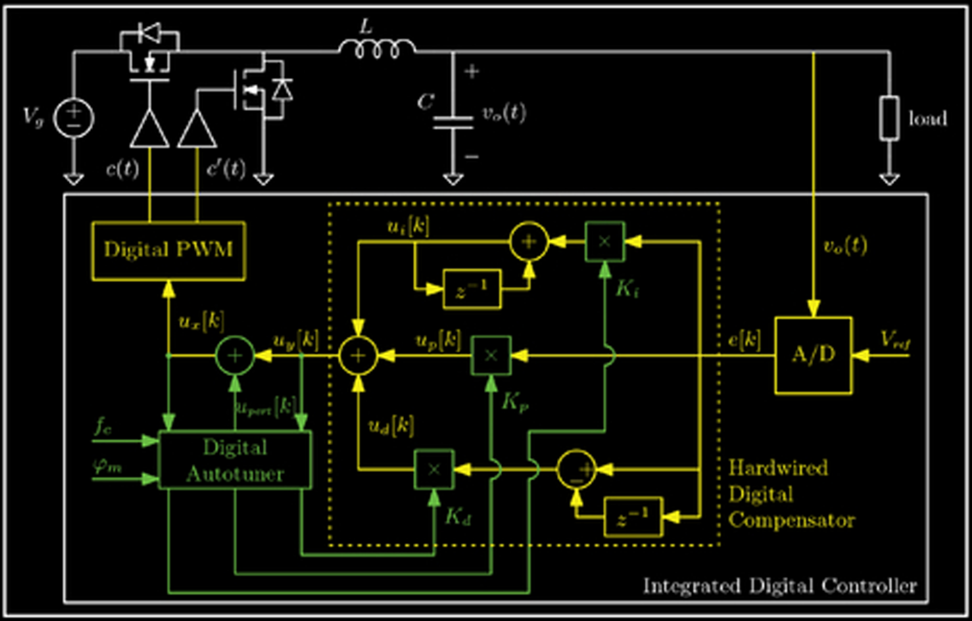
Digital Control of High-Frequency Switched-Mode Power Converters



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CONTENTS

PREFACE

Power electronics fundamentals have been established within the framework of continuous-time analysis, averaged modeling of switched-mode power converters, and analog control theory [1–5]. Ever more often, control and management functions around power converters are implemented digitally, expanding the field of fundamentals to discrete-time modeling and digital control concepts specific to power electronics. Standard textbooks and courses dedicated to digital control of dynamic systems in general provide the necessary background but seldom, if ever, address the specifics necessary to fully understand and successfully practice the design of digitally controlled power converters. We attempt, in this book, to fill the gap by treating the fundamental aspects of digital control of high-frequency switched-mode power converters in a systematic and rigorous manner. Our objectives are to put the reader in the position to understand, analyze, model, design, and implement digital feedback loops around power converters, from system-level transfer function formulations to coding practical designs in one of the mainstream hardware description languages (HDLs) such as VHDL or Verilog.

The book is intended for graduate students of electrical engineering pursuing a curriculum in power electronics and as a reference for engineers and researchers who seek to expand on the expertise in design-oriented knowledge of digital control of power electronics. It is assumed that the reader is well acquainted with foundations of the power electronics discipline, along with associated continuous-time modeling and control techniques [1–6]. Familiarity with sampled-data and discrete-time system analysis topics is helpful but not absolutely essential. Key concepts are developed from the beginning, including a brief review of the necessary discrete-time system fundamentals in Appendix A. For a more comprehensive background, the reader is referred to one of the standard textbooks, such as [7, 8].

The book is composed of eight chapters, structured as follows. The introductory chapter provides an overview of digital control of high-frequency switched-mode power converters, the motivation behind the surge of interest in the area, a summary of analysis, modeling, control, and implementation issues, as well as a summary of recent advances demonstrating potential advantages of digital controllers, including system power management interfaces, programmability of control functions, dynamic response and efficiency improvements, and practical autotuning techniques.

Chapter 1 provides a review of the continuous-time averaged modeling approach for switched-mode power converters. Averaged small-signal modeling is extensively covered by a number of authoritative textbooks [1–3], and the intention is not to replicate this subject in its entirety. Rather, the purpose of Chapter 1 is to focus the reader's attention on the methodology and *assumptions* behind the averaging approach. Understanding of the philosophy and limitations of the averaging technique is essential to appreciate the need for a different approach when it comes to digitally controlled converters.

Chapter 2 introduces the main elements of a digitally controlled converter, with the purpose of providing the reader with a quick overview of the main differences between analog and digital control without immediately entering into detailed modeling issues. This chapter ends with a discussion about the use of continuous-time averaged modeling for designing digital loops, an approach often employed in practice but which can only account for sampling effects and digital control delays in an approximate manner.

The discussion motivates the formulation of a *discrete-time* modeling approach, developed in Chapter 3, which correctly treats the digitally controlled converter as a sampled-data system and formulates its small-signal dynamics in the z-domain rather than in the Laplace domain. In addition to providing the theoretical framework of discrete-time modeling, a number of modeling examples are discussed in Chapter 3. Furthermore, it is shown that a direct link can be established between continuous-time modeling and discrete-time modeling for the converters that are *topologically invariant*, such as the Buck converter. In such cases, a simple and straightforward discretization rule can be formulated, which translates the converter averaged small-signal model into the exact discrete-time model.

Chapter 4 is devoted to direct digital compensator design, based on the discrete-time models developed in Chapter 3. Among many techniques discussed in the literature, the emphasis is given here to the so-called bilinear transform method. The main advantage of the approach is that the entire design procedure is formulated in an equivalent continuous-time domain, in which both the digitally controlled converter and the compensator under design assume the form of continuous-time systems. As a result, the direct digital design can take advantage of the familiar analog control design techniques with the design specifications formulated in the frequency domain. Standard digital proportional-integral (PI) and proportional-integral-derivative (PID) compensator designs are addressed in a number of examples, including voltage-mode, current-mode, and multiloop control of dc–dc converters and power factor correction (PFC) rectifiers.

Amplitude quantization effects introduced by analog-to-digital (A/D) converters and digital pulse width modulators (DPWMs) are discussed in Chapter 5. This chapter first clarifies how limit cycle oscillations can arise in a digitally controlled dc–dc converter in relation to the existence of a dc operating point for the closed-loop system. Secondly, basic design guidelines—referred to as *no-limit-cycling conditions*—are presented, which aim at preventing such generally undesired phenomena to occur. This chapter ends with a brief overview of DPWM and A/D architectures and associated implementation trade-offs.

The issue of compensator implementation is covered in Chapter 6. Scaling and quantization of compensator coefficients are treated first, with the goal of quantifying the quantization-induced errors on the loop gain magnitude and phase at the desired crossover frequency. Secondly, this chapter addresses implementation of the control law in a fixed-point arithmetic environment, providing a methodology for word length determination of the various signals inside the control structure. Given the focus of the book on high-frequency switched-mode power converter applications, the emphasis is on *hardwired* implementations of the control law, together with VHDL and Verilog coding examples. Nevertheless, the principles that apply to software-based, microprogrammed realizations are highlighted as well.

Autotuning is an advanced application of digital control, which brings up intriguing potentials and additional challenges. Because of the importance of this emerging topic, Chapter 7 is devoted to an overview of digital autotuning techniques for high-frequency switched-mode power converters. After a brief discussion about digital autotuning basics, two autotuning techniques are presented in more detail: an injection-based approach and a relay-based approach.

An objective in writing the book has been to emphasize the distinction between the fundamental, theoretical aspects of digital control design on one side and the application of these techniques on the other, demystifying the perception about discrete-time models or digital control as being exceedingly complex and difficult to employ in practice. In line with such philosophy, Matlab® script examples are systematically developed alongside the theory. A few Matlab® commands allow, in most situations, to straightforwardly carry out system-level compensator designs and rapidly proceed to HDL coding and implementation steps. Furthermore, throughout the book, a number of design examples are fully worked out and verified by simulations in the Matlab® environment.

INTRODUCTION

Efficient processing and control of electric power is required in applications ranging from submilliwatt on-chip power management to hundreds of kilowatt and megawatt power levels in motor drives and utility applications. The objectives of high efficiency, as well as static and dynamic control of inputs or outputs under a range of operating conditions, are accomplished using power electronics, that is, switched-mode power converters consisting of passive (capacitive and inductive) components, and power semiconductor devices operated as switches. In high-power applications, control and monitoring tasks are often more complex, while the power semiconductor devices are operated at relatively low switching frequencies, for example, up to tens of kilohertz. The controller cost and power consumption are relatively low compared to the overall system cost and power rating. In these applications, digital control offers clear technical and economic advantages in addressing complex control, management, and monitoring tasks. As a result, for many years now, digital control methods and digital controllers based on general-purpose or dedicated microprocessors, digital signal processor (DSPs), or programmable logic devices have been widely adopted in power electronics applications at relatively high power levels.

In ubiquitous low-to-medium power switched-mode power supply (SMPS) applications, including point-of-load (POL) regulators, nonisolated and isolated dc–dc converters, single-phase power factor correction (PFC) rectifiers, single-phase inverters, and lighting applications, adoption of digital power management and digital control has been slower. In these applications, switching frequencies are often in the range from hundreds of kilohertz to multiple megahertz, and much faster dynamic responses are required. The controller cost and the controller power consumption can easily present significant portions of the system cost and power dissipation. Furthermore, in many applications, control challenges have been successfully met by continuous advances of readily available analog controllers, using well-established analog analysis, modeling, and design techniques [1–5]. Nevertheless, practical digital control of high-frequency switched-mode power converters has moved from proof-of-concept demonstrations to digital pulse width modulation (DPWM) controller chips commercially available from multiple vendors, with growing adoption rates in many applications. Several factors have

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contributed to the increasing penetration of the concept of "digital power" in high-frequency power electronics applications:

- Ongoing advances in digital integrated-circuit processes have continued to increase processing capabilities while bringing the cost down.
- The needs for improved system integration and increasingly complex power management and monitoring functions have translated into the needs for digital interfaces and programmability in switched-mode power conversion applications [9–11].
- Practical high-performance digital control techniques have been introduced and demonstrated, together with innovative approaches offering performance gains or entirely new capabilities that would be difficult or impractical to realize using traditional analog techniques [12–14].

The "digital power" concept encompasses several aspects:

1. *Digital power management*, which refers to system-level control and monitoring of power conversion and distribution, usually over a serial communication bus [9–11]. Power management functions include turning on and off or sequencing system power rails, adjusting setpoints for converter control loops, programming control loop parameters, monitoring and reporting of measured status or variables, and so on [15, 16]. These functions are typically performed at timescales that are relatively long compared to a switching period.
2. *Digital control*, which includes time-domain and frequency-domain converter modeling and control techniques, with control actions performed at timescales comparable to a switching period.
3. *Digital implementation techniques*, which can be classified into two main groups:
   - *Software-based controllers*, where control algorithms are designed and implemented in code executed on general-purpose or specialized microcontrollers or DSP chips. An early example of application of microprogrammed digital control to power factor preregulators is presented in [17].
   - *Hardware-based controllers*, based on custom-integrated circuits or programmable logic devices such as field-programmable gate arrays (FPGAs) [18, 19]. Early examples of such hardware-based digital controllers can be found in [20–22].

This book is focused on the fundamental aspects of analysis, modeling, and design of digital control loops around high-frequency switched-mode power converters in a systematic and rigorous manner. The objectives are to enable the reader to understand, analyze, model, design, and implement digital feedback loops around power converters, from system-level transfer function formulations to practical implementation details. The purpose of this chapter is to introduce the topics covered in the book and to motivate the reader to pursue the theoretical and practical concepts covered in the remaining chapters of this book. Furthermore, this introductory chapter points to some of the more advanced digital techniques reported in the literature, including approaches to dynamic response improvements, system identification, autotuning of digital control loops, and on-line efficiency optimization.

DIGITALLY CONTROLLED SWITCHED-MODE CONVERTERS

A number of DPWM controller architectures and implementation strategies have been investigated and realized in practice. Many standard microcontrollers and DSP chips are now available, featuring multiple high-resolution PWM and analog-to-digital (A/D) channels, which allow software-based implementation of control and management functions. While advances in this area have been rapid, the software-based approaches are still better suited for applications where switching converters operate at relatively low switching frequencies. On the other hand, at switching frequencies in the hundreds of kilohertz to megahertz range, specialized hardware-based control loops are often preferred. This approach is illustrated in the architecture shown in Fig. 1 [12, 13]. The control loop is digital, using specialized, programmable A/D, DPWM, and compensator blocks to achieve high-performance closed-loop dynamic responses, while programmability, power management, and system interface functions are delegated to a microcontroller core. Similar combinations of programmable hardware peripherals specialized for switched-mode power converter applications, with software-based realizations of higher-level management and communication functions are often found in commercially available DPWM controllers.

Controllers of the type shown in Fig. 1 can be developed, realized, and tested using standard digital VLSI design flow starting from logic functions described using

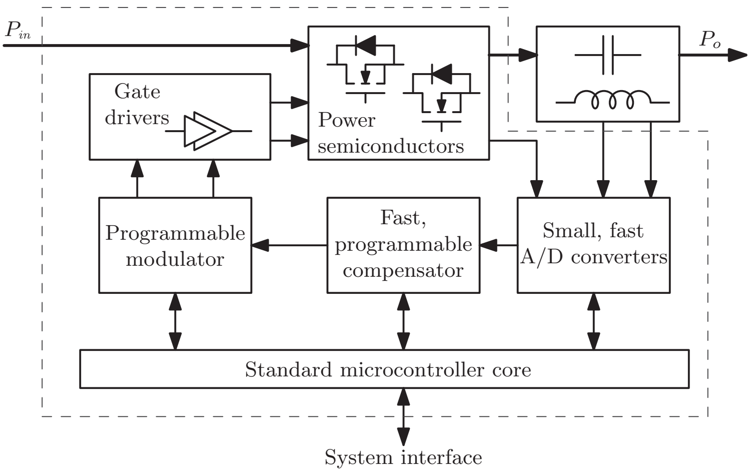

Figure 1 Digital controller architecture for high-frequency switched-mode power converters [13].

hardware description language (VHDL or Verilog), followed by prototyping and experimental verifications using FPGA development platforms, ultimately targeting relatively small, relatively low-gate-count integrated circuits capable of matching or surpassing the state-of-the-art analog solutions in terms of dynamic performance, power consumption, and cost. At the same time, digital PWM controllers offer digital system interface, programmability and flexibility, power management functions, reductions in the number of passive components, reduced sensitivity to process and temperature variations, and potentials for practical realizations of more advanced features.

Figure 2 shows a more detailed block diagram of a hardware-based digital controller around a POL synchronous Buck converter. Output voltage $v_o(t)$ is sampled by an A/D converter and compared to a setpoint reference V_{ref} to produce a digital voltage error signal $e[k]$. The error signal is processed by a discrete-time digital proportional-integral-derivative (PID) compensator to generate a duty cycle command $u_x[k]$. In the basic version of the controller, the compensator gains K_p, K_i, and K_d are found by design to meet control loop specifications, such as the crossover frequency and phase margin, as detailed in Chapters 1–6 of this book. Once a compensator is designed, the gains can be realized using digital multipliers, as shown in Fig. 2. As only a few bits are sufficient to represent the error signal $e[k]$, the entire compensator can also be implemented as a lookup table [21–24]. In a more advanced case, as illustrated by the digital autotuner block in Fig. 2, the compensator gains can

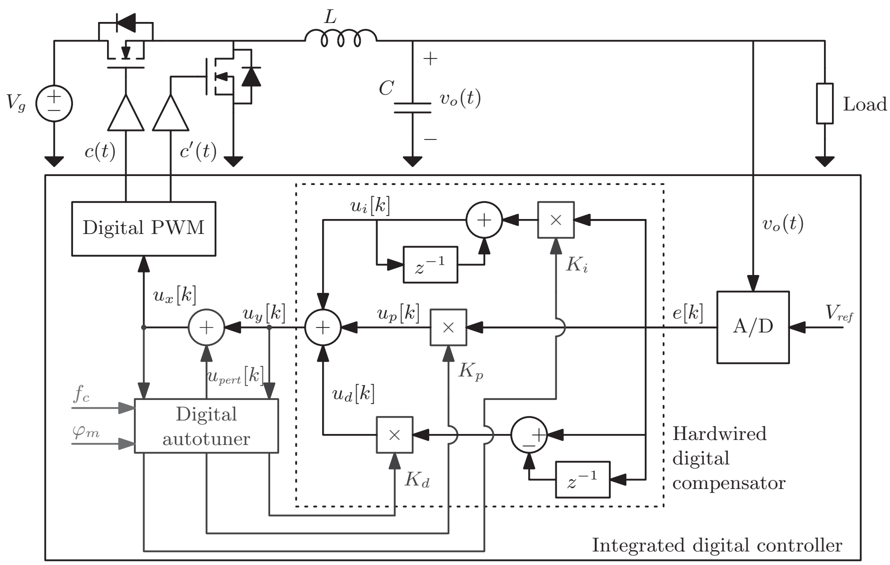

Figure 2 Digitally controlled point-of-load (POL) synchronous Buck dc–dc converter example. Analysis, modeling, design, and implementation of digital control loops are addressed in Chapters 1–6. An additional injection-based autotuning block is shown, which is further discussed in Chapter 7.

be tuned in response to the actual system dynamics to meet the desired specifications. Autotuning is addressed in Chapter 7. Finally, a DPWM block generates the complementary gate-drive control signals $c(t)$ and $c'(t)$ with duty cycle set by the digital command and with appropriate dead times. Together with various enhancements, such a controller can be realized in about 10,000 equivalent logic gates, which translates to about one-third of a square millimeter in a standard 0.35 $\mu\mathrm{m}$ CMOS process. Furthermore, higher-density CMOS processes with high-voltage extensions suitable for power electronics applications are now readily available, making power and cost-effective digital controllers for high-frequency switched-mode power converters a reality. Examples of integrated digital controllers can be found in [21, 22, 24–33].

It is of interest to examine a practical example. Following the block diagram shown in Fig. 2 (without the digital autotuner), a digitally controlled 5 to 1.6 V synchronous Buck converter prototype is described in [34]. The filter component values are $L = 1.1\ \mu\mathrm{H}$, $C = 250\ \mu\mathrm{F}$, and the switching frequency is $500\ \mathrm{kHz}$. The A/D converter is a windowed converter [26], using threshold inverter quantization approach [35]. The A/D conversion range is approximately $200\ \mathrm{mV}$, centered around the reference $V_{ref} = 1.6$ V, for an equivalent output voltage quantization step of $3\ \mathrm{mV}$. A hybrid counter/ring oscillator DPWM is employed, with a time quantization of about 390 ps, that is, with 0.02% duty cycle resolution. A digital PID compensator designed for $f_c \approx f_s/10 = 100\,\mathrm{kHz}$ crossover frequency is VHDL coded and implemented on an FPGA. Figure 3 illustrates an experimental 0 to 8 A load step response, with the voltage deviation and the response time comparable to responses expected from high-performance analog PWM controllers.

Analysis, Modeling, and Control Techniques

Referring to the example in Fig. 2, one may observe that the basic digital control loop is conceptually similar to the standard voltage-mode analog PWM control

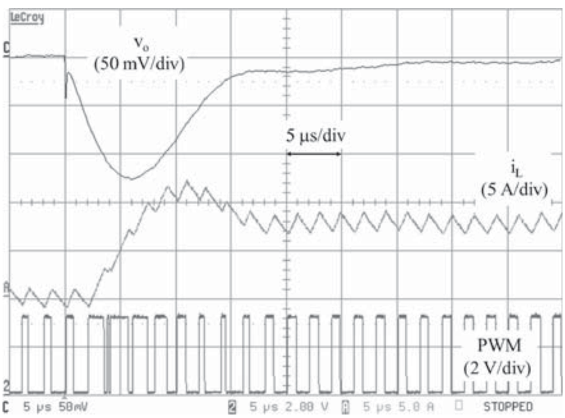

Figure 3 Experimental 0 to 8 A load step response in a digitally controlled POL Buck converter with conventional PID compensation [34] (v_o, $50\ \mathrm{mV}$/div; i_L, 5 A/div; and timescale, 5 μs/div). © 2009 IEEE.

loop, with analog control techniques based on averaged converter models briefly reviewed in Chapter 1. As discussed further in Chapter 2, digital control differs from analog control in two key aspects: *time quantization* and *amplitude quantization*. Time quantization refers to the fact that the controller is a discrete-time system that processes *sampled* versions of sensed analog signals to be regulated and produces a discrete-time control output. In order to design high-performance control loops, it is necessary to understand and consider the resulting delays and aliasing effects. As discussed in Chapters 2 and 3, the use of continuous-time averaged modeling for designing digital loops, an approach often employed in practice, can only account for sampling effects and digital control delays in an approximate manner. A more rigorous approach is based on discrete-time modeling [36], which is described in detail in Chapter 3. This modeling approach enables direct-digital design of compensator transfer functions, which is presented in Chapter 4. The design specifications are presented in frequency domain, in terms of the quantities familiar to the analog designer: the loop-gain crossover frequency f_c and the phase margin φ_m.

Implementation Techniques

In digitally controlled converters, regulation precision and accuracy are determined by the resolutions of A/D and DPWM blocks, which introduce amplitude quantization effects discussed in Chapter 5. These nonlinear effects can lead to steady-state disturbances commonly referred to as limit cycling [37, 38]. Design guidelines to avoid limit cycling are also presented in Chapter 5, together with a brief summary of high-resolution DPWM and A/D implementation techniques.

Digital compensator implementation is addressed in Chapter 6. Scaling and quantization of compensator coefficients are treated first, with the goal of quantifying the errors introduced by coefficient quantization on the loop gain magnitude and phase at the desired crossover frequency. Chapter 6 then addresses PID compensator structures, such as the parallel PID realization shown in Fig. 2, and the implementation of the control law in a fixed-point arithmetic environment, providing a methodology for word length determination of the various signals inside the control structure. Given the focus of the book on high-frequency switched-mode power converter applications, the emphasis is on *hardwired* implementations of the control law, together with VHDL and Verilog coding examples, while the principles that apply to software-based, microprogrammed realizations are also highlighted.

The objectives of Chapters 1–6 are to enable the reader to successfully analyze, model, design, and implement voltage, current, or multiloop digital feedback loops around switched-mode power converters. Alongside, based on the theoretical concepts, Matlab® scripts are systematically developed, which allow one to rapidly perform discrete-time modeling and system-level compensator design steps, and proceed to implementation steps. Practical examples are used throughout the book to illustrate applications of the techniques developed.

SYSTEM AND PERFORMANCE GAINS VIA DIGITAL CONTROL

Increased flexibility, programmability, and integration of system interface and power management functions have been recognized as important advantages of digital controllers. Furthermore, as digital controller implementation opened opportunities for practical implementations of more sophisticated control approaches, considerable advances have been made in various directions. This section highlights some of the gains enabled by digital control in the areas of improved dynamic responses, integration of frequency-response measurements, autotuning of digital control loops, and on-line efficiency optimization.

Improved Dynamic Responses

Standard analog or digital converter controller design techniques are based on linear small-signal models and frequency-domain-based compensator designs. It has been recognized that considering the switching nature of the power stage directly, and operating with large-signal instantaneous state variables to provide on–off control actions accordingly, can result in improved dynamic responses. The switching surface control [39] and many other time-domain based approaches have been investigated both in analog and in digital domains. Digital implementation is particularly well suited for explorations of control techniques targeting improved dynamic responses. A case of special interest is a sequence of switching actions that result in minimum time, that is, time-optimal response to an external disturbance such as a step load transient. For example, for a Buck converter, a time-optimal response to a step load transient consists of a single precisely timed on/off sequence. Various digital control methods have been proposed to implement the time-optimal control [34, 35, 40–60].

These controllers have demonstrated step load transient responses that approach limits imposed by the converter passive LC filter components. For example, Fig. 4 illustrates the step load transient response of the parameter-independent time-optimal controller described in [34], for the same synchronous Buck prototype as in Fig. 3. The single switching action occurring immediately after the step load is visible, which quickly restores the output voltage to regulation. Compared to the response with the standard PID compensator in Fig. 3, both the voltage deviation and the response time are significantly reduced. As the control action is effectively saturated during time-optimal control events, overshoots may occur in converter internal states such as the inductor current. An extension of digital time-optimal control, including practical inductor current limitations, has been addressed in [57].

A number of other approaches to achieve improved dynamic responses have been explored. Multisampling techniques (where converter waveforms are sampled more than once per switching period [61–63]), asynchronous sampling techniques [35, 64, 65], and mixed-signal control techniques [66, 67] have been proposed to

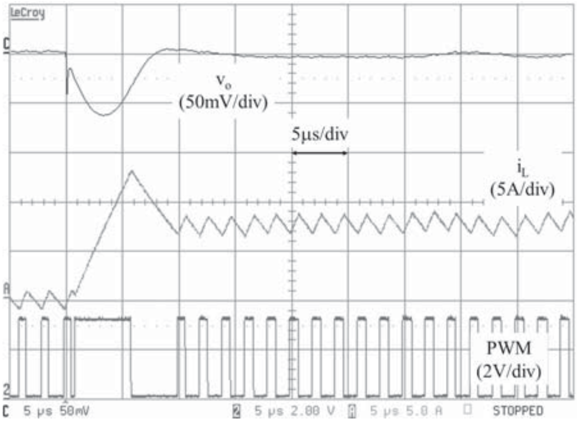

Figure 4 Experimental 0 A-8 A step load response in a digitally controlled POL Buck converter with the time-optimal controller described in [34] (v_o: 50 mV/div, i_L: 5 A/div, timescale: 5 μs/div). © 2009 IEEE.

minimize the control loop delays. Multisampling techniques for constant on-time controllers can also be found in [68–70]. Furthermore, it has been shown that non-linear techniques can lead to dynamic performance improvements in dc–dc [71, 72] and PFC applications [73]. Possibilities for improved dynamic responses in multi-phase architectures have been investigated in [74, 75]. More complex controllers in conjunction with power stage modifications present another interesting direction in dynamic responses improvements. For example, it has been shown how digitally controlled power stages with additional auxiliary switches can offer significant dynamic response improvements [76], while [77] takes this approach a step further and proposes a much tighter load-controller interaction.

Integration of Frequency-Response Measurements

An important experimental verification step in traditional frequency-response-based controller designs includes measuring controller small-signal frequency responses using network analyzers [1, 78]. Feasibility of integrating such nonparametric frequency-domain system identification (system-ID) functionality into digital controllers has been demonstrated in [79, 80]. To briefly summarize the technique, Fig. 5 shows a block diagram of a digitally controlled converter with additional system-ID functions. The identification process consists of perturbing the duty cycle command with a pseudo-random binary sequence (PRBS), cross-correlating the perturbation with the measured output responses to obtain the system impulse response, and performing fast Fourier transform (FFT) to obtain frequency responses. Cross-correlation can be implemented efficiently using the fast Walsh–Hadamard transform (FWHT). Note that the approach reuses the DPWM and the A/D block already present in the digital control loop. To mitigate the effects of switching and quantization noise, the system-ID approach includes a sweep to optimize the perturbation magnitude, pre-emphasis and de-emphasis filters applied to the injected and the sensed signals, respectively, as well as smoothing in the frequency domain. This approach has been applied to a number of converter examples in [80],

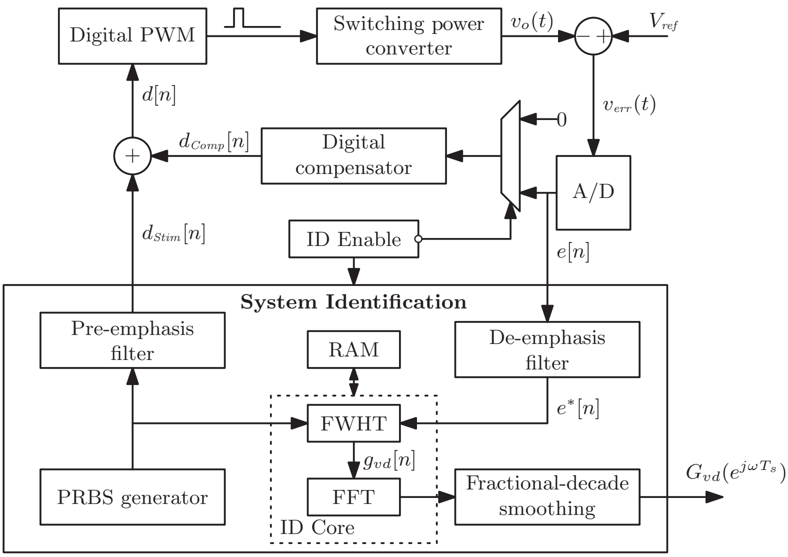

Figure 5 Digitally controlled PWM converter with integrated frequency response measurement capabilities [80]. © 2008 IEEE.

including a 15- to 30-V Boost dc–dc converter operating at $f_s = 195$ kHz. Figure 6 shows time-domain waveforms during the identification process, while Fig. 7 shows the identified frequency responses, which closely match discrete-time modeling predictions.

The resulting on-line identified frequency response can be used for design, diagnostic, or tuning purposes [81]. The success of these applications depends on the fidelity of the identified frequency responses and the degree to which the process is automated, as well as the costs, in terms of gate count or complexity, time duration of identification, and effects on output voltage, incurred to obtain the results. Reference [80] demonstrates the feasibility of incorporating fully automated frequency response measurement capabilities in digital PWM controllers at relatively low additional cost. The identification process can be typically accomplished in several hundred milliseconds, and the output voltage can be kept within narrow bounds during the entire process.

Autotuning

Taking advantage of the digital controller programmability, the overall objective of autotuning is to automatically tune the controller parameters in response to the actual system dynamics. An autotuning digital controller ideally becomes a “plug and play” unit capable of identifying the key characteristics of the power converter and the load and adjusting the controller parameters to achieve specified performance goals. This capability represents a significant departure from the conventional design flow.

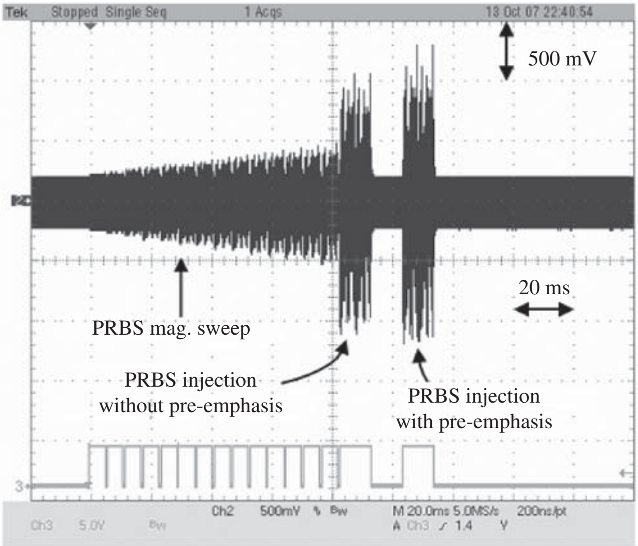

Figure 6 Output voltage during system identification process in a 15- to 30-V Boost dc–dc converter operating at $f_s = 195$ kHz [80]. © 2008 IEEE.

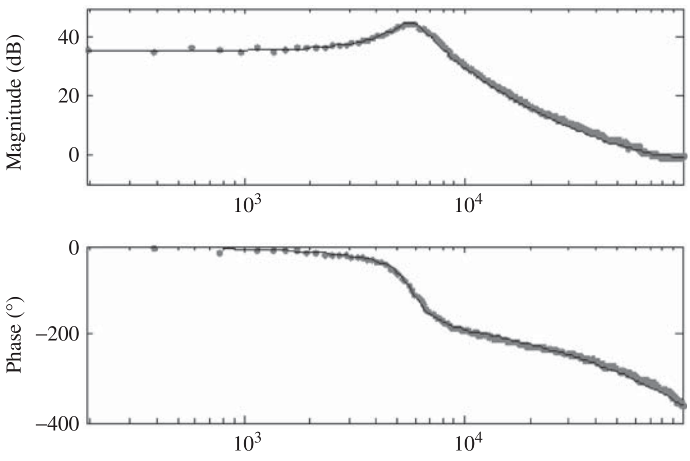

Figure 7 Control-to-output magnitude and phase responses determined by the system identification process illustrated in Figs. 5 and 6 [80]. © 2008 IEEE.

Numerous advances have been made in the area of practical autotuning digital control algorithms and implementation techniques [81–101], and the area is subject to ongoing research and development efforts. Chapter 7 presents an overview of digital autotuning techniques for high-frequency switched-mode power converters. Two autotuning techniques are presented in more detail: an injection-based approach and a relay-based approach.

The controller block diagram in Fig. 2 shows how a digital autotuner can be incorporated into the digital controller based on the injection approach [92, 93, 95, 97]. The autotuning system injects a digital perturbation signal $u_{pert}[k]$ into the feedback loop, superimposed to the PID compensator output $u_y[k]$. The overall control command $u_x[k] = u_y[k] + u_{pert}[k]$ modulates the converter. Simultaneously, signals $u_y[k]$ and $u_x[k]$ before and after the injection point are monitored and the tuning process adjusts the compensator gains until the correct amplitude and phase relationships are established between the ac components of $u_x[k]$ and $u_y[k]$ at the injection frequency, indicating that the loop gain meets the specifications given in terms of the desired crossover frequency f_c and phase margin φ_m. The hardware requirements for the entire adaptive tuning system are relatively modest, making it a practical solution.

Efficiency Optimization

Power conversion efficiency is a key performance metric in most applications. In the efficiency improvements area, potential advantages of digital controllers include abilities to precisely adjust switching frequency or other timing parameters of switch control waveforms [27, 102–107], abilities to reconfigure the power stage on-line either through power-switch segmentation [108, 109] or other gate-drive parameters [109, 110], phase shedding [58, 111], or control of current distribution in multiphase architectures [111–113], as well as abilities to implement algorithmic or preprogrammed approaches to on-line efficiency optimization [102, 103, 114–117].

The shift from traditional analog techniques to digital control in high-frequency power electronics is making an impact on standard design practices, as well as in various applications. Programmability, monitoring, digital system interfaces, and system-level power management are becoming ubiquitous in power systems ranging from mobile electronics to desktop computing, data centers, and communication infrastructure. In these systems, digital control further brings opportunities for improved dynamic responses and correspondingly reduced size of passive filters, together with new approaches to converter-level and system-level efficiency optimizations. In response to increasing energy cost and environmental concerns, various energy efficiency initiatives and programs are addressing power conversion efficiency and power quality in data centers and computer power supplies. It is expected that future energy efficiency program specifications will be even more demanding in terms of efficiency, power factor, and harmonic distortion requirements for off-line power supplies over even wider load ranges. Further significant impact can also be expected in renewable energy applications. For example, distributed module integrated converters or micro-inverters in photovoltaic power systems can take advantage of digital control algorithms for improved maximum power point tracking, fault detection, and efficiency optimization. Similar impact can be foreseen in electric-drive vehicles, not just in inverter controls where digital control is already ubiquitous, but also in battery management and battery charger systems.

CHAPTER 1

CONTINUOUS-TIME AVERAGED MODELING OF DC–DC CONVERTERS

Converter systems rely on feedback loops to achieve the desired regulation performance. For example, in a typical dc–dc converter application, the objective is to maintain tight regulation of the output voltage in the presence of input voltage or load current variations. An accurate small-signal description of the converter control-to-output dynamics is the starting point for feedback loop design techniques based on frequency-domain concepts of loop gain, crossover frequency, phase margin, and gain margin.

The most successful and widespread modeling technique for switched-mode converters is based on *averaged small-signal modeling* [1, 118–120]. This technique is based on first averaging the converter behavior over a switching period with the purpose of smoothing the discontinuous, time-varying nature of the converter into a continuous, time-invariant nonlinear system model. A successive linearization step yields a linear, time-invariant model that can be treated using standard tools of linear system theory. The converter is described by a *continuous-time linear system*, often presented in the form of a linear equivalent circuit model, a natural representation in the context of analog control design.

The averaging approach is currently the most widely accepted way of *understanding* dynamics of switched-mode power converters. In addition to the relative simplicity and straightforwardness, popularity of the averaging approach has been reinforced by the success of innumerable practical designs supported by robust and easy-to-use integrated circuits for analog converter control.

The main purpose of this chapter is to revisit the main aspects of analysis and modeling techniques for switched-mode power converters. Averaged small-signal modeling, in particular, is reviewed in detail, highlighting the main assumptions behind the approach. This prepares the background necessary to understand the limitations of the averaged small-signal modeling in the context of digital control design and to allow subsequent developments of discrete-time models where these limitations are removed.

Digital Control of High-Frequency Switched-Mode Power Converters, First Edition.
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A brief review of pulse width modulated (PWM) dc–dc converters is presented in Section 1.1, followed by a summary of steady-state analysis and modeling techniques in Section 1.2. Section 1.3 explains the need for dynamic modeling in the design of control loops around switched-mode power converters and introduces the small-signal averaged modeling approach. The method of state-space averaging [119, 120], a general approach to modeling switched-mode power converters, is summarized in Section 1.4. Analog control design examples are presented in Section 1.5. In the subsequent chapters, these examples are revisited to illustrate modeling and digital control design principles. To complete the background necessary to engage in developments of analysis, modeling and control techniques in the context of digitally controlled PWM converters, a discussion related to the nature of duty cycle, the control variable in PWM converters, is presented in Section 1.6. The key points are summarized in Section 1.7.

1.1 PULSE WIDTH MODULATED CONVERTERS

The focus of this book is on *PWM* converters, which are operated so as to alternate between two or more distinct subtopologies in a periodic fashion, with a fundamental *switching period* T_s. The Boost converter depicted in Fig. 1.1, for instance, operates with the switch in position 1 for a fraction DT_s of the switching period and with the switch in position 0 for the remaining fraction $D'T_s \triangleq (1 - D)T_s$. The quantity $0 \leq D \leq 1$ is the *duty cycle*, which determines the fraction of a switching period the switch is kept in position 1. In PWM converters, D is the *control input* for the system, which is adjusted by a controller in order to regulate a converter voltage or current.

Typical waveforms of a PWM converter are shown in Fig. 1.2, which exemplifies the gate driving signal $c(t)$ and one of the converter state variables, such as the output voltage $v_o(t)$. Assuming that the converter duty cycle is sinusoidally modulated at a frequency $f_m \ll f_s$, the output voltage consists of a low-frequency component $\overline{v}_o(t)$, plus a high-frequency *switching ripple*. The low-frequency component of $v_o(t)$ contains a dc term V_o and a spectral component at the modulation frequency f_m. Using the terminology of modulation theory, $\overline{v}_o(t)$ is the *baseband* component of $v_o(t)$. The high-frequency content, on the other hand, contains the switching frequency f_s and its harmonics, as well as all the modulation sidebands

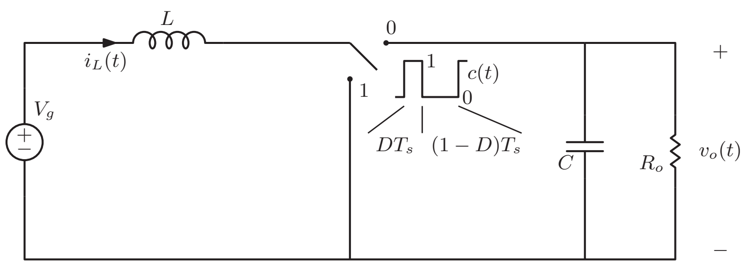

Figure 1.1 Pulse width modulated Boost converter.

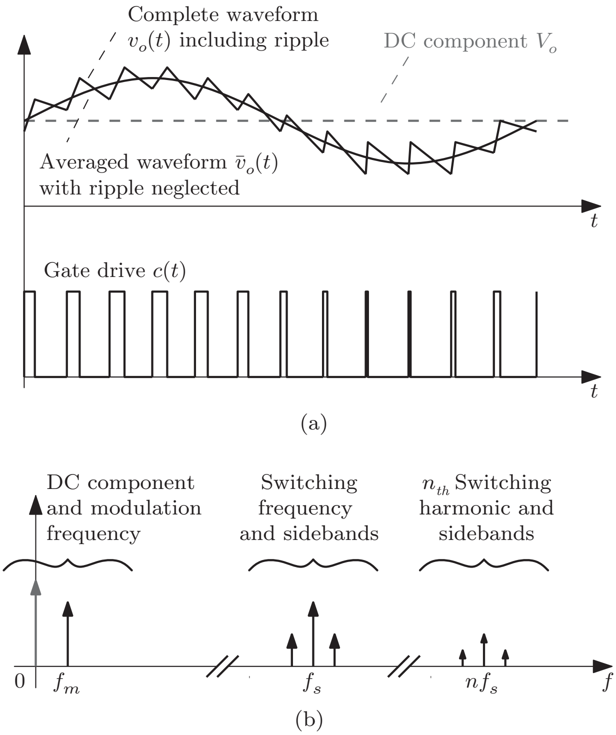

Figure 1.2 (a) Converter waveforms with duty cycle modulation and (b) qualitative spectrum of a pulse width modulated signal.

originating from nonlinear interactions between f_m and f_s components occurring as a result of the modulation process.

In the context of the averaged modeling approach, the separation between low-frequency and high-frequency portions of the converter signals is of central importance. To be more precise, the *moving average operator* $\langle \cdot \rangle_T$ is introduced,

$$\boxed{\langle x(t) \rangle_T \triangleq \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau)\, d\tau}, \tag{1.1}$$

which averages signal $x(t)$ over a period T. With this definition, the low-frequency component $\overline{v}_o(t)$ of $v_o(t)$ illustrated in Fig. 1.2 is defined as its moving average over the switching period T_s,

$$\boxed{\overline{v}_o(t) \triangleq \langle v_o(t) \rangle_{T_s}}. \tag{1.2}$$

The fundamental simplification at the basis of the averaging method consists of describing the small-signal dynamics of $\overline{v}_o(t)$ rather than $v_o(t)$, therefore neglecting high-frequency components of the converter waveforms. Both the power and the limitations of the method reside in the averaging approximation.

1.2 CONVERTERS IN STEADY STATE

When a converter is operating in steady state, every converter state variable—and therefore every voltage and current—is *periodic* in time, with a period equal to the converter switching period T_s. Steady-state operation is reached when all the converter inputs are constant—including the duty cycle—and after all transients are extinguished. In the following text, the basic ideas behind steady-state analysis of PWM converters are summarized. More extensive and detailed treatments can be found in power electronics textbooks [1–5].

Steady-state analysis of switched-mode power converters consists of expressing the dc values of all the voltages and currents in terms of the converter inputs. The analysis is founded on two basic principles, which are direct consequences of the periodicity of the system waveforms:

- *Inductor volt-second balance.* As all the inductor currents are periodic, no net flux variation can occur in any inductor over a switching period,

$$L(i_L(T_s) - i_L(0)) = \int_0^{T_s} v_L(\tau)\, d\tau = 0. \tag{1.3}$$

  This is equivalent to stating that the average inductor voltage over a switching interval is zero,

$$\boxed{\overline{v}_L(t) = 0}. \tag{1.4}$$

- *Capacitor charge (ampere-second) balance.* By a dual argument, as all the capacitor voltages are periodic, no net charge can be absorbed or delivered by any capacitor over a switching period,

$$C(v_C(T_s) - v_C(0)) = \int_0^{T_s} i_C(\tau)\, d\tau = 0. \tag{1.5}$$

  This is equivalent to stating that the average capacitor current over a switching interval is zero,

$$\boxed{\overline{i}_C(t) = 0}. \tag{1.6}$$

The two above-mentioned conditions, combined with conventional circuit analysis, are sufficient to solve the steady-state problem. In practice, the calculations are greatly simplified by introducing the *small-ripple approximation*. By *switching ripple*, one refers to the ac component of a converter voltage or current. In steady state, the switching ripple is a periodic function with a fundamental frequency equal to the converter switching rate. The ripple peak-to-peak amplitudes of a capacitor voltage $v_C(t)$ and an inductor current $i_L(t)$ are denoted as Δv_C and Δi_L, respectively.

The small-ripple approximation states that the dc converter quantities can be approximately determined by neglecting both capacitors voltage ripples and inductors current ripples. This corresponds to considering every capacitor C as an ideal dc