Regression Analysis
By Example
Dedicated to:

Allegra, Martha, and Rima – S. C.

The memory of my parents – A. S. H.

It’s a gift to be simple . . .

Old Shaker hymn

True knowledge is knowledge of why things are as they are, and not merely what they are.

Isaiah Berlin
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It is with great pleasure that we introduce the fifth edition of *Regression Analysis by Example* first published in 1977. The statistical community has been most supportive, and we have benefitted greatly from their suggestions in improving the text.

Regression analysis has become one of the most widely used statistical tools for analyzing multifactor data. It is appealing because it provides a conceptually simple method for investigating functional relationships among variables. The standard approach in regression analysis is to take data, fit a model, and then evaluate the fit using statistics such as $t$, $F$, and $R^2$. Our approach is much broader. We view regression analysis as a set of data analytic techniques that examine the interrelationships among a given set of variables. The emphasis is not on formal statistical tests and probability calculations. We argue for an informal analysis directed toward uncovering patterns in the data.

We utilize most standard and some not so standard summary statistics on the basis of their intuitive appeal. We rely heavily on graphical representations of the data, and employ many variations of plots of regression residuals. We are not overly concerned with precise probability evaluations. Graphical methods for exploring residuals can suggest model deficiencies or point to troublesome observations. Upon further investigation into their origin, the troublesome observations often turn out to be more informative than the well-behaved observations. We notice often that more information is obtained from a quick examination of a plot of residuals than from a formal test of statistical significance of some limited null
hypothesis. In short, the presentation in the chapters of this book is guided by the principles and concepts of exploratory data analysis.

Our presentation of the various concepts and techniques of regression analysis relies on carefully developed examples. In each example, we have isolated one or two techniques and discussed them in some detail. The data were chosen to highlight the techniques being presented. Although when analyzing a given set of data it is usually necessary to employ many techniques, we have tried to choose the various data sets so that it would not be necessary to discuss the same technique more than once. Our hope is that after working through the book, the reader will be ready and able to analyze his/her data methodically, thoroughly, and confidently.

The emphasis in this book is on the analysis of data rather than on formulas, tests of hypotheses, or confidence intervals. Therefore no attempt has been made to derive the techniques. Techniques are described, the required assumptions are given and, finally, the success of the technique in the particular example is assessed. Although derivations of the techniques are not included, we have tried to refer the reader in each case to sources in which such discussion is available. Our hope is that some of these sources will be followed up by the reader who wants a more thorough grounding in theory.

We have taken for granted the availability of a computer and a statistical package. Recently there has been a qualitative change in the analysis of linear models, from model fitting to model building, from overall tests to clinical examinations of data, from macroscopic to the microscopic analysis. To do this kind of analysis a computer is essential and we have assumed its availability. Almost all of the analyses we use are now available in software packages. We are particularly heartened by the arrival of the package R, available on the Internet under the General Public License (GPL). The package has excellent computing and graphical features. It is also free!

The material presented is intended for anyone who is involved in analyzing data. The book should be helpful to those who have some knowledge of the basic concepts of statistics. In the university, it could be used as a text for a course on regression analysis for students whose specialization is not statistics, but, who nevertheless use regression analysis quite extensively in their work. For students whose major emphasis is statistics, and who take a course on regression analysis from a book at the level of Rao (1973), Seber (1977), or Sen and Srivastava (1990), this book can be used to balance and complement the theoretical aspects of the subject with practical applications. Outside the university, this book can be profitably used by those people whose present approach to analyzing multifactor data consists of looking at standard computer output \( t, F, R^2, \) standard errors, etc.), but who want to go beyond these summaries for a more thorough analysis.

The book has a Website: http://www.aucegypt.edu/faculty/hadi/RABE5. This Website contains, among other things, all the data sets that are included in this book and more.
Major changes have been made in streamlining the text, removing ambiguities, and correcting errors pointed out by readers and others detected by the authors. New examples of data sets have been added in Chapter 1. The material on centering and scaling has been moved from Chapter 9 to Section 3.6. Chapters 9 and 10 have been rearranged, so the updated material flows more smoothly. The Appendix to Chapter 10 now includes a brief description of surrogate ridge regression, a recently introduced topic in the literature. New references have also been added. We have rewritten some of the exercises, and increased the number of exercises at the end of the chapters. We feel that the exercises reinforce the understanding of the material in the preceding chapters.

We have attempted to write a book for a group of readers with diverse backgrounds. We have also tried to put emphasis on the art of data analysis rather than on the development of statistical theory.

We are fortunate to have had assistance and encouragement from several friends, colleagues, and associates. Some of our colleagues at New York University and Cornell University have used portions of the material in their courses and have shared with us their comments and comments of their students. Special thanks are due to our friend and former colleague Jeffrey Simonoff (New York University) for comments, suggestions, and general help. The students in our classes on regression analysis have all contributed by asking penetrating questions and demanding meaningful and understandable answers. Our special thanks go to Nedret Billor (Cukurova University, Turkey) and Sahar El-Sheneity (Cornell University) for their very careful reading of an earlier edition of this book. We also thank Amy Hendrickson for preparing the Latex style files and for responding to our Latex questions, and Dean Gonzalez for help with the production of some of the figures.

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Ali S. Hadi

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Cairo, Egypt
CHAPTER 1

INTRODUCTION

1.1 WHAT IS REGRESSION ANALYSIS?

Regression analysis is a conceptually simple method for investigating functional relationships among variables. A real estate appraiser may wish to relate the sale price of a home from selected physical characteristics of the building and taxes (local, school, county) paid on the building. We may wish to examine whether cigarette consumption is related to various socioeconomic and demographic variables such as age, education, income, and price of cigarettes. The relationship is expressed in the form of an equation or a model connecting the response or dependent variable and one or more explanatory or predictor variables. In the cigarette consumption example, the response variable is cigarette consumption (measured by the number of packs of cigarette sold in a given state on a per capita basis during a given year) and the explanatory or predictor variables are the various socioeconomic and demographic variables. In the real estate appraisal example, the response variable is the price of a home and the explanatory or predictor variables are the characteristics of the building and taxes paid on the building.

We denote the response variable by $Y$ and the set of predictor variables by $X_1, X_2, \ldots, X_p$, where $p$ denotes the number of predictor variables. The true relationship between $Y$ and $X_1, X_2, \ldots, X_p$ can be approximated by the regression
INTRODUCTION

where $\varepsilon$ is assumed to be a random error representing the discrepancy in the approximation. It accounts for the failure of the model to fit the data exactly. The function $f(X_1, X_2, \cdots, X_p)$ describes the relationship between $Y$ and $X_1, X_2, \cdots, X_p$. An example is the linear regression model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \varepsilon,$$

where $\beta_0, \beta_1, \cdots, \beta_p$, called the regression parameters or coefficients, are unknown constants to be determined (estimated) from the data. We follow the commonly used notational convention of denoting unknown parameters by Greek letters.

The predictor or explanatory variables are also called by other names such as independent variables, covariates, regressors, factors, and carriers. The name independent variable, though commonly used, is the least preferred, because in practice the predictor variables are rarely independent of each other.

1.2 PUBLICLY AVAILABLE DATA SETS

Regression analysis has numerous areas of applications. A partial list would include economics, finance, business, law, meteorology, medicine, biology, chemistry, engineering, physics, education, sports, history, sociology, and psychology. A few examples of such applications are given in Section 1.3. Regression analysis is learned most effectively by analyzing data that are of direct interest to the reader. We invite the readers to think about questions (in their own areas of work, research, or interest) that can be addressed using regression analysis. Readers should collect the relevant data and then apply the regression analysis techniques presented in this book to their own data. To help the reader locate real-life data, this section provides some sources and links to a wealth of data sets that are available for public use.

A number of data sets are available in books and on the Internet. The book by Hand et al. (1994) contains data sets from many fields. These data sets are small in size and are suitable for use as exercises. The book by Chatterjee, Handcock, and Simonoff (1995) provides numerous data sets from diverse fields. The data are included in a diskette that comes with the book and can also be found at the Website.1

Data sets are also available on the Internet at many other sites. Some of the Websites given below allow the direct copying and pasting into the statistical package of choice, while others require downloading the data file and then importing them into a statistical package. Some of these sites also contain further links to yet other data sets or statistics-related Websites.

The Data and Story Library (DASL, pronounced “dazzle”) is one of the most interesting sites that contains a number of data sets accompanied by the “story” or

1 http://www.stern.nyu.edu/~jsimonof/Casebook
background associated with each data set. DASL is an online library\(^2\) of data files and stories that illustrate the use of basic statistical methods. The data sets cover a wide variety of topics. DASL comes with a powerful search engine to locate the story or data file of interest.

Another Website, which also contains data sets arranged by the method used in the analysis, is the Electronic Dataset Service.\(^3\) The site also contains many links to other data sources on the Internet.

Finally, this book has a Website\(^4\), which contains, among other things, all the data sets that are included in this book and more. These and other data sets can be found at the book's Website.

### 1.3 SELECTED APPLICATIONS OF REGRESSION ANALYSIS

Regression analysis is one of the most widely used statistical tools because it provides simple methods for establishing a functional relationship among variables. It has extensive applications in many subject areas. The cigarette consumption and the real estate appraisal, mentioned above, are but two examples. In this section, we give a few additional examples demonstrating the wide applicability of regression analysis in real-life situations. Some of the data sets described here will be used later in the book to illustrate regression techniques or in the exercises at the end of various chapters.

#### 1.3.1 Agricultural Sciences

The Dairy Herd Improvement Cooperative (DHI) in upstate New York collects and analyzes data on milk production. One question of interest here is how to develop a suitable model to predict current milk production from a set of measured variables. The response variable (current milk production in pounds) and the predictor variables are given in Table 1.1. Samples are taken once a month during milking. The period that a cow gives milk is called lactation. Number of lactations is the number of times a cow has calved or given milk. The recommended management practice is to have the cow produce milk for about 305 days and then allow a 60-day rest period before beginning the next lactation. The data set, consisting of 199 observations, was compiled from the DHI milk production records. The Milk Production data can be found at the book’s Website.

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\(^2\) [http://lib.stat.cmu.edu/DASL](http://lib.stat.cmu.edu/DASL)

\(^3\) [http://www-unix.oit.umass.edu/~statdata](http://www-unix.oit.umass.edu/~statdata)

\(^4\) [http://www.aucegypt.edu/faculty/hadi/RABE5](http://www.aucegypt.edu/faculty/hadi/RABE5)
Table 1.1 Variables in Milk Production Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>Current</td>
<td>Current month milk production in pounds</td>
</tr>
<tr>
<td>Previous</td>
<td>Previous month milk production in pounds</td>
</tr>
<tr>
<td>Fat</td>
<td>Percent of fat in milk</td>
</tr>
<tr>
<td>Protein</td>
<td>Percent of protein in milk</td>
</tr>
<tr>
<td>Days</td>
<td>Number of days since present lactation</td>
</tr>
<tr>
<td>Lactation</td>
<td>Number of lactations</td>
</tr>
<tr>
<td>179</td>
<td>Indicator variable (0 if Days ≤ 79 and 1 if Days &gt; 79)</td>
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</table>

Table 1.2 Variables in Right-To-Work Laws Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>COL</td>
<td>Cost of living for a four-person family</td>
</tr>
<tr>
<td>PD</td>
<td>Population density (person per square mile)</td>
</tr>
<tr>
<td>URate</td>
<td>State unionization rate in 1978</td>
</tr>
<tr>
<td>Pop</td>
<td>Population in 1975</td>
</tr>
<tr>
<td>Taxes</td>
<td>Property taxes in 1972</td>
</tr>
<tr>
<td>Income</td>
<td>Per capita income in 1974</td>
</tr>
<tr>
<td>RTWL</td>
<td>Indicator variable (1 if there are right-to-work laws in the state and 0 otherwise)</td>
</tr>
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</table>

1.3.2 Industrial and Labor Relations

In 1947, the United States Congress passed the Taft-Hartley Amendments to the Wagner Act. The original Wagner Act had permitted the unions to use a Closed Shop Contract unless prohibited by state law. The Taft-Hartley Amendments made the use of Closed Shop Contract illegal and gave individual states the right to prohibit union shops as well. These right-to-work laws have caused a wave of concern throughout the labor movement. A question of interest here is: What are the effects of these laws on the cost of living for a four-person family living on an intermediate budget in the United States? To answer this question a data set consisting of 38 geographic locations has been assembled from various sources. The variables used are defined in Table 1.2. The Right-To-Work Laws data are given in Table 1.3 and can also be found at the book's Website.

5 Under a Closed Shop Contract provision, all employees must be union members at the time of hire and must remain members as a condition of employment.

6 Under a Union Shop clause, employees are not required to be union members at the time of hire, but must become a member within two months, thus allowing the employer complete discretion in hiring decisions.
## Table 1.3 The Right-To-Work Laws Data

<table>
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<tr>
<th>City</th>
<th>COL</th>
<th>PD</th>
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### Table 1.4 Variables in Study of Domestic Immigration

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<td>Unemployment rate in the civilian labor force in 1994</td>
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<td>Wage</td>
<td>Average hourly earnings of production workers in manufacturing in 1994</td>
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<td>Crime</td>
<td>Violent crime rate per 100,000 people in 1993</td>
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<td>Income</td>
<td>Median household income in 1994</td>
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<td>Metrop</td>
<td>Percentage of state population living in metropolitan areas in 1992</td>
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<tr>
<td>Poor</td>
<td>Percentage of population who fall below the poverty level in 1994</td>
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<tr>
<td>Taxes</td>
<td>Total state and local taxes per capita in 1993</td>
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<td>Percentage of population 25 years or older who have a high school degree or higher in 1990</td>
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<td>The number of business failures divided by the population of the state in 1993</td>
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<td>Temp</td>
<td>Average of the 12 monthly average temperatures (in degrees Fahrenheit) for the state in 1993</td>
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<td>Region</td>
<td>Region in which the state is located (northeast, south, midwest, west)</td>
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### 1.3.3 Government

Information about domestic immigration (the movement of people from one state or area of a country to another) is important to state and local governments. It is of interest to build a model that predicts domestic immigration or to answer the question of why do people leave one place to go to another? There are many factors that influence domestic immigration, such as weather conditions, crime, taxes, and unemployment rates. A data set for the 48 contiguous states has been created. Alaska and Hawaii are excluded from the analysis because the environments of these states are significantly different from the other 48, and their locations present certain barriers to immigration. The response variable here is net domestic immigration, which represents the net movement of people into and out of a state over the period 1990–1994 divided by the population of the state. Eleven predictor variables thought to influence domestic immigration are defined in Table 1.4. The data are given in Tables 1.5 and 1.6, and can also be found at the book’s Website.

### 1.3.4 History

A question of historical interest is how to estimate the age of historical objects based on some age-related characteristics of the objects. For example, the variables
### Table 1.5  First Six Variables of Domestic Immigration Data

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Table 1.7 Variables in Egyptian Skulls Data

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<td>Year</td>
<td>Approximate year of skull formation (negative = B.C.; positive = A.D.)</td>
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<tr>
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<td>Basibregmatic height of skull</td>
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<td>Basialveolar length of skull</td>
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<tr>
<td>NH</td>
<td>Nasal Height of skull</td>
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in Table 1.7 can be used to estimate the age of Egyptian skulls. Here the response variable is Year and the other four variables are possible predictors. There are 150 observations in this data set. The original source of the data is Thomson and Randall-Maciver (1905), but they can be found in Hand et al. (1994), pp. 299–301. An analysis of the data can be found in Manly (1986). The Egyptian Skulls data can be found at the book’s Website.

1.3.5 Environmental Sciences

In a 1976 study exploring the relationship between water quality and land use, Haith (1976) obtained the measurements (shown in Table 1.8) on 20 river basins in New York State. A question of interest here is how the land use around a river basin contributes to the water pollution as measured by the mean nitrogen concentration (mg/liter). The data are shown in Table 1.9 and can also be found at the book’s Website.

1.3.6 Industrial Production

Nambe Mills in Santa Fe, New Mexico, makes a line of tableware made from sand casting a special alloy of metals. After casting, the pieces go through a series of shaping, grinding, buffing, and polishing steps. Data was collected for 59 items produced by the company. The relation between the polishing time and the product diameters and the product types (Bowl, Casserole, Dish, Tray, and Plate) are used to estimate the polishing time for new products which are designed or suggested for design and manufacture. The data are given in Table 1.10. The variables representing product types are coded as binary variables (1 corresponds to the type and 0 otherwise). Diam is the diameter of the item (in inches), polishing time is measured in minutes, and price in dollars. The polishing time is the major item in the cost of the product. The production decision will be based on the estimated time of polishing. The data is obtained from the DASL library, can be found there, and also at the book’s Website.
Table 1.8  Variables in Study of Water Pollution in New York Rivers

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Table 1.9  New York Rivers Data

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### Table 1.11  Number of O-rings Damaged and Temperature (Degrees Fahrenheit) at Time of Launch for 23 Flights of Space Shuttle Challenger

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### 1.3.7 The Space Shuttle Challenger

The explosion of the space shuttle Challenger in 1986 killing the crew was a shattering tragedy. To look into the case a Presidential Commission was appointed. The O-rings in the booster rockets, which are used in space launching, play a very important part in its safety. The rigidity of the O-rings is thought to be affected by the temperature at launching. There are six O-rings in a booster rocket. Table 1.11 gives the number of rings damaged and the temperature at launchings of the 23 flights. The data set can also be found at the book’s Website. The analysis performed before the launch did not include the launches in which no O-ring was damaged and came to the wrong conclusion. A detailed discussion of the problem is found in The Flight of the Space Shuttle Challenger in Chatterjee, Handcock, and Simonoff (1995, pp. 33–35). Note here that the response variable is a proportion bounded between 0 and 1.

### 1.3.8 Cost of Health Care

The cost of delivery of health care has become an important concern. Getting data on this topic is extremely difficult because it is highly proprietary. These data were collected by the Department of Health and Social Services of the State of New Mexico and cover 52 of the 60 licensed facilities in New Mexico in 1988. The variables in these data are the characteristics which describe the facilities size, volume of usage, expenditures, and revenue. The location of the facility is also indicated, whether it is in the rural or nonrural area. Specific definitions of the variables are given in Table 1.12 and the data are given in Table 1.13 and also at the book’s Website. There are several ways of looking at a body of data and extracting various kinds of information. For example, (a) Are rural facilities different from