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Artificial Transmission Lines for RF *and* Microwave Applications

Ferran Martín

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To Anna, Alba and Arnau

CONTENTS

PREFACE

Transmission lines and waveguides are essential components in radiofrequency (RF) and microwave engineering for the guided transmission of electromagnetic (EM) energy (power and information signals) between two points. Moreover, transmission lines and waveguides are key building blocks for the implementation of passive and active RF/microwave devices of interest in wireless communications (filters, diplexers, splitters, couplers, amplifiers, oscillators, mixers, etc.). In planar technology, low-cost devices can be fabricated by etching patterns (a set of transmission lines and stubs providing certain functionality) in a printed circuit board (PCB), avoiding the use of lumped components, such as capacitors, inductors, or resonators. Transmission line-based circuits are usually designated as distributed circuits, since transmission lines can be described by a network of distributed parameters. In certain designs (e.g., amplifiers and mixers), it is necessary to combine distributed and lumped active elements, such as diodes or transistors. Nevertheless, the main relevant aspect of distributed components is their capability to mimic lumped-reactive elements or a combination of them (e.g., resonators or even more complex reactive circuits). It is thus possible to design fully planar functional devices on the basis of the distributed approach, or to minimize the number of lumped elements (unavoidable in certain components, e.g., active circuits) in the designs.

Distributed circuits have two main drawbacks: (1) their dimensions scale with frequency, and (2) transmission lines exhibit very limited design flexibility. Typically, the required transmission lines in the designs have a length of the order of the wavelength, which means that dimensions may be too extreme if the operating frequencies are moderate or low. Concerning the second aspect, distributed circuits are designed by means of transmission lines with certain phase (at the operating frequency) and characteristic impedance. The phase varies linearly with the length of the line and frequency (to a first-order approximation transmission lines are dispersionless). Therefore, the functionality of distributed circuits is limited to a certain band; namely, the required nominal phase is lost if frequency deviates from the operating value, which means that distributed circuits are bandwidth limited by nature.

The limitations of ordinary transmission lines as building blocks for device design are in part originated from the fact that these lines exhibit a limited number of free parameters for design purposes (by excluding losses, ordinary lines are described by a distributed network with two reactive elements in the unit cell). However, by truncating the uniformity (in the longitudinal direction), by etching patterns in the ground plane, by loading the line with reactive elements, or by using a combination of these (or other) strategies to increase the degrees of freedom of the lines, many possibilities to reduce device size, to improve performance, or to achieve novel functionalities, are open. This book has been mainly conceived to introduce and study alternatives to ordinary lines for the design and implementation of RF/microwave components with superior characteristics in the above cited aspects. We refer to these lines as artificial transmission lines, and the term is as wide as the number of strategies that one can envision to improve the size, performance, or functionality of ordinary lines. The book is devoted to the analysis, study, and applications of artificial lines mostly implemented by means of a planar transmission line (host line) conveniently modified (e.g., with modulation of transverse dimensions, with etched patterns in the metallic layers, and with reactive loading), in order to achieve certain functionality, superior performance, or reduced size. Nevertheless, it will be shown that in certain artificial waveguiding structures, such as electroinductive and magnetoinductive delay lines, the host line is not present. Waveguide-based components are not included in this book, entirely focused on artificial transmission lines in planar technology. Obviously, it is not possible to cover all the material available in the literature, related to the topic of artificial transmission lines, in a single book. Necessarily, the contents of this book are influenced by the personal experience and background of the author. However, many RF/microwave devices and applications of artificial transmission lines reported by other researchers are included in this book, or properly referenced.

The book is devoted to readers that are already familiar with RF/microwave engineering. The aim of writing this book has been to provide an up-to-date state of the art in artificial transmission lines, and an in-depth analysis and study of those aspects, structures, devices, and circuits that are more relevant (according to the criterion of the author) for RF/microwave engineering, including design guidelines that can be useful to researchers, engineers, or students involved in the topics covered by this book. Nevertheless, Chapter 1 is dedicated to the fundamentals of planar transmission lines for coherence and completeness, since most of the concepts of this chapter are used in the subsequent chapters, and are fundamental to understand the principles and ideas behind the design and applications of artificial transmission lines.

Chapter 2 is focused on artificial transmission lines based on periodic structures, where periodicity plays a fundamental role and is responsible for the presence of band gaps in the transmission spectrum of these lines. The Floquet analysis (leading to the concept of space harmonics), complemented by the coupled mode theory (from which useful expressions for the design of periodic artificial lines are derived), and the transfer matrix method (useful to obtain the dispersion relation of the fundamental space harmonic), are included in the chapter. The last part is devoted to the applications, which have been divided into those of periodic nonuniform transmission lines (e.g., harmonic and spurious suppression), and those of reactively loaded lines, where not only the reflection properties of periodic structures but also the inherent slow-wave effect associated to reactive loading, are exploited.

Chapters 3 and 4 are dedicated to artificial transmission lines inspired by metamaterials, or based on metamaterial concepts. The importance of these artificial lines in this book has forced the author to separate the fundamentals/theory and applications into different chapters in order to avoid an excessive chapter length. Thus, Chapter 3 is focused on the theory, circuit models, and main implementations of metamaterial transmission lines, whereas Chapter 4 deals with the applications. Many applications of metamaterial transmission lines are based on the superior controllability of the characteristic impedance and dispersion of these lines, as compared to ordinary lines, related to the presence of reactive elements loading the line. Indeed, metamaterial transmission lines have opened a new way of "thinking" in the design of microwave components, where tailoring the dispersion diagram, and not only the characteristic impedance, is the key aspect (we may accept that metamaterial transmission lines have given rise to microwave circuit design on the basis of impedance and dispersion engineering). The further controllability of the relevant line parameters (phase constant and characteristic impedance) in metamaterial transmission lines, as compared to ordinary lines, has a clear parallelism with the further controllability of the constitutive parameters (permittivity and permeability) in effective media metamaterials (periodic artificial structures exhibiting controllable EM properties, different from those of the materials which they are made). Indeed, we can define an effective permittivity and permeability in metamaterial transmission lines despite that these lines are one-dimensional structures, and we can design the lines in order to support backward (or left-handed) wave propagation (as occurs in metamaterials with simultaneous negative effective permittivity and permeability). However, whereas in effective media metamaterials periodicity and homogeneity (satisfied if the period is much smaller than the wavelength) are necessary conditions to properly define an effective permeability and permittivity, periodicity, and homogeneity are not requirements for impedance and dispersion engineering with metamaterial transmission lines.

The former metamaterial transmission lines were implemented by loading a host line with series capacitors and shunt inductors (CL-loaded approach), or by loading the host lines with electrically small resonators, formerly used for the implementation of bulk effective media metamaterials (metamaterial resonators). This latter approach has been called resonant-type approach. Both approaches are included in this book (and many other latter developments), but special emphasis is put on the resonant-type approach. Moreover, in Chapter 4 there are several applications where, rather than the controllability of the impedance and dispersion of the artificial lines, the working principle is the resonance of a transmission line (host line) loaded with metamaterial resonators (these lines are designated as transmission lines with metamaterial loading in Chapter 4). Since metamaterial transmission lines are inspired by metamaterials, an introduction to these artificial media and the former implementation are included in Chapter 3. Chapter 3 includes also a section devoted to study the main electrically small resonators useful for the synthesis of metamaterials and microwave circuits based on them (resonant-type approach). In Chapter 4, the applications include enhanced bandwidth components, multiband components, filters and diplexers, active devices with novel functionalities (e.g., distributed amplifiers), novel antennas (e.g., leaky wave antennas and antennas for RFID tags), microwave sensors, and so on.

In Chapter 5, the focus is on reconfigurable components based on tunable artificial lines and nonlinear transmission lines. Several materials, components, and technologies (including varactors, RF-MEMS, ferroelectrics, and liquid crystals) for the implementation of tunable components are introduced. Then the chapter focuses on the design of tunable artificial transmission lines and their applications, mostly, although not exclusively, devoted to filters. The last part of the chapter deals with the topic on nonlinear transmission lines, structures that support the propagation of solitons and are of interest for harmonic multiplication.

Finally, other advanced transmission lines or, more generally, waveguiding structures are presented and studied in Chapter 6, including applications. The covered topics are electroinductive and magnetoinductive wave delay lines, common-mode suppressed differential lines, lattice network-based transmission lines, transmission lines loaded with non-Foster components, and metamaterial-based substrate-integrated waveguides. Grouping these topics in a single chapter does not obey to a thematic reason, but to the fact that most of them have been recently proposed and/or are still under development, or even to the fact that they are very specific to be included in the previous chapters (e.g., the electroinductive and magnetoinductive wave delay lines and the substrate-integrated waveguides).

It is the author's hope that the present manuscript constitutes a reference book in the topic on artificial transmission lines and their RF and microwave applications, and that the book can be of practical use to researchers, students and engineers involved in RF and microwave engineering, especially to those active in planar circuit and antenna design.

Ferran Martín
Santa Maria d'Oló (Barcelona)
September 2014

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FERRAN MARTÍN

1

FUNDAMENTALS OF PLANAR TRANSMISSION LINES

1.1 PLANAR TRANSMISSION LINES, DISTRIBUTED CIRCUITS, AND ARTIFICIAL TRANSMISSION LINES

In radiofrequency (RF) and microwave engineering, transmission lines are two-port networks used to transmit signals, or power, between two distant points (the source and the load) in a guided (in contrast to radiated) way. There are many types of transmission lines. Probably, the most well-known transmission line (at least for nonspecialists in RF and microwave engineering) is the coaxial line (Fig. 1.1), which consists of a pair of concentric conductors separated by a dielectric, and is typically used to feed RF/microwave components and to connect them to characterization and test equipment. Other planar transmission lines are depicted in Figure 1.2. There are many textbooks partially or entirely focused on transmission lines and their RF and microwave applications [1–8]. The author recommends these books to those readers interested in the topic of the present book (artificial transmission lines), which are not familiar with conventional (or ordinary) transmission lines. Nevertheless, the fundamentals of planar transmission lines are considered in this chapter for completeness and for better comprehension of the following chapters. As long as waveguides (and even optical fibers) do also carry electromagnetic (EM) waves and EM energy between two points, they can also be considered transmission lines. However, this book is entirely devoted to planar structures; and for this reason, waveguides are out of the scope of this chapter.

Artificial Transmission Lines for RF and Microwave Applications, First Edition. Ferran Martín.

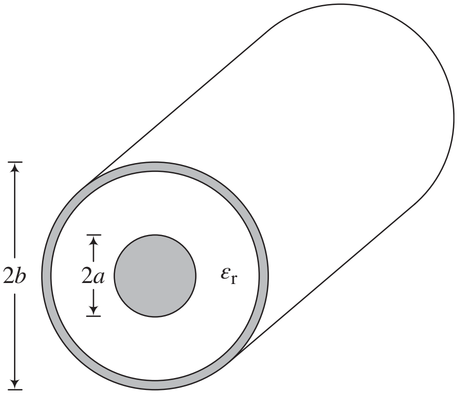

FIGURE 1.1 Perspective three-dimensional view of a coaxial transmission line. The relevant geometry parameters of the line are indicated, and ε_r is the relative permittivity (or dielectric constant) of the dielectric material.

Obviously, there are not transmission lines in natural form.[1] Transmission lines must be fabricated in order to satisfy certain requirements or specifications; in this sense, they are actually artificial (i.e., man-made) structures. However, the term *artificial transmission line* is restricted to a specific type of transmission lines, to distinguish them from the conventional ones.[2] Before discussing the definition and scope of the term *artificial transmission line*, let us now point out the different approaches for the study of planar (conventional) transmission lines. If the physical length of the transmission line is much smaller than the wavelength of the transmitted signals, the voltages and currents in the line are uniform, that is, they do not depend on the position in the line.[3] Under these conditions, the voltages and currents are dictated by the Kirchhoff's current and voltage laws and by the terminal equations of the lumped elements present at the input and output ports of the line, or at any position in the line. This is the so-called lumped element approach, which is generally valid up to about 100 MHz, or even further for planar structures (or circuits) including transmission lines not exceeding the typical sizes of printed circuit boards or PCBs (i.e., various centimeters). At higher frequencies, typically above 1 GHz, the finite propagation velocity of the transmitted signals (of the order of the speed of light) gives rise to variations of voltage and current along the lines, and the lumped circuit approach is no longer valid. At this regime, transmission lines can be analyzed by means of field theory, from Maxwell's equations. However, most planar transmission lines can alternatively be studied and described by means of an intermediate approach between lumped circuits and field equations: the distributed circuit approach. Indeed, for

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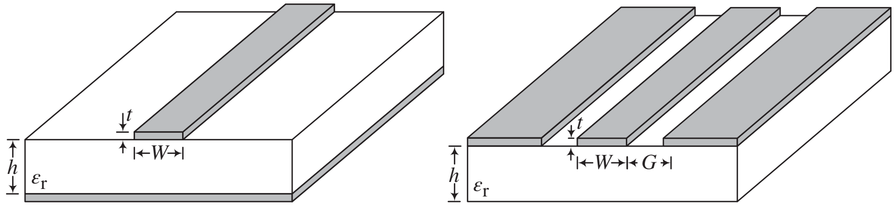
[1] Exceptions to this are, for instance, the axons, which transmit nerve signals in brain neurons.

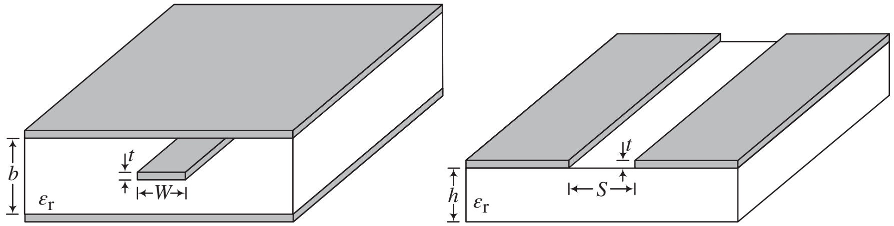
[2] Conventional (or ordinary) transmission lines are uniform along the propagation direction (see Fig. 1.2).

[3] Strictly speaking, this is true if losses are negligible. The effects of losses in transmission lines will be discussed later in detail.



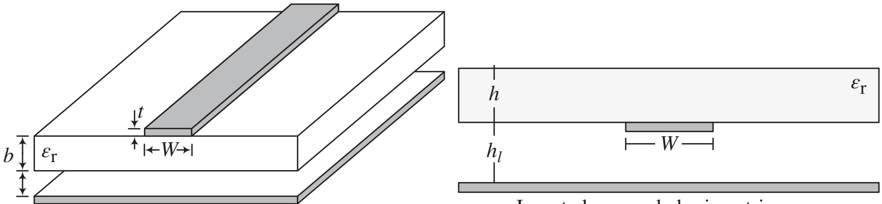
Microstrip

CPW



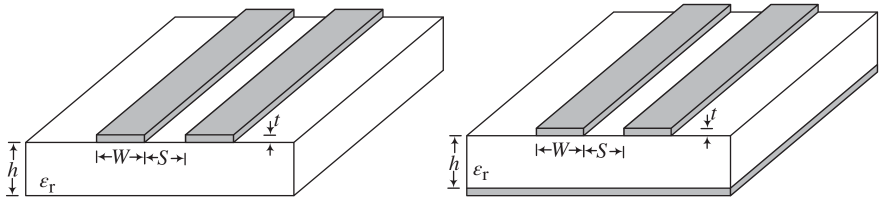
Stripline

Slot line



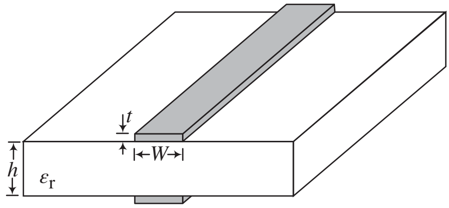
Suspended microstrip

Inverted suspended microstrip



Coplanar strips

Coupled microstrip



Paired strips

FIGURE 1.2 Perspective three-dimensional view of the indicated planar transmission lines, and relevant geometry parameters. These transmission lines are used for the implementation of distributed circuits, where the shape and transverse dimensions (W, S, G) of the line (or set of lines and stubs) are determined in order to obtain the required line functionality.

transverse electric and magnetic (TEM),[4] or quasi-TEM, wave propagation in planar transmission lines (i.e., the fundamental modes), there is a link between the results inferred from the distributed analysis and field theory. Nevertheless, this connection is discussed and treated in Appendix A, since it is not necessary to understand the contents of the present and the next chapters.

The most intriguing aspect of transmission lines operating at microwave frequencies and beyond is the fact that such lines can replace lumped elements, such as capacitances and inductances, in planar circuits, thus avoiding the use of lumped components which increase cost and circuit complexity. Hence, in RF and microwave engineering, transmission lines are not only of interest for signal or power transmission, but they are also key elements for microwave device and component design on the basis of the distributed approach. Thus, the constituent building blocks of distributed circuits are transmission lines and stubs,[5] which are implemented by simply etching metallic patterns on a microwave substrate (such patterns define a set of transmission lines and stubs providing certain functionality).

Distributed circuits are typically low cost since they are implemented in planar technology. However, the design flexibility, performance, or functionality of planar microwave circuits can be enhanced (and/or their dimensions can be reduced) by loading the lines with reactive elements (not necessarily planar),[6] or by breaking the uniformity of the lines in the direction of propagation, or by considering specific arrangements able to provide certain advantages as compared to ordinary lines. In the context of this book, the term *artificial transmission line* is used to designate these lines with superior characteristics, and to distinguish them from their conventional counterparts (ordinary lines). Hence, notice that the term *artificial transmission line* is not only restricted to designate artificial structures mimicking the behavior of ordinary lines (e.g., an LC ladder network or a capacitively loaded line acting as a slow wave transmission line).[7] In this book, the definition of *artificial transmission line* is

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[4] Transmission lines supporting TEM modes require at least two conductors separated by a uniform (homogeneous) dielectric, and the electric and magnetic field lines must be entirely contained in such dielectric. In such modes, the electric and magnetic field components in the direction of propagation are null. A coaxial line is an example of transmission line that supports TEM modes. Microstrip and CPW transmission lines (see Fig. 1.2) are nonhomogeneous open lines, and hence do not support pure TEM modes, but quasi-TEM modes.

[5] Stubs are short- or open-circuit transmission line sections, shunt or series connected to another transmission line, intended to produce a pure reactance at the attachment point, for the frequency of interest.

[6] Notice that this loading refers to line loading along its length, not at the output port (as considered in Section 1.3 in reference to ordinary lines). A line with a load at its output port is usually referred to as terminated line.

[7] Artificial lines that mimic the behavior of ordinary lines are sometimes referred to as synthetic lines. Synthetic lines can be implemented by means of lumped, semilumped, and/or distributed components (combination of transmission lines and stubs). Synthetic lines purely based on the distributed approach (e.g., stub-loaded lines) are out of the scope of this book since they are indeed implemented by combining ordinary lines. Other artificial lines that can be considered to belong to the category of synthetic lines (e.g., capacitively loaded lines) are included in this book; but obviously, it is not possible to include all the realizations of synthetic lines reported in the literature. Artificial lines able to provide further functionalities than ordinary lines (e.g., metamaterial transmission lines with multiband functionality) are not considered to be synthetic transmission lines.

very broad and roughly covers all those lines that cannot be considered ordinary lines. Nevertheless, in many applications of artificial transmission lines, these lines simply replace ordinary lines, and the design approach of microwave circuits based on such artificial lines is similar to the one for ordinary lines, based on the control of the main line parameters. Therefore, in the next subsections, we will focus the attention on the study and analysis of ordinary lines, including the main transmission line parameters, reflections at the source and load (mismatching), losses in transmission lines, a comparative analysis of the most used planar transmission lines, and examples of applications. Most of these contents will be useful in the following chapters. Other useful contents for this chapter and chapters that follow (and in general for RF/microwave engineering), such as the Smith Chart and the scattering S-matrix, are included for completeness in Appendix B and C, respectively.

1.2 DISTRIBUTED CIRCUIT ANALYSIS AND MAIN TRANSMISSION LINE PARAMETERS

Planar transmission lines can be described by cascading the lumped element two-port network unit cell depicted in Figure 1.3, corresponding to an infinitesimal piece of the transmission line of length Δz, and C', L', R', and G' are the line capacitance, line inductance, line resistance, and line conductance per unit length, respectively. R' is related to conductor losses, whereas G' accounts for dielectric losses. From Kirchhoff's circuit laws applied to the network of Figure 1.3, the following equations are obtained:

$$v(z,t) - R'\Delta z \cdot i(z,t) - L'\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0 \tag{1.1a}$$

$$i(z,t) - G'\Delta z \cdot v(z+\Delta z,t) - C'\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0 \tag{1.1b}$$

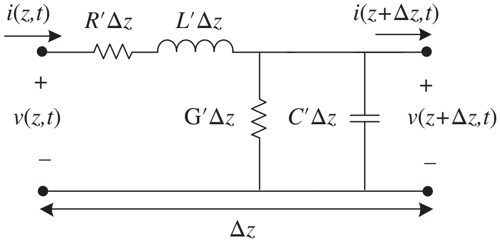

FIGURE 1.3 Lumped element equivalent circuit model (unit cell) of an ordinary transmission line.

By dividing these equations by Δz, and taking the limit as $\Delta z \rightarrow 0$, it follows:

$$\frac{\partial v(z,t)}{\partial z} = -R' i(z,t) - L' \frac{\partial i(z,t)}{\partial t} \tag{1.2a}$$

$$\frac{\partial i(z,t)}{\partial z} = -G' v(z,t) - C' \frac{\partial v(z,t)}{\partial t} \tag{1.2b}$$

Equations 1.2 are known as the telegrapher equations. If we now consider sinusoidal steady-state conditions (i.e., $v(z, t) = V(z)\cdot e^{j\omega t}$ and $i(z, t) = I(z)\cdot e^{j\omega t}$), the time variable in the previous equations can be ignored:

$$\frac{dV(z)}{dz} = -(R' + j\omega L')I(z) \tag{1.3a}$$

$$\frac{dI(z)}{dz} = -(G' + j\omega C')V(z) \tag{1.3b}$$

and the well-known wave equations result

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \tag{1.4a}$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \tag{1.4b}$$

where $\gamma = \alpha + j\beta$ is the complex propagation constant, given by

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} \tag{1.5}$$

and α and β are the attenuation constant and the phase constant, respectively. Notice that if conductor and dielectric losses can be neglected ($R' = G' = 0$), $\alpha = 0$, and the phase constant is proportional to the angular frequency and given by

$$\beta = \omega\sqrt{L'C'} \tag{1.6}$$

The general solutions of the wave equations are traveling waves of the form:

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z} \tag{1.7a}$$

$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z} \tag{1.7b}$$

where the first and second terms correspond to wave propagation in $+z$ and $-z$ directions, respectively. By combining (1.3) and (1.7), it follows that the relation between voltage and current for the traveling waves, also known as the characteristic impedance, is given by

$$Z_\mathrm{o} = \frac{V_\mathrm{o}^+}{I_\mathrm{o}^+} = \frac{-V_\mathrm{o}^-}{I_\mathrm{o}^-} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \tag{1.8}$$

For lossless lines, the voltage and current in the line are in phase, and the characteristic impedance is a real number:

$$Z_\mathrm{o} = \sqrt{\frac{L'}{C'}} \tag{1.9}$$

Although losses may limit the performance of distributed microwave circuits, losses are usually neglected for design purposes, and the propagation constant and characteristic impedance are approximated by (1.6) and (1.9), respectively. According to (1.6), the dispersion relation β–ω is linear. The phase velocity, v_p, and the group velocity, v_g, are thus identical and given by

$$v_\mathrm{p} = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}} \tag{1.10}$$

$$v_\mathrm{g} = \left(\frac{d\beta}{d\omega}\right)^{-1} = \frac{1}{\sqrt{L'C'}} \tag{1.11}$$

and the wavelength in the line is given by:

$$\lambda = \frac{2\pi v_\mathrm{p}}{\omega} = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{L'C'}} \tag{1.12}$$

That is, it is inversely proportional to frequency.[8] Sometimes, the length of a transmission line (for a certain frequency) is given in terms of the wavelength, or expressed as electrical length, $\phi = \beta l$, where l is the physical length of the line, and ϕ is an angle indicating whether distributed effects should be taken into account or not (as a first-order approximation, distributed effects are typically neglected if $\phi < \pi/4$). In many distributed circuits, transmission lines and stubs are $\lambda/4$ or $\lambda/2$ long at the operating frequency, corresponding to electrical lengths of $\phi = \pi/2$ and $\phi = \pi$, respectively.

For plane waves in source-free, linear, isotropic, homogeneous, and lossless dielectrics, the wave impedance, defined as the ratio between the electric and magnetic fields, and the phase velocity, are given by [1, 2] (see Appendix A):

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \tag{1.13}$$

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[8] As will be shown, for artificial transmission lines expressions (1.10–1.12) are not necessarily valid. Indeed, for certain artificial lines, the wavelength either increases or decreases with frequency depending on the frequency regions.

$$v_\mathrm{p} = \frac{1}{\sqrt{\mu\varepsilon}} \tag{1.14}$$

where ε and μ are the dielectric permittivity and magnetic permeability, respectively. These expressions, derived from Maxwell's equations, do also apply to TEM wave propagation in planar transmission lines, and therefore the main line parameters can be expressed in terms of the material parameters.[9] Notice that for nonmagnetic materials $\mu = \mu_\mathrm{o}$, the permeability of vacuum, and hence the phase velocity can be rewritten in the usual form:

$$v_\mathrm{p} = \frac{1}{\sqrt{\mu_\mathrm{o}\varepsilon}} = \frac{1}{\sqrt{\mu_\mathrm{o}\varepsilon_\mathrm{o}\varepsilon_\mathrm{r}}} = \frac{c}{\sqrt{\varepsilon_\mathrm{r}}} \tag{1.15}$$

where c is the speed of light in vacuum, and $\varepsilon = \varepsilon_\mathrm{o}\varepsilon_\mathrm{r}$ (ε_o and ε_r being the permittivity of vacuum and the dielectric constant, respectively). However, for open nonhomogeneous lines, such as microstrip or coplanar waveguide (CPW) transmission lines, where pure TEM wave propagation is not possible, the previous expression does not hold. Nevertheless, the phase velocity in open lines can be expressed as (1.15) by simply replacing the dielectric constant of the substrate material, ε_r, with an effective dielectric constant, ε_re, which takes into account the presence of the electric field lines in both the substrate material and air[10]:

$$v_\mathrm{p} = \frac{c}{\sqrt{\varepsilon_\mathrm{re}}} \tag{1.16}$$

1.3 LOADED (TERMINATED) TRANSMISSION LINES

A uniform (in the direction of propagation) transmission line is characterized by the phase constant β (or by the electrical length βl), and by the characteristic impedance, Z_o. In a semi-infinitely long transmission line with a traveling wave generated by a source, the characteristic impedance expresses the relation between voltage and current at any transverse plane of the line. If losses are neglected, it follows that the power carried by the traveling wave along the line is given by

$$P^+ = \frac{1}{2}\frac{|V_\mathrm{o}^+|^2}{Z_\mathrm{o}} \tag{1.17}$$

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[9] However, the wave impedance should not be confused with the characteristic impedance, Z_o, of transmission lines supporting TEM waves, which relates the voltage and current in the line and depends not only on the material parameters but also on the geometry of the line (see Appendix A).

[10] See at the end of Appendix A for more details.