orbits (UPOs) embedded in chaos attractors is fast. Using this performance, DC–DC converters working in chaos may have a fast dynamic response [19]. Moreover, it has been actually proved that the application of...
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Chaotic power spectral characteristics can effectively reduce the EMI and improve the electromagnetic compatibility (EMC) of DC–DC converters [20–25].

Although spectrum analysis of chaotic switching converters has resulted in some research achievements, there are still fewer quantitative indicators and comprehensive assessments of the characteristics of the chaotic spectrum because the switching converter is a complex nonlinear, nonautonomous, and time varying system. First of all, we need to know about the nonlinear behavior of switching converters and some basic dynamical concepts. In this chapter, we will begin with an introduction to DC–DC converters and their operation modes. Next, the conventional modeling methods are presented. Some basic nonlinear dynamics knowledge is then introduced in the rest of this chapter.

1.2 Overview of PWM DC–DC Converters

1.2.1 Principle of Pulse Width Modulation

DC–DC converters supply a regulated DC output voltage to a variable-load resistance effectively. Switch-mode DC–DC converters convert one DC input voltage $V_{\text{in}}$ to a desired output voltage $V_{\text{out}}$ by storing the input energy temporarily and then releasing that energy to the output load. The average value of voltage is moderated by controlling the switch on and off durations ($t_{\text{on}}$ and $t_{\text{off}}$). In Figure 1.1, the input voltage $V_{\text{in}}$ is chopped to an output voltage $V_{\text{out}}$. The longer the switch is on compared to the off duration, the higher the power supplied to the load. So, the duty cycle, describing the ratio of on duration to the switching time period $T$, is defined as

$$d = \frac{t_{\text{on}}}{T} \quad (1.1)$$

Obviously, the average output voltage varies depending on $d$. By varying $d$, $V_{\text{out}}$ can be controlled. So, this method is called Pulse Width Modulation (PWM).

The simplest way to generate a PWM signal is by comparison, which requires only a sawtooth or a triangle waveform and a comparator. When the value of the control signal ($v_{\text{con}}$, the dash line, commonly comes from the error of the actual output voltage and reference voltage) is greater than the modulation waveform ($v_{\text{ramp}}$, the sawtooth with fixed frequency $T$ and peak value $V_{U}$), the comparator output becomes high,
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Figure 1.2  Pulse width modulation concept

Figure 1.3  Closed-loop controlled PWM system to regulate the output voltage

otherwise it is in the low state shown in Figure 1.2 [4]. In practical terms, a PWM controller consists of three main components: a clock for setting the switching frequency, an output voltage error amplifier, and a sawtooth signal synchronized with the clock.

To illustrate, a PWM step-down converter and feedback loop block is shown in Figure 1.3 as an example. A closed-loop converter contains three ports: a switching main circuit, sampling circuit, and a control circuit that is introduced to regulate the output voltage. It is desired to design the feedback system in such a way that the output voltage is accurately regulated, and is insensitive to disturbances in $v_g$ or in the load current [3]. A control system can be constructed by causing the output voltage to follow a given reference voltage by varying the duty cycle, because the output voltage is a function of the switch’s duty cycle.

1.2.2 Basic Topologies of DC–DC Converters

According to incomplete statistics, there have been more than 500 prototypes of DC–DC converters developed over the past seven decades. But all of them come
Figure 1.4 Basic topologies of nonisolated DC–DC converters including (a) buck, (b) boost, (c) buck–boost, and (d) Ćuk converters

from the several basic topologies shown in Figure 1.4, where switch $S$ and diode $D$ are alternately on and off. In these topologies, converters, are the two most basic topologies. The buck–boost converter carries out both the stepping up and down action. The Ćuk converter is a duality buck–boost converter.

If the output is required to be electrically isolated from the input, isolated DC–DC converters, whose isolation is provided by a high frequency isolation transform, are needed. There are two kinds of isolated topologies, unidirectional and bidirectional core excitation based on the way they use the transformer core. The unidirectional
Figure 1.5 Isolated DC–DC converters including (a) forward, (b) flyback, (c) half-bridge, and (d) full-bridge converters
1.2.3 Operation Modes of DC–DC Converters

DC–DC converters operate in one of two modes depending on the characteristics of the inductor current [2, 4]:

1. continuous conduction mode (CCM),
2. discontinuous conduction mode (DCM).

As shown in Figure 1.6, the continuous conduction mode is defined by the continuous output current (greater than zero) over the entire switching period, whereas the DCM is defined by the discontinuous output current (equal to zero) during any portion of the switching period. Each mode is discussed in relation to the buck–boost converters in subsequent sections [4].

1.2.3.1 Continuous Conduction Mode

The operation of CCM in steady state consists of two states – switch-on and switch-off modes – whose equivalent circuits of boost converter are illustrated in Figure 1.7. When the switch is on for a time duration $t_{on}$, the switch conducts the source power and the diode becomes reverse biased, then the inductor current linearly increases. When the switch is off for a time duration $t_{off}$, the diode becomes forward biased, and the inductor current increases.

1.2.3.2 Discontinuous Conduction Mode

The operation of DCM in steady state consists of three states besides the two modes mentioned above, and the third one is that $S$ and $D$ are all off and the inductor current

![Figure 1.6 Inductor current waveforms at (a) CCM and (b) DCM](image-url)
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Figure 1.7 CCM circuit states of boost converter when (a) $S$ on, $D$ off and (b) $S$ off, $D$ on

Figure 1.8 DCM circuit state of boost converter ($S$ off, $D$ off)

stays zero. Figure 1.8 shows the waveforms for the DCM. In this mode, the inductor current drops all the way to zero some time after the switch is turned off, and then remains at zero, with the transistor and diode both off, until the transistor is turned on again.

1.2.4 State-Space Model of DC–DC Converters

A feedback controlled buck converter is illustrated in Figure 1.3. In order to design a stable feedback system, and properties such as transient overshoot, settling time, and steady state regulation should meet special demands, and the model of converter should be established first. The dynamic model of a DC–DC converter can be one of several [2, 4]:

1. circuit model,
2. linearized model,
3. state-space model and averaged state-space model,
4. discrete time sampled data model.

The state-space model is the most popular among those mentioned above, and is the key step in modeling PWM converters with the small-signal linearization model. We will use the buck converter to illustrate the state-space modeling method of DC–DC converters.

A state-space model is a model of a system that may be represented by a differential equation. Here the state is a collection of variables summarizing the past of a system for the purpose of predicting the future. The system is called time-invariant if the
different equations do not explicitly depend on time \( t \). However, it is possible to have more general time-varying systems because the functions depend on time.

A system is called linear if the functions are linear in \( x \) and \( u \). A linear state-space system can thus be represented as state equations in the matrix forms of

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &=Cx + Du
\end{align*}
\]  

(1.2)

where \( x \) is the state variables, which are a minimum number of variables to uniquely specify the state of the system, \( u \) is the independent input to the system, \( A, B, C, \) and \( D \) are constant matrices. Such a system is said to be linear time invariant (LTI).

For the buck DC–DC converter, choosing the inductor current and capacitor voltage as natural state variables, and picking the input voltage source as \( u \) and resistor voltage as the output \( v_o \), there are two state matrices for CCM operation based on the circuit theory.

**Mode 1:** Switch \( S \) is on, at duration \( dT \):

\[
\begin{align*}
\dot{x} &= A_1 x + B_1 u \\
v_o &= C_1 x
\end{align*}
\]  

(1.3)

**Mode 2:** Switch \( S \) is off, at duration \((1-d)T\):

\[
\begin{align*}
\dot{x} &= A_2 x + B_2 u \\
v_o &= C_2 x
\end{align*}
\]  

(1.4)

Here, \( x = [v_o \ i_L]^T \), \( A_1 = A_2 = \begin{pmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{pmatrix} \), \( B_1 = [1/L \ 0]^T \), \( B_2 = [0 \ 0]^T \), and \( C_1 = C_2 = [0 \ 1]^T \).

The state-space averaged model is a method of deriving the average value of state variables in one period. For switching converters, averaging and weighting the state variables by duty cycle over a switching period \( T \), we obtain a state-space averaged equation

\[
\begin{align*}
\dot{x} &= [A_2 d + A_2 (1-d)] x + [B_1 d + B_2 (1-d)] u \\
v_o &= [C_1 d + C_2 (1-d)] x
\end{align*}
\]  

(1.5)

Next, disturbance and linearization methods are utilized near the working points to obtain a small-signal model. Finally, the transfer function may be obtained using the small-signal model to design the linear controller through classic control theory.

Generally speaking, although the proponents of a given method may prefer to express the end result in a specific form, the end results of nearly all methods are equivalent.