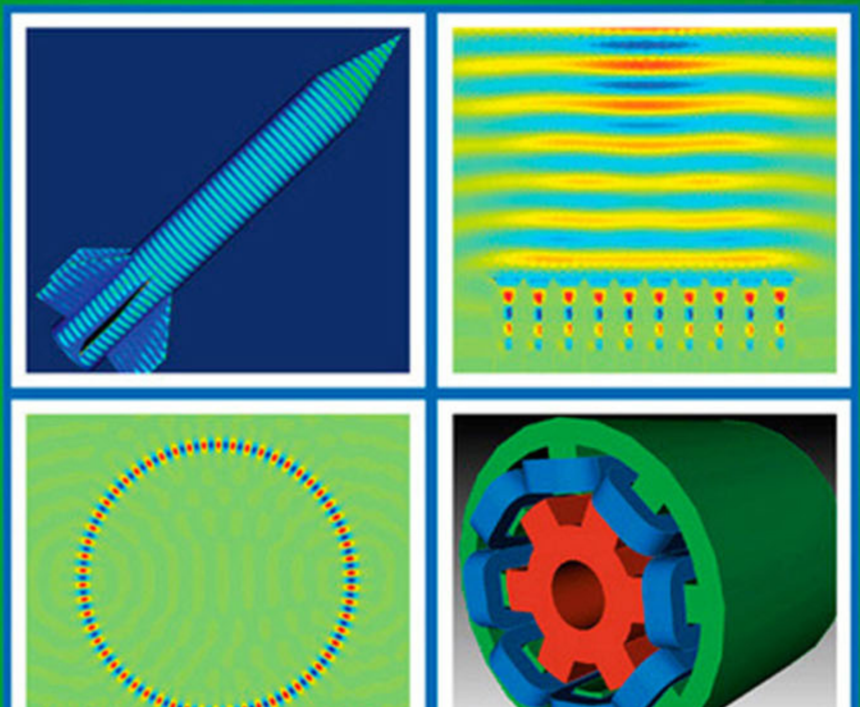
The Finite Element Method in Electromagnetics

THIRD EDITION



JIAN-MING JIN

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Third Edition

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To my wife, Joanna,
and our children, Angela,
David, and Joy

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Chapter 8: Vector Finite Elements

Chapter 9: Absorbing Boundary Conditions

PREFACE

It has been more than 20 years since the publication of the first edition and nearly 12 years since the publication of the second edition of this book. The finite element method has experienced phenomenal growth in the field of computational electromagnetics during these periods. While the early effort was focused on developing the method to model electromagnetic fields correctly and solve electromagnetic problems accurately, the recent effort has been focused more on enhancing the power and expanding the capability of the method to deal with large-scale, complex, and real-life engineering electromagnetic problems through the development of new, more efficient algorithms and effective utilization of new computer technologies. The development of the time-domain finite element method, including the discontinuous Galerkin time-domain method, and a variety of domain decomposition methods during the past decade, together with a variety of new applications, serves as a great testimonial to the success of the recent effort. The purpose of this third edition is to reflect these new developments and applications.

The major changes in the third edition include a significant revision of Chapter 12 on the time-domain finite element method, the addition of two new chapters on the finite element analysis of periodic structures (Chapter 13) and on the domain decomposition methods for finite element analysis of large-scale electromagnetic problems (Chapter 14), the addition of two new sections on higher-order hierarchical vector basis functions (Section 8.7) and on symmetric finite element–boundary integral formulations (Section 10.7), and the removal of the chapter on the method of moments and fast solvers. In addition, there are many relatively minor changes to improve presentation and to add new application examples. Because young generations are no longer familiar with Fortran programming, all the Fortran codes have been converted into pseudo codes.

Chapter 12 on the time-domain finite element method is now organized as follows. The chapter starts with the finite element formulation in the time domain and derives time-stepping equations by finite differencing, such as forward, backward, and central differencing, and the Newmark-beta method. It then presents a scheme to analyze the stability of the time-stepping equations. After that, it deals with modeling of both electrically and magnetically frequency-dispersive media, and discusses a variety of important mesh truncation techniques, which include absorbing boundary conditions, perfectly matched layers, boundary integral equations, and waveguide port boundary conditions. Perfectly matched layers and boundary integral equations provide effective truncation for analysis of unbounded open-region scattering and radiation problems, and waveguide port boundary conditions provide perfect absorption and excitation at waveguide ports, which are important for analysis of microwave devices, antennas, and circuits. In this chapter we also discuss how to employ the time-domain finite element method for a hybrid field–circuit analysis to effectively model and simulate complex microwave circuits with embedded lumped circuits. Finally, we deal with one of the most challenging problems for the time-domain finite element method, which is its efficiency. To overcome this challenge, we present a physics-motivated dual-field domain decomposition method and its special version called the element-level time-domain method. We also formulate two popular discontinuous Galerkin time-domain methods and compare them with the element-level time-domain method to evaluate their accuracy and performance.

Chapter 13 on the finite element analysis of periodic structures is added because of widespread applications of periodic structures in electromagnetics, microwave engineering, radar, and optics. In this chapter we first present the finite element formulation of the field in the unit cell of an infinitely periodic structure, and illustrate its application in the dispersion analysis of one- and two-dimensional periodic media. Next we deal with the finite element analysis of two-dimensional scalar fields, with a focus on the truncation of the computational domain in the nonperiodic direction with a Floquet absorbing boundary condition. We then extend the analysis from the frequency to the time domain by introducing a transformed wave function. After that, we formulate the finite element analysis for three-dimensional vector fields, both in the frequency and time domains. Finally, we describe the finite element analysis of scattering and radiation by an angular periodic structure or a discrete body of revolution. Throughout the entire chapter, we give examples to illustrate the application and demonstrate the capability of the finite element analysis.

Chapter 14 is added to cover the important subject of domain decomposition methods for finite element analysis of large-scale electromagnetic problems. Domain decomposition is a general concept, under which many numerical schemes have been developed, ranging from overlapping to nonoverlapping domain decomposition and from simple iteration to complicated simultaneous coupling between subdomains. Instead of presenting every domain decomposition method, we focus on those that have demonstrated the best performance with respect to accuracy, efficiency, and scalability. In particular, we first present the basic ideas of the Schwarz and Schur complement domain decomposition methods for simple scalar problems. These include the alternating and additive overlapping Schwarz methods, the nonoverlapping optimized Schwarz method, and the primal, dual, and dual-primal Schur complement domain decomposition methods. After that, we present the three most robust and powerful nonoverlapping domain decomposition methods for solving Maxwell's equations. The first one is the dual-primal finite element tearing and interconnect method based on one Lagrange multiplier for static, quasistatic, and low-frequency electromagnetic problems. The second one is the dual-primal finite element tearing and interconnect method based on two Lagrange multipliers for more challenging high-frequency electromagnetic problems. The method is extended to handle problems for which the finite element meshes on subdomain interfaces do not conform to each other. The last one is the nonoverlapping and nonconformal optimized Schwarz domain decomposition method, which is modified to yield a global interface system for a more effective solution. The three methods are compared to assess their performance. Throughout the entire chapter, we present many highly challenging problems to demonstrate the power and capabilities of the domain decomposition methods.

Besides one significantly expanded chapter and two new chapters, the third edition has expanded the range of application of the finite element method to more electromagnetics-related problems. The first two editions were mostly focused on the analysis of electromagnetic scattering problems and, to a limited extent, antenna radiation and microwave device problems. The third edition has included more analysis examples of antennas and microwave devices and added coverage of application to electrical machines, microwave integrated circuits, periodic structures, phased-array antennas, and photonic devices. For example, a new section (Section 4.6) has been added to demonstrate the analysis of multiconductor transmission lines using the two-dimensional finite element method. Another section (Section 4.7.4) has been added to show the analysis of axisymmetric antennas. Chapters 12–14 present many examples on modeling of complex antennas, phased arrays, frequency selective surfaces, microwave integrated circuits, dielectric gratings, photonic crystal cavities and waveguides, and electromechanical devices.

To keep this book more focused on the finite element method, I have removed the chapter on the method of moments and fast solvers in this new edition. An expanded and updated version of the removed chapter can be found in my book *Theory and Computation of Electromagnetic Fields* (Wiley, 2010). Whereas some argued that the method of moments should be considered as a subset of the finite element method, others argued for the contrary, all because both methods can be formulated based on the method of weighted residuals. However, in the computational electromagnetics community, the two have been regarded as different methods because the finite element method is primarily employed to solve partial differential equations, whereas the method of moments is mostly used to deal with integral equations formulated with the aid of Green's functions. Both methods play important roles in computational electromagnetics; in fact, they are considered two of the three building blocks for the field of computational electromagnetics.

Just like the second edition, which increased from 442 to 753 pages, this edition has also significantly increased in size even after the removal of the chapter on the method of moments and fast solvers. Whereas the size might be overwhelming for beginners, I have maintained all the basic materials and the presentation style that made the first two editions a success. The new materials are added simply to introduce more advanced finite element-based techniques and to illustrate more applications of the finite element method. I hope that this new edition will continue to serve as a useful and easy-to-read introductory text on the finite element method for electromagnetic analysis and that it will also introduce interested readers to the development of more powerful techniques and inspire more research on applications of the finite element method for computational electromagnetics.

As always, I am most grateful to my colleagues, friends, former and current graduate students and postdoctoral fellows who have contributed, directly or indirectly, to the research reported in this book and shared joy and frustration with me along the journey. I have been very fortunate and blessed to have the opportunity to participate in the research and development of the finite element method in electromagnetics and witness its rapid progress and phenomenal growth from dealing with simple two-dimensional problems discretized with a few hundred variables in the 1980s to solving highly complicated three-

dimensional real-life engineering problems modeled with billions of variables today. Finally, I would like to thank Jamie Hutchinson of our Publications Office, who read the entire manuscript of the third edition and checked for editorial correctness.

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PREFACE TO THE FIRST EDITION

In spite of the fact that the finite element method has been applied to the analysis of problems in electromagnetics for nearly thirty years and, as a result, a large number of research articles have been published during this period, few texts and research books have been written on the subject. This is particularly true in the areas of scattering and radiation. The situation is in sharp contrast to other engineering fields, such as structural analysis and fluid dynamics, where many textbooks have been published with emphasis ranging from theory to applications.

For two reasons, the author feels that a book is necessary describing the theory and applications of the finite element method in electromagnetics. The first is related to the fact that there has been no suitable text on the finite element method for use in computational electromagnetics courses at the graduate level.[1] Such courses have been offered in many universities around the world in response to the increasing use of the finite element method for analysis and design of electromagnetic devices. The second reason is due to the growing demand for a comprehensive reference book on the finite element method which would provide researchers in electromagnetics a broad view of the topic. The lack of a suitable reference book has been an inconvenience for many researchers in the area.

This book is written to address the two concerns mentioned above and is intended to serve as a text for graduate students as well as a reference for electromagnetics researchers. The book has several unique features that distinguish it from other books:

1. *Systematic treatment of the finite element method.* The book starts with an easy-to-read introduction to the method and proceeds to one-, two-, and three-dimensional problems. For each class of problems, a rigorous finite element solution is first developed in general form, from which the solution to specific problems can be deduced. Therefore, the finite element analysis for these problems is not repeated.

2. *Extensive applications in electromagnetics.* The book describes the application of the finite element method to many electromagnetic problems ranging from scalar to vector fields and static to time-harmonic cases. It addresses problems such as electro-and magnetostatic field computations, microwave and optical waveguide characterization, cavity resonances, two- and three-dimensional scattering computations, and analysis of microstrip patch antennas. These are aided by many examples, most of which were developed by the author.

3. *Analysis of open-region scattering and radiation problems.* The book places an emphasis on the treatment of open-region scattering and radiation problems, an important and difficult topic not covered by any of the previous texts. Various approaches, both approximate and exact, are discussed with many specific examples.

4. *Inclusion of recent developments.* The book includes the most recent developments of the finite element method in electromagnetics. These include vector elements and hybrid methods, which again have not been covered by previous texts.

5. *Formulation of variational problems.* The book devotes one chapter to the variational principles used to establish the required functionals for variational finite element formulations. The variational formulation is often considered to be the most difficult obstacle for the finite element method. In most books, the required functional is usually stated first, then proved, rather than derived from the original problem. As a result, students are always puzzled about its origin. Hopefully, this will not be the case in reading this book.

The book contains eleven chapters and three appendices. Chapter 1 presents a brief review of some basic equations and concepts of electromagnetic theory. Particular attention is paid to the differential equations and boundary conditions, including radiation conditions, since the finite element method deals with them directly.

The finite element method is introduced in Chapter 2. We first briefly review the two classic methods—the Ritz variational method and Galerkin's method—which form the foundation of the finite element method. A simple example is then used to first illustrate their application procedures and then to introduce the concept of the finite element method. It is felt that such an introduction is more natural and easily comprehensible to the beginner. We next describe the basic steps of the finite element method in general, without reference to a specific problem. Finally, an alternative formulation is presented so that the basic principle of the finite element method stands out more clearly.

The finite element analysis of a general one-dimensional problem is described in Chapter 3. The result is then applied to the problem of plane wave reflection by a metal-backed dielectric slab, and also to the differential equation arising from the application of the on-surface radiation condition method to scattering by an impedance cylinder. The first problem illustrates the analysis on a straight line, and the second demonstrates the method's capability in treating a curved line. The chapter concludes with an introduction of the concept of higher-order elements and a performance evaluation of linear, quadratic, and cubic elements.

Chapter 4 deals with the finite element method and its application in two-dimensional space. Again, a general two-dimensional boundary-value problem is considered first and the result is then applied to specific electromagnetic problems. These include the computation of electro- and magnetostatic potentials and fields, characterization of discontinuities in parallel plate waveguides, and computation of scattering by cylindrical structures in conjunction with absorbing boundary conditions. This is followed by the formulation and performance evaluation of higher-order elements.

Chapter 5 describes the finite element analysis of a general three-dimensional scalar problem and its application to electrostatic problems. It also contains the formulation of the solution to magnetostatic and time-harmonic electromagnetic problems, which require the treatment of vector quantities. However, the formulation has not found many applications in electrodynamic cases because of the unique features of electromagnetic fields. The associated difficulties are discussed at the end of the chapter.

Chapter 6 is devoted to the variational formulation of electromagnetic problems. Discussed are various variational principles often employed to establish the required variational expressions or functionals. The generalized variational principle turns out to be very simple and very useful and can be employed for most problems encountered in electromagnetics. This enables us to formulate the finite element solution for the rest of the book based on variational models.

Chapter 7 describes the finite element analysis of eigenvalue problems, which in contrast to the problems analyzed in the previous chapters, are nondeterministic. The specific problems considered concern various waveguide structures including dielectric-filled waveguides, microstrip transmission lines and optical fibers, as well as three-dimensional cavities. The treatment of open structures and the appearance of spurious solutions are also discussed.

Vector finite elements are introduced in Chapter 8 in order to address the unique features of electromagnetic fields. These elements are relatively new, and hence have not been discussed in previous finite element books. However, it is these elements that make the finite element analysis useful in terms of electric and magnetic field vectors. As we will see, they are particularly suitable for electromagnetic field vectors and are important for future applications of the method in electromagnetics. In this chapter we first formulate edge-based triangular, rectangular, and quadrilateral elements for two-dimensional applications and then consider edge-based brick, tetrahedral, and hexahedral elements for three-dimensional applications. Examples are given to demonstrate the advantages of this new class of elements.

Chapter 9 addresses one of the most difficult aspects of electromagnetic scattering and radiation problems—open regions or open boundaries. A hybrid procedure is introduced which combines the finite element method with boundary integral equations appropriate for unbounded space. The chapter begins with two-dimensional scattering problems and proceeds to three-dimensional scattering and antenna problems. It ends with discussions of some important issues of the hybrid method, including the famous interior resonance problem.

In Chapter 10 we discuss an alternative approach to open region problems which involves the use of eigenfunction expansions. The problem of waveguide discontinuities is considered first, followed by various formulations for scattering problems, including the well-known unimoment method.

The last chapter presents several solution methods and algorithms for linear algebraic equations arising from the finite element discretization. These include the banded matrix method, the profile storage method, and the conjugate and biconjugate gradient methods. This chapter also contains a brief discussion of the solution of the standard and generalized eigenvalue problems.

There are three appendices covering related topics. Appendix A lists some vector identities and integral theorems that are commonly used in the finite element formulation of electromagnetic problems. Appendix B describes the Ritz procedure as applied to the functionals involving the complex conjugation of functions. In Appendix C we discuss the derivation and performance of various absorbing boundary conditions often employed for open-region scattering and radiation problems.

The time convention adopted in this book is $e^{j\omega t}$, suppressed throughout. The references are listed at the end of each chapter. The book also contains a limited number of exercises to further supplement and reinforce the concepts and ideas presented.

Finally, the author invites his readers to point out any errors that come to their attention. He also welcomes any comments and suggestions.

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[1]For undergraduate courses, two excellent texts are available: P. P. Silvester and R. L. Ferrari, *Finite Elements for Electrical Engineers*, 2nd ed. (Cambridge: Cambridge University Press, 1990) and S. R. H. Hoole, *Computer Aided Analysis and Design of Electromagnetic Devices* (New York: Elsevier Press, 1989).

PREFACE TO THE SECOND EDITION

Since the publication of the first edition of this book about eight years ago, much progress has been made in development of the finite element method for the analysis of electromagnetic problems, especially in five areas. The first is the development of higher-order vector finite elements, which make it possible to obtain highly accurate and efficient solutions of vector wave equations. The second is the development of perfectly matched layers as an absorbing boundary condition. Although the perfectly matched layers were intended primarily for the time-domain finite-difference method, they have also found applications in the finite element simulations. The third is perhaps the development of hybrid techniques that combine the finite element and asymptotic methods for the analysis of large, complex problems that were unsolvable in the past. The fourth is further development of the finite element–boundary integral methods that incorporate fast integral solvers, such as the fast multipole method, to reduce the computational complexity associated with the boundary integral part. The last, but not the least, is the development of the finite element method in the time domain for transient analysis. As a result of all these efforts, the finite element method has gained more popularity in the computational electromagnetics community and has become one of the preeminent simulation techniques for electromagnetic problems.

In this second edition, we have updated the subject matter and introduced new advances in finite element technology. In Chapter 1, which presents the basic electromagnetic equations and concepts, we have added a brief review of vector analysis because of its importance in the finite element formulation of electromagnetic problems. We have also added sections on field–source relations, Huygens's principles, and definitions of radar cross section, since they are used frequently in the subsequent chapters.

Chapter 2 introduces the basic concepts of the finite element method after a brief review of classical methods for boundary-value problems. Minor changes have been made to improve clarity. In the next three chapters we develop the finite element method in one, two, and three dimensions and its application to electromagnetic problems. We have added sections on isoparametric elements that can provide a superior geometrical modeling, in addition to accurate representation of unknown functions to be computed, and sections on dispersion analysis to illustrate the convergence of higher-order finite elements.

In Chapter 6 we discuss various variational principles to establish the variational expression for a given electromagnetic boundary-value problem. We have added one section to present the most general variational principle and illustrate its application to electromagnetic problems involving anisotropic media. This topic is useful since anisotropic media have been used widely in electronic and electro-optical devices.

Chapter 7, which describes the finite element analysis of eigenvalue problems, remains unchanged except for some minor modifications to update the topic.

In Chapter 8 we introduce vector finite elements for the modeling of electromagnetic vector wave equations. Since the first edition of this book presented only the lowest-order vector elements in two and three dimensions, major revisions have been made to cover the developments of higher-order vector elements with examples to demonstrate their superior performance. The higher-order vector elements are expected to significantly affect application of the finite element method to electromagnetic problems.

Chapter 9 is a new chapter devoted to the important topic of absorbing boundary conditions. This topic was addressed briefly in Appendix C of the first edition. It is now fully expanded to cover two-dimensional scalar and three-dimensional vector absorbing boundary conditions, adaptive absorbing boundary conditions, fictitious absorbers, and perfectly matched layers. The adaptive absorbing boundary condition is a relatively new approach that can systematically improve the accuracy of the solution obtained using an absorbing boundary condition. The concept of perfectly matched layers was proposed only a few years ago; hence, it is an entirely new topic. Efforts have been made to present it to suit finite element applications since it has been used mostly for time-domain finite-difference simulations. A section has been included for the finite element analysis of scattering and radiation by complex body-of-revolution structures using perfectly matched layers.

In Chapter 10 we address the development of a hybrid technique that combines the finite element and boundary integral methods for open-region scattering and radiation problems. Efforts have been made to improve treatment of the interior resonance problem in both two and three dimensions. A section has been added to present a highly effective preconditioner to accelerate the iterative solution of the finite element –boundary integral method, with numerical examples to demonstrate its great potential. Coupled with fast integral solvers, the hybrid finite element–boundary integral method is promising for dealing with large-scale problems involving complex structures and inhomogeneous materials.

Chapter 11 covers the use of eigenfunction expansions for the finite element analysis of open-region problems. New material has been added to present eigenfunction expansions on elliptical boundaries that can significantly reduce the size of the computational domain in comparison to circular ones. New examples have been included to demonstrate the capability of the method to simulate microwave devices such as circulators, filters, and junctions.

Chapter 12 is another new chapter, in which we describe development of the finite element method for the time-domain analysis of transient electromagnetic problems. Time-domain simulations are important because of their ability to model nonlinear materials. In this chapter we cover basic time-marching schemes and their stability analysis. We also discuss the modeling of dispersive media, the formulation of orthogonal vector basis functions, and application of the time-domain finite element method to open-region scattering and radiation problems with the aid of absorbing boundary conditions, perfectly matched layers, and boundary integral equations.

In Chapter 13 we present solution methods and algorithms for linear algebraic equations arising from the finite element discretization which include the banded matrix method, the profile storage method, and the conjugate and biconjugate gradient methods. In this new edition we have added other useful direct and iterative solvers and discussed preconditioning techniques to speed up iteration convergence and reordering schemes for the bandwidth reduction. A new section has also been added to describe the asymptotic waveform evaluation method for fast frequency-sweep analysis.

Chapter 14 is also a new chapter and was suggested by a reviewer. It presents another very powerful computational method in electromagnetics—the method of moments—and its fast solvers. The inclusion of this chapter is justifiable because the method of moments is very closely related to the finite element method, at least in terms of basic principles. Moreover, good understanding of the method of moments can help a great deal in development of the hybrid finite element–boundary integral method presented in Chapter 10. The fast solvers, including the FFT-based method, adaptive integral method, and fast multipole method, can all be incorporated into the hybrid finite element–boundary integral method to reduce the computational complexity associated with the boundary integral part and thus further expand the capability of the hybrid method.

Appendix A has been slightly expanded to list some formulas and integral theorems for vector analysis on surfaces which are useful for the manipulation of absorbing boundary conditions, boundary integrals, and higher-order vector basis functions on curved surfaces. Appendix B remains unchanged.

The remaining appendices (C–E) are all new. To be specific, Appendix C presents a definition and derivation of Green's functions and their applications in electromagnetics. We have included this because Green's functions are the basis for boundary integral equations and near-to-far-field calculations. In Appendix D we describe a numerical procedure to evaluate singular integrals, which is essential for the method of moments and the finite element–boundary integral method. Finally, Appendix E lists the definitions and useful properties of some special functions used in this book.

As a result of the revision, more than one-third of this second edition is new. The book is still intended as a textbook for use in computational electromagnetics courses at the graduate level and as a research reference for scientists and electrical engineers. It is detailed enough for self-study as well. We have included some exercises to supplement and reinforce the concepts and ideas presented and to facilitate its use as a textbook. For teaching purposes I have developed a set of PowerPoint viewgraphs about the finite element method and will be happy to make them available to those teaching the method for electromagnetic analysis.

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