<table>
<thead>
<tr>
<th>Monograph Number</th>
<th>Title</th>
<th>Editors</th>
</tr>
</thead>
<tbody>
<tr>
<td>161</td>
<td>Circulation in the Gulf of Mexico: Observations and Models</td>
<td>Wilton Sturges and Alexis Lugo-Fernandez (Eds.)</td>
</tr>
<tr>
<td>162</td>
<td>Dynamics of Fluids and Transport Through Fractured Rock</td>
<td>Boris Faybishenko, Paul A. Witherspoon, and John Gale (Eds.)</td>
</tr>
<tr>
<td>163</td>
<td>Remote Sensing of Northern Hydrology: Measuring Environmental Change</td>
<td>Claude R. Duguay and Alain Pietroniro (Eds.)</td>
</tr>
<tr>
<td>164</td>
<td>Archean Geodynamics and Environments</td>
<td>Keith Benn, Jean-Claude Mareschal, and Kent C. Condie (Eds.)</td>
</tr>
<tr>
<td>165</td>
<td>Solar Eruptions and Energetic Particles</td>
<td>Natchimuthukonar Gopalswamy, Richard Mewaldt, and Jarno Torst (Eds.)</td>
</tr>
<tr>
<td></td>
<td>Physical Interactions</td>
<td></td>
</tr>
<tr>
<td>167</td>
<td>Recurrent Magnetic Storms: Corotating Solar Wind Streams</td>
<td>Bruce Tsurutani, Robert McPherron, Walter Gonzalez, Gang Lu, José H. A. Sobral, and Natchimuthukonar Gopalswamy (Eds.)</td>
</tr>
<tr>
<td>168</td>
<td>Earth's Deep Water Cycle</td>
<td>Steven D. Jacobsen and Suzan van der Lee (Eds.)</td>
</tr>
<tr>
<td>169</td>
<td>Magnetospheric ULF Waves: Synthesis and New Directions</td>
<td>Kazue Takahashi, Peter J. Chi, Richard E. Denton, and Robert L. Lysal (Eds.)</td>
</tr>
<tr>
<td>170</td>
<td>Earthquakes: Radiated Energy and the Physics of Faulting</td>
<td>Rachel Abercrombie, Art McGarr, Hiroo Kanamori, and Giulio Di Toro (Eds.)</td>
</tr>
<tr>
<td>171</td>
<td>Subsurface Hydrology: Data Integration for Properties and Processes</td>
<td>David W. Hyndman, Frederick D. Day-Lewis, and Kamini Singh (Eds.)</td>
</tr>
<tr>
<td>172</td>
<td>Volcanism and Subduction: The Kamchatka Region</td>
<td>John Eichelberger, Evgeni Gordeev, Minoru Kasahara, Pavel Izbekov, and Johnathan Lees (Eds.)</td>
</tr>
<tr>
<td>173</td>
<td>Ocean Circulation: Mechanisms and Impacts—Past and Future Changes</td>
<td>Andreas Schmittner, John C. H. Chiang, and Sidney R. Hemming (Eds.)</td>
</tr>
<tr>
<td></td>
<td>of Meridional Overturning</td>
<td></td>
</tr>
<tr>
<td>174</td>
<td>Post-Perovskite: The Last Mantle Phase Transition</td>
<td>Kei Hirose, John Brodholt, Thorne Lay, and David Yuen (Eds.)</td>
</tr>
<tr>
<td>175</td>
<td>A Continental Plate Boundary: Tectonics at South Island, New Zealand</td>
<td>David Okaya, Tim Stem, and Fred Davey (Eds.)</td>
</tr>
<tr>
<td>176</td>
<td>Exploring Venus as a Terrestrial Planet</td>
<td>Larry W. Esposito, Ellen R. Stefan, and Thomas E. Cravens (Eds.)</td>
</tr>
<tr>
<td>177</td>
<td>Ocean Modeling in an Eddyng Regime</td>
<td>Matthew Hecht and Hiroyasu Hasumi (Eds.)</td>
</tr>
<tr>
<td>178</td>
<td>Magma to Microbe: Modeling Hydrothermal Processes at Oceanic</td>
<td>Robert P. Lowell, Jeffrey S. Seewald, Anna Metaxas, and Michael R. Perfit (Eds.)</td>
</tr>
<tr>
<td></td>
<td>Spreading Centers</td>
<td></td>
</tr>
<tr>
<td>179</td>
<td>Active Tectonics and Seismic Potential of Alaska</td>
<td>Jeffrey T. Freymueller, Peter J. Hauegler, Robert L. Wesson, and Göran Ekström (Eds.)</td>
</tr>
<tr>
<td>180</td>
<td>Arctic Sea Ice Decline: Observations, Projections, Mechanisms, and</td>
<td>Eric T. DeWeaver, Cecilia M. Bitz, and L-Bruno Tremblay (Eds.)</td>
</tr>
<tr>
<td></td>
<td>Implications</td>
<td></td>
</tr>
<tr>
<td>181</td>
<td>Midlatitude Ionospheric Dynamics and Disturbances</td>
<td>Paul M. Kintner, Jr., Anthea J. Coster, Tim Fuller-Rowell, Anthony J. Mannucci, Michael Mendillo, and Roderick Heelis (Eds.)</td>
</tr>
<tr>
<td>182</td>
<td>The Stromboli Volcano: An Integrated Study of the 2002–2003 Eruption</td>
<td>Sonia Calvari, Salvatore Inguggiato, Giuseppe Puglisi, Maurizio Ripepe, and Mauro Rosi (Eds.)</td>
</tr>
<tr>
<td>183</td>
<td>Carbon Sequestration and Its Role in the Global Carbon Cycle</td>
<td>Brian J. McPherson and Eric T. Sundquist (Eds.)</td>
</tr>
<tr>
<td>184</td>
<td>Carbon Cycling in Northern Peatlands</td>
<td>Andrew J. Baird, Lisa R. Belyea, Xavier Comas, A. S. Reeve, and Lee D. Slater (Eds.)</td>
</tr>
<tr>
<td>185</td>
<td>Indian Ocean Biogeochemical Processes and Ecological Variability</td>
<td>Jerry D. Wiggert, Raleigh R. Hood, S. Wajih A. Naqvi, Kenneth H. Brink, and Sharon L. Smith (Eds.)</td>
</tr>
<tr>
<td>186</td>
<td>Amazonia and Global Change</td>
<td>Michael Keller, Mercedes Bustamante, John Cash, and Pedro Silva Dias (Eds.)</td>
</tr>
<tr>
<td>187</td>
<td>Surface Ocean–Lower Atmosphere Processes</td>
<td>Corinne Le Quèrè and Eric S. Saltzman (Eds.)</td>
</tr>
<tr>
<td>188</td>
<td>Diversity of Hydrothermal Systems on Slow Spreading Ocean Ridges</td>
<td>Peter A. Rona, Colin W. Devey, Jérôme Dyment, and Bramley J. Murton (Eds.)</td>
</tr>
<tr>
<td>189</td>
<td>Climate Dynamics: Why Does Climate Vary?</td>
<td>De-Zheng Sun and Frank Bryan (Eds.)</td>
</tr>
<tr>
<td>191</td>
<td>Rainfall: State of the Science</td>
<td>Firat Y. Testik and Mekonnen Gebremichael (Eds.)</td>
</tr>
<tr>
<td>192</td>
<td>Antarctic Subglacial Aquatic Environments</td>
<td>Martin J. Siegert, Mahlon C. Kennicutt II, and Robert A. Bindschadler (Eds.)</td>
</tr>
<tr>
<td>193</td>
<td>Abrupt Climate Change: Mechanisms, Patterns, and Impacts</td>
<td>Harunur Rashid, Leonid Polyak, and Ellen Mosley-Thompson (Eds.)</td>
</tr>
<tr>
<td>194</td>
<td>Stream Restoration in Dynamic Fluvial Systems: Scientific</td>
<td>Andrew Simon, Sean J. Bennett, and Janine M. Castro (Eds.)</td>
</tr>
<tr>
<td></td>
<td>Approaches, Analyses, and Tools</td>
<td></td>
</tr>
<tr>
<td>195</td>
<td>Monitoring and Modeling the Deepwater Horizon Oil Spill</td>
<td>Yonggang Liu, Amy MacFadyen, Zhen-Gang Ji, and Robert H. Weisberg (Eds.)</td>
</tr>
</tbody>
</table>
# CONTENTS

**Preface**  
*A. Surjalal Sharma, Armin Bund, Vijay P. Dimri, and Daniel N. Baker* ................................................................. vii

**Complexity and Extreme Events in Geosciences: An Overview**  

## Section I: Solid Earth

**Earthquakes: Complexity and Extreme Events**  
*M. R. Yoder, D. L. Turcotte, and J. B. Rundle* ................................................................. 17

**Patterns of Seismicity Found in the Generalized Vicinity of a Strong Earthquake: Agreement With Common Scenarios of Instability Development**  
*M. V. Rodkin* ......................................................................................................................... 27

**Characterizing Large Events and Scaling in Earthquake Models With Inhomogeneous Damage**  
*Rachele Dominguez, Kristy Tiampo, C. A. Serino, and W. Klein* ................................................................. 41

**Fractal Dimension and \( b \) Value Mapping Before and After the 2004 Megathrust Earthquake in the Andaman-Sumatra Subduction Zone**  
*Sohini Roy, Uma Ghosh, Sugata Hazra, and J. R. Kayal* ......................................................................................................................... 55

**Stress Pulse Migration by Viscoelastic Process for Long-Distance Delayed Triggering of Shocks in Gujarat, India, After the 2001 \( M_w \) 7.7 Bhuj Earthquake**  
*B. K. Rastogi, Pallabee Choudhury, Rakesh Dumka, K. M. Sreejith, and T. J. Majumdar* ................................................................. 63

**Extreme Seismic Events in Models of Lithospheric Block-and-Fault Dynamics**  
*A. T. Ismail-Zadeh, J.-L. Le Mouël, and A. A. Soloviev* ......................................................................................................................... 75

## Section II: Oceans

**Investigation of Past and Future Polar Low Frequency in the North Atlantic**  
*Matthias Zahn and Hans von Storch* ......................................................................................................................... 99

**Variability of North Atlantic Hurricanes: Seasonal Versus Individual-Event Features**  
*Álvaro Corral and Antonio Turiel* ......................................................................................................................... 111

**Large-Scale Patterns in Hurricane-Driven Shoreline Change**  
*Eli D. Lazarus, Andrew D. Ashton, and A. Brad Murray* ......................................................................................................................... 127

## Section III: Atmosphere

**Precipitation and River Flow: Long-Term Memory and Predictability of Extreme Events**  
*Armin Bund, Mikhail I. Bogachev, and Sabine Lennartz* ......................................................................................................................... 139

**Extreme Events and Trends in the Indian Summer Monsoon**  
*V. Krishnamurthy* ......................................................................................................................... 153
Empirical Orthogonal Function Spectra of Extreme Temperature Variability Decoded From Tree Rings of the Western Himalayas  

On the Estimation of Natural and Anthropogenic Trends in Climate Records  
S. Lennartz and A. Bunde ..........................................................................................177

Climate Subsystems: Pacemakers of Decadal Climate Variability  
Anastasios A. Tsonis ..........................................................................................191

Dynamical System Exploration of the Hurst Phenomenon in Simple Climate Models  
O. J. Mesa, V. K. Gupta, and P. E. O’Connell..............................................................209

Low-Frequency Weather and the Emergence of the Climate  
S. Lovejoy and D. Schertzer..................................................................................231

Section IV: Geospace  
Extreme Space Weather: Forecasting Behavior of a Nonlinear Dynamical System  
D. N. Baker........................................................................................................255

Supermagnetic Storms: Hazard to Society  
G. S. Lakhina, S. Alex, B. T. Tsurutani, and W. D. Gonzalez ..............................................267

Development of Intermediate-Scale Structure in the Nighttime Equatorial Ionosphere  
A. Bhattacharyya .....................................................................................................279

Complex Analysis of Polar Auroras for 1996  
James Wanliss and Joshua Peterson ........................................................................293

On Self-Similar and Multifractal Models for the Scaling of Extreme Bursty Fluctuations in Space Plasmas  
N. W. Watkins, B. Hnat, and S. C. Chapman .............................................................299

Section V: General  
Extreme Value and Record Statistics in Heavy-Tailed Processes With Long-Range Memory  
Aicko Y. Schumann, Nicholas R. Moloney, and Jörn Davidsen ........................................315

Extreme Event Recurrence Time Distributions and Long Memory  
M. S. Santhanam .....................................................................................................335

Dealing With Complexity and Extreme Events Using a Bottom-Up, Resource-Based Vulnerability Perspective  
Roger A. Pielke Sr., Rob Wilby, Dev Niyogi, Faisal Hossain, Koji Dairaku, Jimmy Adegoke, George Kallos, Timothy Seastedt, and Katharine Suding ......................................................345

AGU Category Index .............................................................................................361

Index.......................................................................................................................363
Understanding extreme natural events and their societal consequences is among the most pressing scientific challenges of our time. Like most of the major scientific challenges in the Earth and space sciences, there is increasing recognition that an integrated approach involving multiple disciplines will be needed to advance the science underlying extreme events that lead to natural hazards. Complexity science, with its multidisciplinary nature and its focus on dynamical instability and sudden changes, provides a natural framework for this effort. The main goal of this volume is to bring together the research in extreme events, complexity science, and natural hazards and to explore new ways to advance our knowledge and develop strategies.

The need for an integrated approach to the understanding of extreme events and the resulting natural hazards was highlighted by the devastating consequences of the March 2011 Japan earthquake of magnitude 9.0 and the resulting tsunami. The severe damage done by the tsunami was further compounded by the exposure of the nuclear power plants to potential accidents with unprecedented consequences. The possibility of such a potential confluence of events also brings into focus the effects of other extreme events such as severe space weather. The technological infrastructure we depend on, such as telecommunications and electric power networks, are susceptible to disruption in severe space weather, and the scenarios of such conditions during other natural hazards need serious study.

The low-probability, high-impact nature of extreme events makes their understanding a continuing imperative, and complexity science with its systems-based approach provides an important complement to the traditional first-principles studies. The nature of the distribution function of the events is essential to the characterization of extreme events, and recent studies have shown many interesting results. This could lead to better quantification of their likelihood. The distributed nature of the components and the strong interaction among them is another feature common to systems exhibiting extreme events. This is evident in many branches of geosciences, e.g., atmospheric, hydrologic, oceanic, and space sciences, and has led to the characterization of the long-range nature of the correlations among the components. The combination of the frequency of extreme events and the strong correlation among them is an important feature in assessing their potential hazard to society. Such advances make a strong case for pursuing the approaches based on the developments in complexity science.

Responding to extreme events and natural hazards depends strongly on timely warning with specified likelihoods. The development of the capability to provide such warnings is a major objective of the research efforts. The preparedness of our society for natural disasters depends on the accomplishments of such research, and we hope that the insights gained from this volume will stimulate new initiatives.

This volume derives from the Chapman Conference on Complexity and Extreme Events in Geosciences, held in Hyderabad, India, in February 2010. This conference was truly inter-disciplinary, as is evident from the coverage of the many disciplines in this volume, and provided a forum for exploring new research ideas and collaborations. The National Geophysical Research Institute, Hyderabad, hosted the conference, and the National Science Foundation supported it with a travel grant.

The editors would like to thank the colleagues who participated in the evaluation of the papers submitted to this volume. We owe the high quality of the articles in this volume to the diligence, expertise, and rigorous standards of these reviewers.
Complexity and Extreme Events in Geosciences: An Overview

A. Surjalal Sharma,1 Daniel N. Baker,2 Archana Bhattacharyya,3 Armin Bunde,4 Vijay P. Dimri,5 Harsh K. Gupta,5 Vijay K. Gupta,6 Shaun Lovejoy,7 Ian G. Main,8 Daniel Schertzer,9 Hans von Storch,10,11 and Nicholas W. Watkins12,13,14

Extreme events are an emergent property of many complex, nonlinear systems in which various interdependent components and their interaction lead to a competition between organized (interaction dominated) and irregular (fluctuation dominated) behavior. Recent advances in nonlinear dynamics and complexity science provide a new approach to the understanding and modeling of extreme events and natural hazards. The main connection of extreme events to nonlinear dynamics arises from the recognition that they are not isolable phenomena but must be understood in terms of interactions among different components, within and outside the specific system. A wide range of techniques and approaches of complexity science are directly relevant to geosciences, e.g., nonlinear modeling and prediction, state space reconstruction, statistical self-similarity and its dynamical origins, stochastic cascade models, fractals and multifractals, network theory, self-organized criticality, etc. The scaling of processes in geosciences has been one of the most active areas of studies and has the potential to provide better tools for risk assessment and analysis. Many studies of extreme events in geosciences are also contributing to the basic understanding of their inherent properties, e.g., maximum entropy production and criticality, space-time cascades, and fractional Lévy processes. The need for better data for extreme events is evident in the necessity for detailed statistical analysis, e.g., in marine storms, nonlinear correlations, etc. The Chapman Conference on Complexity and Extreme Events held (2010) in Hyderabad,
India, was focused on the understanding of natural hazards mainly from the perspective of complexity science. The emerging theme from the conference was the recognition of complexity science as the interdisciplinary framework for the understanding of extreme events and natural hazards.

1. INTRODUCTION

Complexity refers to the behavior of systems with many interdependent components that lead to organized as well as irregular (irreducibly stochastic) features. In such systems the knowledge of the parts does not necessarily lead to the predictable behavior of the entire system. The coupling among the components is essentially nonlinear, and this leads to a rich variety of dynamical behavior, geometrical patterns, and statistical distributions that are seen in virtually all disciplines. In geosciences the studies of nonlinear dynamical process and complexity have been an active field of research for many decades [Lovejoy et al., 2009]. The basic ideas underlying complexity science have now matured to the point that they are generating new approaches to addressing problems in many different disciplines, including geosciences. One such area is the nature of extreme events, in particular the natural hazards, whose connection to complexity arises from the recognition that they are not isolable phenomena but must be understood in terms of interactions among different components, inside and outside the specific system.

Extreme events are of both natural and anthropogenic origin and are of widespread concern mainly because of their damaging consequences [Lubchenco and Karl, 2012]. Although there is no single definition, at least in the physical sciences, of extreme events [Bunde et al., 2002; Jentsch et al., 2006; Moffat and Shuckburgh, 2011], there is a significant, widely accepted body of work [Coles, 2001], since that of Fisher and Tippett [1928], which informs much statistical practice and goes by the name of extreme value theory. In science and applications, however, the practical interpretation of the degree of “extremeness” often mixes the statistical attributes of infrequent occurrence and low probability, the physics- or prediction-related property of being unexpected, and the societal or economic aspect of strong impact, etc. In general, it is not so clear that extreme events can be characterized or “marked” by only one or even a few measures. However, it is, nonetheless, clear that extreme events are typically rare, and in the distribution of events of all magnitudes they are identified as those outside the bulk; that is, they occur in the tail of the distribution. A main objective in the analysis of extreme events thus relates directly to the understanding of the distribution functions of the events, in particular the values in the tail. Even when a single probability distribution suffices to capture a system’s amplitude variation, however, we may still distinguish at least two scenarios. One is the case where the distribution is relatively “short tailed” in its fluctuations, the model for this being the Gaussian. In this framework, extreme events really are rare and, in the Gaussian example, will only very rarely exceed three standard deviations from the mean. Conversely, if the underlying distribution is actually heavy tailed, with examples being the power law, lognormal, and stretched exponential families, the mean will be a much less adequate characterization of the expected behavior, and we will see effects like the 80–20 rule of thumb from actuarial science, where 80% of losses come from only 20% of claims [e.g., Embrechts et al., 1997].

Another feature of extreme events is that they occur suddenly, often without clear warning, and on large spatial scales compared to the system size. Such long-range order, i.e., the value of a physical variable at an arbitrary point is correlated with its value at a point located far away, is also a property of thermodynamic phase transitions. In the case of second-order phase transitions the correlation length reaches the system size, giving rise to arbitrarily large extreme events at the critical point because of the competition between random fluctuations and cooperative dynamical interactions. This leads to the recognition that long-range correlations are important indicators of the emergence of extreme events. In view of these features the dynamical and statistical approaches of complexity science provide a natural framework for the study of extreme events [Sharma et al., 2010]. An aspect of correlation of particular importance to extremes, and not always recognized, is that while a low probability of occurrence must indeed imply that such an event will be, on average, rare, the correlations in time between extreme events can mean that several such “black swan” tail events may still follow each other in close succession (dubbed “bunched black swans” by Watkins et al. [2011]). The recurrence time distribution is not simply prescribed by the frequency distribution, a point which becomes progressively more significant as temporal correlations become longer ranged.

A widely known property of many natural phenomena is the dynamical instability of nonlinear systems, which leads to limits of predictability because of sensitivity to the initial conditions, even for systems with few degrees of freedom. This behavior in deterministic systems such as the Lorenz equations [Lorenz, 1963] is a key dynamical origin for irregularity in nature and is now known as chaos. The Lorenz
The interplay of dynamics, geometry, and information and their effects on the scaling properties of the population and the predictability of individual events is an important feature of complexity science in general and in the studies of extreme events in particular. This chapter is an overview of recent advances in the understanding of extreme events in the natural world, in particular those made through the use of nonlinear dynamics, statistical physics, and related approaches. The topics covered include the recent studies of earthquakes, river flows, and climate variations, which have demonstrated long-range dependences (clustering) in the appearance of extreme events. These have important practical implications for climate change, natural hazard forecasting, and risk management. Frequency-magnitude statistics of many natural hazards follow power laws, exhibiting features of complex behavior, but robust power law statistics for a population often occur in hand in hand with restricted predictability of individual events. This poses significant challenges in developing and communicating operationally useful forecasts of such events. The interdisciplinary nature of the research leads naturally to strong connections with a wide range of applications. Here we focus on the relationship between the science underlying the behavior of a complex system and the emergence of intermittent, perhaps clustered, extreme events, and identify approaches that may lead to significant advances in understanding in future.

2. NONLINEAR DYNAMICS, COMPLEXITY, AND EXTREME EVENTS

The complex behavior in deterministic systems with a few degrees of freedom, such as the Lorenz attractor, led to models of low-dimensional chaos for dynamical systems. In mathematical terms such systems are represented by a small number of first-order ordinary differential equations. These were complemented by approaches that include more complex spatio-temporal dynamics, namely, in the form of partial differential equations such as Ginzburg-Landau and Kuramoto-Sivashinsky equations. While these models were successful in representing many laboratory systems, they faced many difficulties in describing large-scale open natural systems. The next advance came from the recognition of the nonlinear coupling and the dissipative nature of many dynamical systems, responsible for the contraction of phase space. These two properties underlie the elucidation of strange attractors as the hallmark of chaotic dynamics, leading to many new developments in the dynamical systems theory. One application of dynamical systems with a few degrees of freedom to understanding of the Hurst effect is given by Mesa et al. [this volume]. Another set of applications require the assumption of an effectively low-dimensional nature of large-scale systems and of the applicability.
of the “embedding theorem,” thus enabling the reconstruction of dynamical models from time series data. The converse assumption, of high dimensionality, has been termed “stochastic chaos” and is the assumption used in multifractal and other stochastic approaches [see, e.g., Lovejoy and Schertzer, this volume]. The use of low dimensionality as a paradigm has stimulated many new approaches to the study of seemingly complicated behavior in many natural systems, including in geosciences. In the studies of the dynamics of the geospace environment this approach provided the first predictive models of geomagnetic activity, enabled by the extensive data from ground-based and spaceborne instruments reviewed by Sharma [1995].

Various studies reveal that many of the Earth’s processes show fractal statistics, where the relevant distributions follow power laws with noninteger (“fractal”) exponents. Fractal structures that show self-similarity and are governed by a noninteger dimension are ubiquitous in geosciences. For example, they occur in the frequency size statistics of events and in the behavior of various physical properties such as density, susceptibility, electrical resistivity, and thermal conductivity. Nonlinearity and nonuniqueness are nearly always encountered in the inverse problem in geophysical exploration, including the components of data acquisition, processing, and not least interpretation (often by expert elicitation). Fractal dimension plays a vital role in the design of survey networks with proper sampling and gridding [Dimri, 1998; Srivastava et al., 2007]. The interpretation of geophysical data such as gravity and magnetic data has shown good results assuming a fractal distribution of sources. Theoretical relation between fractal sources and their geophysical response has been used to derive source parameters [Maus and Dimri, 1994]. The fractal theory has led to the development of a wide variety of physical models of seismogenesis including nonlinear dynamics that can be used to characterize the seismicity pattern of a region [Kagan and Knopoff, 1980; Summonu and Dimri, 2000]. Similarly, the “coda” or tail of a seismogram can be modeled well with a fractal distribution of scatterers that emerge in a growing fracture population [Vlastos et al., 2007].

The late Per Bak and colleagues sought to explain the widespread appearance of spatial fractals and temporal “1/f” noise by proposing a new class of avalanche-like models [Bak et al., 1987] that exhibited fractally structured growing instabilities (“avalanches”), obeying heavy-tailed (algebraic) and thus extreme probability distributions. In fact, Bak most tersely expressed his intent a little later (with K. Chen [Bak and Chen, 1989, p. 5]) in the memorably short abstract: “Fractals in nature originate from self-organized critical dynamical processes.” These self-organized critical (SOC) processes not only have extremes but also entire structures determined by them. There are now many models of self-organized criticality and significant developments toward an analytical theory. The applications of the SOC paradigm range from geosciences (including earthquakes, forest fires, solar activity, and rainfall) to the financial markets, while the renewed emphasis on the heavy tails generated by SOC processes has had a direct influence on modern network science and its applications. Although generally the SOC approach does not provide dynamical predictions, it describes the statistical features and thus yields the probabilities of events. For example, SOC provides a single unifying theory for much of what had previously been separate empirical observations of earthquake phenomena [Main, 1996]. While this provides a useful basis for seismic hazard calculation based on the population [Main, 1995], the proximity to criticality and the inherent stochastic element inevitably degrades any predictive power for individual events (see http://www.nature.com/nature/debates/earthquake/equake_frame set.html) [Kagan, 2006]. Similarly, in those (common) cases where the dynamics act over large ranges of scale, multifractal processes provide an interesting paradigm for extremes. In these cascade type processes the variability at any given resolution (i.e., the process averaged at the given scale) is precisely the consequence of the wide scale range of the dynamics. This includes not only the range from the largest scale to the resolution scale, but somewhat surprisingly, it also depends on the smaller scales whose variability is not completely “smoothed out” and which leads to extreme variability in the form of “fat-tailed” (power law) probability distributions. Since they involve both fractal structures and power law distributions, multifractal processes thus provide a “nonclassical” route to SOC.

More generally, the advances in the studies of complexity in many areas of geosciences have led to new predictive and diagnostic approaches that exploit how patterns, processes, and probabilities are mutually coupled. For example, nonlinear dynamics and complexity can be exploited to reduce forecast error, to understand atmospheric flow transitions, to test climate models by analyzing their atmospheric variability [Govindan et al., 2002; Vyushin et al., 2004; Rybski et al., 2008], and to explain atmospheric and oceanic teleconnections [Tsonis, this volume]. The nonlinear aspects of the climate can regulate El Niño’s background state, while the related approaches to data analysis, which are becoming increasingly popular, can be used to check the consistency of general circulation models in the representation of fundamental statistical properties of the atmosphere. In other areas of geosciences the role of nonlinear dynamics and complexity in complex landscape patterns of Earth’s surface and hydrologic processes are well recognized, as presented in this volume.
Recent advances in the studies of extreme events from the viewpoint of nonlinear dynamics and complexity have demonstrated the existence and role of long-term memory in many complex systems [Bunde et al., 2005; Sharma and Veeramani, 2011; Mesa et al., this volume]. For example, when the memory can be described by a linear autocorrelation function that decays algebraically with an exponent \( \gamma \), then the probability density function of the return intervals between events above some threshold no longer show the exponential decay typical for uncorrelated data but a stretched exponential decay characterized by the same exponent \( \gamma \) as the autocorrelation function [Bunde et al., 2005]. Also, the return intervals themselves are long-term correlated, again characterized by the same exponent. This approach provides a new way to understand the clustering of extreme events. When the linear correlations vanish, and long-term memory exists only in the form of nonlinear correlations, such as the volatility bunching seen in finance [e.g., Mantegna and Stanley, 2000], the effect becomes even stronger, and both the probability distribution functions of the return intervals and their autocorrelation function decay as a power law [Bogachev et al., 2007].

When considering complexity in the atmosphere and ocean, and to some extent space plasmas (especially when modeled using magnetohydrodynamics), we must recall the strong historical and theoretical links with the field of fully developed turbulence, in particular the classical turbulence laws associated with the pioneers L. F. Richardson, A. N. Kolmogorov, A. Obukhov, S. Corrsin, and R. Bolgiano. These classical laws were proposed as emergent laws in the field of sufficiently strong hydrodynamic turbulence where “developed turbulence as a new macroscopic state of matter” [Manneville, 2010] appears at high Reynolds number (Re). This new state has been considered as a form of matter with properties that cannot simply be reduced to, or simply deduced from, the governing Navier-Stokes equations. Although fluid (and plasma) geosystems certainly differ from incompressible hydrodynamics in several important respects, we may, nevertheless, expect higher-level laws to emerge and that they will share at least some of the features of fully developed turbulence.

In the atmosphere, key obstacles to applying classical turbulence are the strong anisotropy (especially stratification) and intermittency. However, over the years, new types of models and new symmetry principles have been developed in order to generalize the classical laws so as to handle these issues. For weather the key generalizations are from isotropic to anisotropic notions of scale and from smooth, quasi-Gaussian variability to strong, cascade-generated multifractal intermittency. Lovejoy and Schertzer [this volume, and references therein] argue that this leads to a model of atmospheric and oceanic dynamics as a system of coupled anisotropic cascade processes. They go on to indicate how the same approach can be extended to much longer (climate) time scales.

Extreme events in the Earth’s near-space environment are driven by the solar wind, which brings the energetic plasma and fields from the solar eruptive events such as coronal mass ejections (CME) to geospace. Forecasting of space weather is an active field of research, and recently many new techniques and approaches have been developed. The nonlinear dynamical approach to space weather forecasting played a pioneering role by showing the predictability of space weather. Research in this area is now quite advanced and has provided techniques for dynamical and statistical forecasting [Ukhorskiy et al., 2004]. Many extreme space weather events in the recent past have caused serious damage to technological systems such as satellites, power transmission, etc. [National Research Council (NRC), 2008]. Although these events may not seem devastating by themselves, a confluence of natural hazards in the different regions of the Earth’s environment can make our society and its technological systems highly vulnerable because of the interconnectedness [Baker and Allen, 2000]. In this aspect the nonlinear dynamical framework for the study of the clustering of events, described above, becomes directly relevant to the extended Earth and space system. The studies of these extreme events in nature are summarized in the following sections.

3. EARTH SCIENCES: EARTHQUAKES AND LANDSCAPE DYNAMICS

One aspect of near-critical dynamics anticipated by Bak et al. [1987] and others is that earthquakes can also occur or be induced in the interior of the plates not just at the boundaries. For example, the Bhuj earthquake (\( M_w 7.7 \)) of 26 January 2001 is recognized to be the deadliest among the recorded intraplate earthquakes [Gupta et al., 2001], and many aspects, including its recurrence time, have been studied extensively. This earthquake has attracted further interest as an analog for the New Madrid earthquakes (\( M_w 7.5 – 8.0 \)) that struck the central United States almost two centuries ago [Ellis et al., 2001]. From the standpoint of extreme event studies the recurrence time of such earthquakes is of key interest. In this case the recurrence time is estimated to be \( \sim 1500 \) years. The bounds on recurrence times are naturally limited by the available data, and this estimate suffers from the lack of sufficient data. However, this also highlights the need for integrating the different types of available data with the models to develop better estimates for key features such as the recurrence times, aftershock distributions, etc. The studies of more than 500 aftershocks (\( M > 2.0 \)) of the Bhuj earthquake using the data of 3-D velocity, gravity, magnetic,
GPS, and satellite observations have shown its features such as rupture propagation along two trends in India [Kayal and Mukhopadhyay, 2006]. The studies of the postseismic deformation of this earthquake [Rastogi et al., this volume], focused on the changes in the seismicity of the Gujarat region in space and time, have identified the Kachchh and the Saurashtra regions as more vulnerable, consistent with the observed increase in seismicity. They modeled the postseismic stress changes due to the earthquake and interpreted the deformation in these regions due to the migration of the stress pulse via viscoelastic process in lower crust and upper mantle resulting from the 20 MPa stress drop of the 2001 Bhuj earthquake.

The Koyana Dam located in western India is the most outstanding example of reservoir-triggered seismicity (RTS) where triggered earthquakes have been occurring since the impoundment in 1962 in a restricted area of 20 × 30 km². These include the largest RTS earthquake of M 6.3 on 10 December 1967, 22 earthquakes of M > 5, and about 200 M ~ 4 earthquakes [Narain and Gupta, 1968; Gupta, 2002]. The maximum credible earthquake estimated for the Koyana region is M 6.8, and it is reasonable to infer that the Koyana region was stressed close to critical before the relatively small perturbation to the stress field on impoundment of the reservoir. The impoundment could serve only as a trigger, leading to many small events as well as the extreme, as well as fluctuations in the subsurface pore fluid pressure. So far about 60% of the energy of an M 6.8 earthquake has been released, and the rest of the energy could be released over the next 3 to 4 decades [Gupta et al., 2002]. The occurrence of M > 5 earthquakes is governed by factors like the rate of loading, highest water levels reached, duration of retention of the high water levels, and whether the previous water maxima has been exceeded or not (Kaiser effect). In a very exciting development a borehole to probe the physical state and mechanical behavior of the Koyana fault parameters at the hypocentral depths of 7 km is under advanced stages of planning [Gupta et al., 2011]. Downhole measurements complemented by observations on cores and cuttings, analyses of fluids and gas samples, and geological and geophysical characterization studies would help answer questions related to the genesis of stable continental region earthquakes in general with a special emphasis on RTS.

If Earth’s brittle lithosphere is in a state of self-organized criticality, as implied by these and other observations, that, in turn, begs the question of how the system evolved to such a marginally stable, far-from-equilibrium, stationary state in the first place. Using the information theoretic approach outlined above, Dewar [2003] showed that the most likely (maximum entropy) state for such systems was one where the entropy production rate was also a maximum. This is not a general conclusion (maximum entropy production is not a principle as such, though it is often referred to as one) and depends on key assumptions being met for specific systems, for example, that there exists a single, stable, attractor steady state solution. Main and Naylor [2008, 2010] tested the hypothesis of maximum entropy production (MEP) on synthetic earthquake populations generated by a dissipative cellular automaton model. By tuning the dissipation (entropy production), MEP occurs when the global elastic strain is near critical, with small relative fluctuations in macscopic strain energy expressed by a low seismic efficiency and broad-bandwidth power law scaling of frequency and rupture area. These phenomena, all as observed in natural earthquake populations, are hallmarks of the broad conceptual definition of SOC, though the MEP state is near but strictly subcritical. In the MEP state the strain field retains some memory of past events, expressed as coherent “domains,” implying a degree of predictability, albeit strongly limited in practice by the proximity to criticality and our inability to map the natural stress field at an equivalent resolution to the numerical model. The resulting theoretical and practical limitations have led to the debate on earthquake predictability moving on to what can be done (or not) with the data in real time, in a low-probability, high-impact forecasting environment, and what the consequences might be for operational forecasting, including quantifying the consistency, quality, and utility of any forecast and issues of communication between scientists, the relevant authorities, and the general public in an uncertain, complex, and nonlinear world [Jordan et al., 2011].

### 4. ATMOSPHERIC AND OCEAN SCIENCES

Obviously, it is difficult to accurately define what an extreme event in atmospheric, oceanic, hydrologic, cryospheric, or others disciplinary contexts of the Earth system are. When “climate” is defined as the statistics of atmospheric, oceanic, cryospheric, and hydrologic states and “climate change” is defined as the change of these statistics, this helps, but when enlarging the time scales, other components add to this system [cf. van Andel, 1994]. However, the difficulty of defining extreme events is largely a consequence of the subjective nature of this usual yet vague definition of climate. When the weather and the climate are objectively defined in terms of their type of scaling variability [e.g., Lovejoy and Schertzer, this volume], then precise definitions of extremes are indeed possible. Lovejoy and Schertzer [this volume] argue that an objective scaling approach is needed to clarify the key distinction, occulted in the usual approaches, between low-frequency weather and the climate and that this is a prerequisite for objective definitions of climate states and climate change.
For the time being, we refer to extremes in the present context mostly to short-term events, which extend over a few hours, maybe a few days, but hardly more than a few months. Some of them are caused by mechanisms, which are considered mostly external to the climate system, namely, earthquakes or landslides, which may accompany tsunamis and other catastrophic events. Others are due to the internal dynamics of mainly the oceanic and atmospheric systems, such as extreme rainfall events associated with river flooding, but not all flooding is associated with extreme rainfall because of the role of river networks in aggregating flows as explained in section 5. Marine storms, such as tropical or midlatitude cyclones cause havoc not only because they are associated with rainfall but because of related storm surges and coastal inundations [von Storch and Woth, 2008]. Such events go along with massive societal losses, in particular with loss of life sometimes of the order of 100,000 lives and more, the last time in 2006 when tropical storm Nargis struck the coast of Myanmar [Fritz et al., 2009]. Other important examples are the Sumatra-Andaman $M_w$ 9.2 earthquake of 26 December 2004 and the resultant tsunami that claimed over 250,000 human lives in south and southeast Asian countries and caused immense financial losses [Dimri and Srivastava, 2007; Gupta, 2008; Swaroopa Rani et al., 2011] and the Tohoku, Japan, $M_w$ 9.0 earthquake of 11 March 2011 and the resultant tsunami that caused nuclear accidents [Lay and Kanamori, 2011]. The Japan earthquake has given rise to a global debate on the anticipated maximum size of an earthquake in a given region and the safety of nuclear power plants in the coastal region. As a matter of fact, in the first 11 years of the 21st century the number of human lives lost because of earthquakes has exceeded the total number of human lives lost in the entire 20th century because of earthquakes.

To what extent such extreme events really go along with catastrophes with societal losses depends very much on the vulnerability of the society and of the degree of adaptation. Thus, a tropical cyclone may cause much more damage when hitting the coast of the United States than when the same storm hits the coast of Cuba.

Since climate is presently changing, and will likely continue to do so as a response to ever increasing greenhouse gas concentrations in the atmosphere (also the characteristics of some extreme events are changing and will change in future), the state of knowledge has been assessed recently in the 2011 report of the Intergovernmental Panel on Climate Change (IPCC) [2011]. For the present change the IPCC [2011, p. 7] asserts

There is evidence that some extremes have changed as a result of anthropogenic influences, including increases in atmospheric concentrations of greenhouse gases. It is likely that anthropogenic influences have led to warming of extreme daily minimum and maximum temperatures on the global scale. There is medium confidence that anthropogenic influences have contributed to intensification of extreme precipitation on the global scale. It is likely that there has been an anthropogenic influence on increasing extreme coastal high water due to increase in mean sea level. The uncertainties in the historical tropical cyclone records, the incomplete understanding of the physical mechanisms linking tropical cyclone metrics to climate change and the degree of tropical cyclone variability provide only low confidence for the attribution of any detectable changes in tropical cyclone activity to anthropogenic influences.

For the future, the IPCC [2011, pp. 10, 11, 12] finds the following changes “likely”: “. . . frequency of heavy precipitation or the proportion of total rainfall from heavy falls will increase in the 21st century over many areas of the globe”; “Average tropical cyclone maximum wind speed is . . . to increase, although increases may not occur in all ocean basins. . . the global frequency of tropical cyclones will either decrease or remain essentially unchanged”; and “It is very likely that mean sea level rise will contribute to upward trends in extreme coastal high water levels in the future.”

Although the usual approach to assessing possible anthropogenic climate change is to use numerical models, this can also be done by first understanding the natural variability and then developing statistical tests to assess the probability of any observed changes occurring naturally. This approach is discussed in detail by Lennartz and Bunde [this volume]. By providing probability information as a function of space-time scales, scaling approaches to atmospheric variability thus provide a different, complementary path to studying anthropogenic effects. Pielke et al. [2009, p. 413] give evidence in support of the hypothesis that although the natural causes of climate variations and changes are undoubtedly important, the human influences are significant and involve a diverse range of first-order climate forcings, including, but not limited to, the human input of carbon dioxide. Most, if not all, of these human influences on regional and global climate will continue to be of concern during the coming decades.

When dealing with the issue of real-time warning and prediction as well as the issue of determining present statistics and possible future changes of these statistics, mostly dynamical models are applied: models of the form $\Delta Y/\Delta t = \Sigma$ processes, where some processes are explicitly described by first principles such as mass conservation and others (such as boundary layer turbulence) are parameterized; that is, their net effect on the resolved scales and parameters is empirically closed [cf. Müller and von Storch, 2004]. Such models usually exhibit chaotic behavior, so that the predictability is limited often to only a few days (e.g., in case of the extratropical atmosphere) or even hours (convective rainfall). However, this chaotic feature does not inhibit the model from skillfully representing the statistics of the system.
properly, as well as their dependence on certain external factors; the best example represents the annual cycle, as a response to solar radiation. Thus, contemporary climate models can simulate thousands of years, exhibiting a realistic chaotic behavior and realistic long-term memory [cf. Rybski et al., 2008]. (For a broader discussion about models, their specifics and their role in a societal context, refer to von Storch et al. [2011].) At present, there is a discussion if the low-frequency variability of global circulation models is too low or not compared with paleodata [Lovejoy and Schertzer, this volume, Figures 1a and 1b].

The model simulations include the formation of extreme events, for instance, mesoscale marine storms, such as polar lows in the Atlantic [Zahn and von Storch, 2010] or medicanes (rare hurricane-like small storms in Mediterranean Sea) [Cavicchia and von Storch, 2012]. Their formation is realistic in terms of numbers, across-scales link to large-scale conditions, and dynamical features; they allow the derivation of scenarios of future conditions associated with elevated greenhouse gas concentrations in the atmosphere.

On the longer time scales, there is an emerging realization that abrupt and prevalent temperature variability with significant impacts has recurred in the past when the Earth system was forced beyond threshold, although scaling approaches suggest that they could be the result of internal variability mechanisms repeating scale after scale over wide ranges (e.g., cascades and multifractals). The emerging evidence supports the recognition that Earth’s climate has characteristics of a critical phenomenon and also is sensitive to small changes in solar output on the centennial time scale. Quasi-cyclic manifestations of “critical forcing parameter” like solar cycles appear in the spectra because of their imprint at the time of major changes.

The reconstructed proxy record of temperature variability decoded from tree rings provides suitable data for the study of abrupt changes in temperature. The study by Tiwari et al. [this volume] is based on the spectra of empirical orthogonal functions of a newly reconstructed tree ring temperature variability record decoded from the western Himalaya for the period spanning 1227–2000 A.D., addressing the frequency resolution of interdecadal and interannual oscillatory modes. The spectral analysis of first principal component (PC1) with about 61.46% variance reveals the dominance of some significant solar cycles notably peaking around 81, 32, 22, and 8–14 years. This analysis in the light of the recent ocean-atmospheric model results suggests that even small variation in solar output in conjunction with atmospheric-ocean system and other related feedback processes could have caused abrupt temperature variability at the time of “criticality” through the triggering mechanism. Identification of different natural frequency modes from a complex noisy temperature record is essential for a better understanding of the climate response to internal/external forcing.

5. HYDROLOGIC SCIENCE: FLOODS AND DROUGHTS

The NRC [1991] classified hydrologic science as a distinct geoscience. The importance of the global water cycle and its nonlinear interactions with the natural Earth system and the engineered Earth system was instrumental in devoting an entire chapter to nonlinearity in the NRC [1991] volume titled “Hydrology and Applied Mathematics.” The NRC [2012] has revisited the challenges and opportunities. Our focus here is on hydrologic extremes that include high-frequency flood events and low-frequency drought events. Analyses of paleohydrologic and paleoclimate time series for understanding droughts have a long history in hydrology that dates back to the classic work of Hurst [1951]. Indeed, the “Hurst effect” has become quite well known in the nonlinear science literature [Feder, 1988], and Hurst’s scaling approach has since been greatly generalized by Mandelbrot [1999] and others into a multifractal approach with the implications for the extremes mentioned above, including power law tails on the rain rate and river flow distributions [e.g., Bunde et al., this volume]. Mesa et al. [this volume] give a brief overview of the pertinent literature from hydrologic science, climate science, and dynamical systems theory. Simulations from the Daisyworld model with and without the hydrologic cycle as a simple climate model reveal complex nonstationary behavior that inhibits consistent interpretation of the Hurst exponent using three well-known estimation methods. Challenging problems for future investigations are suggested that offer a broad context for nonlinear geophysics in understanding droughts.

Accurate estimates of the magnitude and frequency of floods are needed for the design of water-use and water-control projects, for floodplain definition and management, and for the design of transportation infrastructure such as bridges and roads. These practical engineering needs have led to the development of regional flood frequency analysis that Davdy et al. [2012] have summarized. For example, Fuller [1914] analyzed peak flows from around the world, but particularly from the United States, and observed that the mean of the maximum annual flood can be related to the drainage area as a power law with an exponent of 0.8. Similar empirical relationships have since been employed to relate discharge to drainage basin characteristics.

Unfortunately, the accuracy of flood quantile estimates is constrained by the data available at a stream gauging site: record lengths are often limited to 100 years and are typically less than 30 years. To overcome these limitations, numerous
statistical methods to estimate flood quantiles have been developed through the years. Furthermore, quantile estimates are often needed for ungauged sites at which no historical stream flow data are available. The challenging problem of prediction in ungauged basins (PUB) led to an international decadal initiative [Sivapalan et al., 2003].

Global warming, and other human influences like large-scale deforestation in the Amazon River basin, is changing the global and regional water cycle and the climate system. Owing to strong nonlinear coupling between climate and the water cycle, future floods are not expected to be statistically similar to the past [Milly et al., 2002; Dawdy, 2007]. As a result, estimates of flood quantiles using historic data would be biased. Can the results in the regional flood frequency analyses be understood in terms of physical mechanisms producing floods? This is a key question. A geophysical understanding of regional flood frequencies is anchored in spatial power law statistics or scaling as summarized below.

How the scaling behavior in the quantiles of annual peak discharge arises and how the slopes and intercepts can be predicted from physical processes require understanding of scaling relations in individual rainfall-runoff events. Experimental research along this line was initiated in the small 21 km² Goodwin Creek experimental watershed in Mississippi [Ogden and Dawdy, 2003]. Event scaling was a major shift in focus from the study of regional annual flood statistics that is well established in the literature. Ogden and Dawdy [2003] first observed scaling relations in peak discharges for 226 individual rainfall-runoff events that spanned hourly to daily time scales and found that the scaling slopes and intercepts vary from one event to another. The mean of 226 event slopes (0.82) was close to the common slope of mean annual and 20 year return peak discharges (0.77), which suggested that it should be possible to predict scaling in annual flood frequencies from event scaling.

Key results in the last 20 years have established the theoretical and observational foundations for developing a nonlinear geophysical theory or the scaling theory of floods in river basins. It has the explicit goal of linking the statistics of space-time rainfall input and the physics of flood-generating processes and self-similar branching patterns of drainage networks with spatial power law statistical relations between floods and drainage areas across multiple scales of space and time. A substantial literature in the last 30 years has developed around stochastic point process models and multifractal approaches to space-time rainfall intensities. Gupta et al. [2007] reviewed the developments in the scaling flood theory. Published results have shown that the spatial power law statistical relations emerge asymptotically from conservation equations of mass and momentum in self-similar (self-affine) channel networks as drainage area goes to infinity. These results have led to a key hypothesis that the physical basis of power laws in floods has its origin in the self-similarity (self-affinity) of channel networks. Self-similarity is also the basis for the widely observed fractal structure and Horton relations in river networks [Rodriguez-Iturbe and Rinaldo, 1997; Gupta et al., 2007; McConnell and Gupta, 2008]. Observed power laws in floods range from hours and days of individual flood events to an annual time scale of flood frequencies. They serve as the foundation for developing a new diagnostic framework to test different assumptions governing multi-scale spatiotemporal variability in physical processes that arise in predicting power law statistical relations between floods and drainage areas [Gupta et al., 2010].

Important new developments are taking place toward generalizing the scaling theory of floods to medium and large basins involving annual flood frequencies. For example, Poveda et al. [2007] discovered a link between mean annual runoff that is estimated from annual water balance and annual flood scaling statistics. Their study included many large river basins in Colombia that have varying hydrology and climates ranging from arid to humid. Lima and Lall [2010] found scaling in annual flood quantiles in large basins of Brazil (~800,000 km²). They developed a Bayesian approach to estimate scaling parameters on interannual time scales. Generalizations to large basins have important relevance to flood prediction under climate change [Milly et al., 2002]. Future work requires an understanding of how scaling slopes and intercepts in annual flood quantiles are modified in a changing climate, which can serve as a basis for making future flood predictions. All these developments and many others not given here have made a substantial contribution to solving the PUB problem [Sivapalan et al., 2003].

6. SPACE WEATHER: SOLAR ACTIVITY, MAGNETOSPHERE, IONOSPHERE, AND SOCIETAL IMPACTS

Extreme events in space weather occur during periods when the magnetosphere is strongly driven by the solar wind, bringing energetic plasma and fields from the solar eruptive events such as coronal mass ejections to geospace. Many extreme space weather events have caused serious damage to technological systems such as satellites, power transmission systems, etc. Some well-known examples are the collapse of the Hydro Quebec power grid during the great geomagnetic storm of March 1989, the Canadian telecommunications satellite outage during a period of enhanced energetic electron fluxes at geosynchronous orbit in January 1994, the electrical breakdowns and satellite malfunctions during the magnetic cloud event of July 2000 (Bastille Day event), the disabling of
GPS-based aviation system during the severe space weather events of October–November 2003 (Halloween storms), the disturbances in commercial airline traffic during several days of enhanced geomagnetic activity in January 2005, etc. [NRC, 2008]. Although these events may not seem devastating by themselves, a confluence of natural hazards in the different regions of the environment of the Earth can make our society and its technological systems highly vulnerable because of their interconnectedness [Baker and Allen, 2000; NRC, 2008]. In this aspect the nonlinear dynamical approach, with its ability to integrate many interacting components, becomes a natural framework for the study of the extreme events in the extended Earth and space system.

The modeling of space weather events relies strongly on the availability of good geospace data, and among the most widely used data are the geomagnetic indices. Geomagnetic field data from ground magnetometer stations around the globe have been recorded for more than one and half centuries, and these data have been used to compute the geomagnetic indices. Among the many indices the auroral electrojet indices (AE, AL, and AU) characterize the substorms, and the ring current index Dst represents the geomagnetic or space storms. The substorms, with a characteristic time of ~1 hour, are episodic in nature and are the essential elements of magnetospheric dynamics. The auroral electrojet indices provide the detailed dynamical features of the global aspects of substorms. On the other hand, the geomagnetic storms, with a typical time scale of ~10 hours, are more global space weather disturbances during which intense substorms occur. The auroral electrojet indices computed from the horizontal component of the magnetic field disturbances reflect the strengths of the large-scale ionospheric currents driven by the reconfiguration of the magnetosphere during substorms. They are highly variable during strongly disturbed periods, with peak values of 1000–2000 nT during extreme events cited earlier.

The substorms with AL index values less than −1000 nT are considered strong disturbances and can be characterized as extreme events. The geomagnetic storms with Dst values less than −100 nT are referred to as intense storms. The substorms occur during the storms, and their episodic nature and high variability are evident in the sharp peaks of the electrojet indices. As is the case with extreme events in general, there is no single measure of the extreme events in space weather. For example, the Dst index for the well-known “Carrington” storm of 1–2 September 1859 is estimated to have reached a value of −1760 nT [Tsurutani et al., 2003], and its effects were felt across the globe. The more recent Bastille Day event of 14–16 July 2000 with a Dst minimum of −300 nT was an extreme space weather event that led to significant damage to satellites and other technological infrastructure. It should be emphasized here that the main objective in the studies of extreme events is the nature of their distribution.

From observed solar-terrestrial dynamics and detailed studies of the complexity of the magnetospheric system, many studies (see reviews of Sharma [1995], Klimas et al. [1996], Vassiliadis [2006], and Baker [2012, this volume]) have shown evidence that the solar wind–magnetosphere–ionosphere realm clearly is characterized by nonlinear dynamics. It is argued that methods borrowed from other branches of physics, chemistry, and engineering can offer useful, if imperfect, analogies on how to deal with nonlinearity in the Sun-Earth system. Ultimately, space weather prediction probably must depart from idealized local “stability” analyses of various plasma domains to consider true global system responses and must incorporate realistic, nonlinear aspects. Forecasting of space weather using data-based nonlinear dynamical models of solar wind–magnetosphere coupling [Vassiliadis et al., 1995; Valdivia et al., 1996; Ukhorskiy et al., 2004; Chen and Sharma, 2006] would be useful not only at high latitudes where geomagnetically induced currents pose a risk to power transmission grids but also at equatorial and low latitudes where the development of irregular structure in the ionosphere can cause degradation and even disruption in the operation of satellite-based communication and navigation systems such as the GPS. A disturbance dynamo set up by enhanced currents in the auroral ionosphere during magnetic substorms can after a few hours give rise to conditions in the equatorial ionosphere that are conducive for the development of such irregular structure [Bhattacharyya, this volume].

In the light of such understandings, there is strong motivation to go away from the usual plasma physics techniques to consider more aggressive and comprehensive approaches. It may be that the magnetotail has a distributed set of interacting control agents, some of which are local plasma conditions and some of which are remote in the ionosphere or solar wind. Thus, it may appear that instabilities are spontaneous or seemingly random. We need to use new analysis tools to address such issues. Incorporating both traditional forecasting methods for solar storms and also using the array of methods embodied in the most modern tools of nonlinear dynamics offers the best hope for dealing with extreme space weather events [Sharma and Veeramani, 2011; Baker, this volume].

The vision of a space weather forecasting plan begins with understanding what is, in effect, space “climate.” It is known that the Sun undergoes an approximately 11 year cycle of increasing and then decreasing sunspot activity. Some solar cycles have been relatively weak, while others in the historical record have been very strong [Baker, this volume]. The
stronger maxima tend to have more episodes of strong solar flares, intense outbursts of ultraviolet light, and (perhaps) greater chances of solar particle “superstorms.” Conversely, weaker maxima may exhibit fewer of these extreme characteristics. However, even these general patterns of behavior are not absolute. Some of the largest historical events, for example, occurred during a relatively weak sunspot maximum period [see Baker, this volume]. On even shorter temporal scales it is known that forecasting space weather events will require continuous and effective observations of the Sun and its corona. Being able to observe the initiation of a powerful solar flare and detecting an earthward bound CME at its early stages gives society the best chance of predicting a powerful geomagnetic storm. Such CME observations can provide perhaps 18 hour (or longer) alerts that a solar-induced geomagnetic storm is imminent.

By using models developed in the research community and driving such models with space-based and ground-based solar observations, the natural hazard community has moved an important step closer to providing tens of hours warning of impending geomagnetic storms. Both ground-based and space-based observations of the Sun can be fed into highly capable models to forecast solar wind conditions near Earth. In the magnetosphere-ionosphere system, models of the complete, coupled geospace system can be used to provide forecasts ranging from tens of minutes to several days. The present hazard response plan still has a long way to go to achieve the levels of accuracy and reliability that are desired by the U.S. industrial, military, and commercial sectors. Moreover, knowing with accuracy the near-Earth solar and solar wind drivers of space weather still demands further understanding of the nonlinear responses of the geospace system that are described here [Baker, this volume]. The complex feedbacks within the solar wind–magnetosphere-ionosphere system may ultimately limit our ability to predict the size, duration, and precise location of magnetospheric current systems that can wreak havoc on human technologies, but we must understand these better to avert severe consequences for humans.

7. OVERARCHING ISSUES: INTERDISCIPLINARY APPROACH, DATA, AND LARGE-SCALE SIMULATIONS

As we have seen above, at least three research threads have all made significant contributions to fundamental complexity science and to the geosciences over the last 40 or more years. One thread has been the use of low-dimensional deterministic models, particularly chaotic ones, based on ordinary differential equations or maps [Lorenz, 1963; May, 1976]. This has been accompanied by data analysis methods such as the recovery of attractors by embedding and has led to many applications [Abarbanel et al., 1993; Kantz and Schrieber, 1999]. In space weather this approach has provided the earliest predictive models and is now the basis for near-real-time forecasting. The irregular behavior of geomagnetic activity, however, cannot be described entirely by complexity in low-dimensional dynamics, and there are strong multiscale features that exhibit long-range behavior.

A second thread, described as stochastic chaos above, has involved the use of additive stochastic models that can exhibit heavy tails and/or long-range dependence. Described as $H$ self-similar by applied mathematicians, these models include Mandelbrot’s Lévy (alpha stable) motion for heavy tails, his fractional Brownian motion for long-range dependence, and his subsequent fractional hyperbolic model of 1969 that combined both effects. Here $H$ is a single self-similarity exponent, which is the same as the more familiar Hurst exponent in the absence of heavy tails [e.g., Mercik et al., 2003]. Related models like autoregressive fractionally integrated moving average, which are not completely self-similar but do show long-range dependence, have been extensively studied in applied statistics [e.g., Beran, 1994]. A third thread has developed from the recognition that the “wild” variability seen in some natural systems, particularly features such as nonlinear correlation in amplitudes that gives rise to observed “volatility bunching,” can be modeled by multiplicative cascade multifractal models rather than additive monofractal ones. These generalize the single scaling exponent to a continuous spectrum of exponents.

Clearly, exactly what sort of extremes a given natural system exhibits may reflect the extent to which the above descriptions apply to it, as, for example, discussed in the contribution by Watkins et al. [this volume], where the implications of monofractal and multifractal models of integrated extreme “bursts” above a threshold are compared. Focusing on the burst problem, we note that several interdisciplinary stochastic approaches exist that provide quantitative information about extreme bursts. Some are more focused on the probability of single large events, while others are more concerned with extended dwell times above a given spatiotemporal threshold. As mentioned above, a statistical extreme value theory is available. In addition, there is a mathematical theory of record breaking [Nevzorov, 2001]; while image processing and fluid dynamics contribute the idea of level sets, stochastic processes offer sojourner times [Berman, 1992]; and as noted above, complexity science has contributed models of “avalanches” in the space-time activity of nonequilibrium systems, where the term “avalanche” refers here to a cascading, spatiotemporal, coherent energy release event. It can be seen that even without historical and disciplinary factors, it would be completely natural for
knowledge relevant to a given burst problem to be siloed and so not yet be “joined up.” In practice, the choice of approach has also differed between and within application domains, sometimes reflecting the data’s own limitations and also reflecting the differing cultures and assumptions of theoretical physics, complexity science, the geosciences, applied stochastic modeling, and statistical inference. Although such knowledge is not yet integrated, particularly for Earth and sciences applications, nonetheless, significant efforts have started along this direction.

The rise of complexity science in geophysics has given currency to the expression “emergent” for higher-level laws, i.e., properties that cannot be predicted easily from the elementary interactions. For example, processes which appear complicated at one scale often produce emergent laws in a suitable limit at a higher scale that are relatively simple. A great deal of complexity science has so far been precisely the elaboration of various nonlinear models or, even paradigms, which allow one to reduce “complexity” to “simplicity” in this way. In many, even most, geophysical cases, these nonlinear models capture a good deal of the phenomenology and physics of processes that had hitherto been impervious to realistic modeling. It is particularly true for cases where ubiquitous assumptions of homogeneity and linearity had reigned. For example, the prototypical example of self-organized criticality, the classical sandpile model, has been widely applied across the geosciences including studies of the atmosphere where cascades can generate a “nonclassical” SOC. By generically describing some phenomena that are so extreme that even their mean behaviors are determined by avalanche-like events, it is frequently the only type of model that is even remotely realistic, e.g., earthquakes and forest fires.

The space-time variability of physical processes on multiple scales is common to many disciplines of geosciences. But this feature presents a huge scientific challenge in developing physically based models. Let us briefly explain this cross-cutting and overarching issue. The first challenge is suitable parameterizations of processes at “appropriate scales” that are required in specifying the coupled conservation equations of mass, momentum, energy, etc. Typically, observations are not available at the scales of interest, which require statistical parameterizations. For example, the coupled conservation equations underlying the scaling theory of floods [Gupta et al., 2010] need how much rainfall becomes runoff involving surface and subsurface processes in each hillslope (~0.05 km²) of a river basin. But each basin has a large number of hillslopes. For example, a basin of area A has 20A hillslopes, so a 100 km² “small basin” has about 2000 hillslopes. There is no known measurement technology that can be used to measure runoff in each hillslope even for a single experimental basin in the world, because runoff varies both in space and time among hillslopes. Furthermore, no known theory is available that can be used to parameterize runoff from hillslopes, so new theories are needed to solve this problem. As new and diverse approaches are developed from different fields of geosciences, a major issue for future conferences and workshops would be to identify if parameterization models have some features in common that can apply across disciplines.

In taking stock, it is clear that complexity science has been most effective in quantifying the dynamical features and scaling laws of spatiotemporal data of various types and in providing an explanation for such data in a unified framework, sometimes when there is no competing theory. It has helped us solve the puzzle of why such scaling laws in a population of events can be robust, while at the same time leading in many cases to pessimism (some would say realism) on the potential for the predictability of the timing of individual extreme events. Principles of self-organization have been shown to have a thermodynamic explanation in terms of entropy production (minimum for a near-equilibrium, linear system, leading to Euclidean geometry, or maximum for a far-from-equilibrium nonlinear one, leading to fractal geometry), though this may not be universal for all systems and is a subject of much debate. Scaling laws are extremely useful in quantifying hazard and risk in an operational sense, but much needs to be done to quantify the uncertainties in forecasting future risk based on a limited sample of past extreme events [Main et al., 2008].

8. CONCLUSION

The long-term dependencies in geophysical systems and their implications for catastrophic extreme events have direct applications to other areas such as life sciences. The elucidation of long-range (scaling) dependencies can be used in forecasting extreme events [Bunde et al., this volume]. Indeed, over the last 30 years, work on general scaling processes has shown that the extremes are precisely the consequence of variability that builds up scale after scale over huge ranges.

An important aspect of extreme events research is the extensive demands for computational resources. Enhanced computational resources have led to investigations of complexity in a wide range of Earth and space systems, and this cyber-enabled discovery will lead to more complex but reliable forward models. The ensemble of model results can then yield statistical properties that reflect the uncertainties and their propagation, providing the scaling behaviors with respect to the external forcing parameters. These statistical approaches are essential for characterizing extreme events accurately or their uncertainties realistically if this is not possible.
The different geoscience disciplines have many important problems that need to be studied using approaches beyond those commonly used within the specific discipline. The underlying features of many of these problems arise from the nonlinear nature, and complexity science provides a framework for studying the seemingly unconnected phenomena. This has been evident from the progress made in the Earth and space sciences based on the understanding of nonlinear dynamics, chaos, fractals, multifractals, scaling, etc.

Currently, the first principle or theoretical models are limited in providing predictable models of extreme events. Among the many reasons for this is the disparate space and time scales associated with extreme events. A promising approach is the data-driven modeling in which the inherent features of the many phenomena contributing to an extreme event can be used together, independent of a priori assumptions, to build an integrative model. We are also entering a new age of data-intensive research, where key information is extracted and new objects and patterns identified from very large, continuously recorded data sets, a process known as “data assimilation” [Hey et al., 2009]. It remains to be seen whether predictive power can be improved by such models or such improved short-term databases and methods of assimilation, but this can only be done by applying rigorous tests that reflect the epistemic (model dependent) and aleatoric (statistical) uncertainties involved.

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Earthquakes: Complexity and Extreme Events

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Earthquakes are clearly complex phenomena; they are chaotic, and they are widely considered to be an example of self-organized criticality. Despite the complexity, earthquakes satisfy several scaling laws to a good approximation. The best known is Gutenberg-Richter (GR) frequency-magnitude scaling. This scaling is valid under a wide range of conditions, including global seismicity. GR scaling is important in seismic hazard assessment because it can be used to estimate the risk of large earthquakes from the rate of occurrence of small earthquakes. Also important in seismic hazard assessment is the concept of characteristic earthquakes (CEs) on mapped faults. In this paper, we address the alternative GR and CE behaviors for faults. We use the sequence of CEs that have occurred on the Parkfield segment of the San Andreas fault. We conclude that the data tend to support the CE hypothesis, but the GR hypothesis cannot be ruled out on the basis of currently available data. We also use numerical simulations to study the CE hypothesis.

1. INTRODUCTION

Earthquakes are one of the most feared of the natural hazards because they incur extensive costs in both life and property and because they occur without warning. Absent reliable precursory phenomena [Turcotte, 1991], the seismic hazard is mitigated by enforcing construction standards and facilitating public preparedness in general. Three earthquakes in 2010 illustrate the effectiveness of properly hardening infrastructure against the seismic hazard. The $M = 7.0$ Haiti earthquake, on 12 January 2010, occurred 25 km west of Port au Prince and caused an estimated 230,000 deaths. The $M = 7.1$ New Zealand earthquake on 4 September 2010 occurred 40 km west of Christchurch; there were no deaths. The great difference in fatalities can be attributed largely to enforced standards of construction in New Zealand. The $M = 8.8$ Chile earthquake on 27
February 2010 caused 250 deaths. The relatively low toll, again, can be attributed to construction codes, earthquake safety education, and preparedness among the general public, enforced following the $M = 9.5$ 1960 Chile earthquake, which is the strongest earthquake to occur since seismographs have been available to quantify earthquake magnitudes. Earthquakes of concern generally have magnitudes $M > 6$. These earthquakes occur on preexisting faults, but only a small fraction of the relevant faults have been mapped. Many have no surface expression and are not defined by seismic activity. It is the purpose of this chapter to discuss the complex nature of large earthquakes and methods by which the earthquake hazard can be assessed. Slider block models are considered as simple analogs for distributed seismicity. A coupled pair of slider blocks have been shown to exhibit deterministic chaos [Huang and Turcotte, 1990], and multiple slider blocks exhibit self-organized critical (SOC) behavior [Carlson and Langer, 1989]. In the slider block model, blocks are pulled over a surface by a driver plate. The blocks are connected to the driver plate and to each other by springs. The frictional interaction between the blocks and the surface leads to stick-slip behavior; sequences of slip events occur. The frequency-area statistics of the SOC behavior are power law (fractal)

$$N(A) \sim A^{D/2},$$  \hspace{1cm} (1)

where $N$ is the number of slip events of area $A$, specifically the number of events involving $A$ discrete block elements, and $D$ is the fractal dimension of the system.

Despite the complexity of seismicity, it is accepted that the frequency-magnitude distribution of earthquakes satisfies Gutenberg-Richter (GR) scaling under a wide variety of conditions. The applicable relation is [Gutenberg and Richter, 1954]

$$\log[N(> M)] = a - bM,$$  \hspace{1cm} (2)

where $N(> M)$ is the cumulative number of earthquakes in a specified spatial area and time window with magnitudes greater than $M$, $b \approx 1$, and the constant $a$ is a measure of seismic intensity. Aki [1981] showed that GR scaling, equation (2), is equivalent to the fractal scaling given in equation (1) if $A$ is the earthquake rupture area and $D = 2b$. The association of earthquakes with SOC behavior has been discussed in some detail by Turcotte [1997, 1999]. The association of this behavior with statistical physics has been outlined by Rundle et al. [2003]. Pavlos et al. [2007] and Iliopoulos and Pavlos [2010] have discussed the association of SOC behavior with low-dimensional chaos.

2. GR SCALING VERSUS CHARACTERISTIC EARTHQUAKES (CES)

It is recognized that GR scaling is generally applicable to regional and global seismicity. There are, however, two limiting hypotheses that can explain this scaling. In the GR hypothesis, each fault has a GR distribution of earthquakes on it. The largest earthquake would be limited by the size of the fault, but earthquakes occurring would satisfy GR frequency-magnitude scaling. This is the behavior associated with the SOC slider block model. The array of blocks has fractal (GR) frequency-magnitude scaling limited by the size of the array.

The second limiting hypothesis for GR scaling of regional seismicity introduces the concept of CEs. Each fault has CEs, of approximately equal magnitude, that rupture the entire fault. Large faults have large earthquakes and small faults have small earthquakes. It is recognized that faults have a fractal number-area distribution [Turcotte, 1997], and this distribution is responsible for the GR scaling of regional seismicity. An example of a CE would be the 1906 San Francisco earthquake on the northern San Andreas fault. Very few earthquakes have occurred on or near this fault since 1906. However, we do not have a sufficiently long record to study the distributions during an entire earthquake cycle. A focus of this chapter will be to consider the alternative GR and CE hypotheses for the sequence of earthquakes that have occurred on the Parkfield segment of the San Andreas fault.

Clearly, actual seismicity will not satisfy either of the extreme hypotheses, but will have characteristics of both. The relative importance of GR scaling versus CEs has important implications for earthquake hazard assessment. GR scaling provides a seismicity-based approach to earthquake hazard assessment. The GR hypothesis is that large earthquakes ($M > 6$) occur where large numbers of small earthquakes $(2 < M < 6)$ occur. The GR scaling given in equation (2) extrapolates the numbers of small earthquakes to forecast the probabilities of occurrences of large earthquakes. The direct use of GR scaling for risk assessment is known as the relative intensity method [Holliday et al., 2006; Shcherbakov et al., 2010]. Modifications of the GR approach form the basis of the regional earthquake likelihood models that are currently being tested for California [Field, 2007a].

The second approach to the understanding of the earthquake hazard is to consider sequences of large ($M > 6$) CEs that occur on mapped faults. The fault-based CE hypothesis is that a sequence of large earthquakes occurs on mapped faults with a specified statistical distribution of recurrence times. This approach is the basis of the Working Groups on California Earthquake Probabilities hazard assessment [Field,
in which rates and magnitudes of earthquakes, mean slip rates, measured strain, and similar data are evaluated with respect to known fault maps.

The alternative GR and CE approaches have also been applied to earthquakes on specified faults or fault segments. It is accepted that small earthquakes on or near a fault obey GR scaling. In the GR hypothesis, all earthquakes on or near the fault satisfy GR scaling. In the competing CE hypothesis, the large “characteristic” earthquakes lie above GR scaling, which is to say they occur more frequently than predicted by the GR distribution of small events in the region. Arguments favoring the CE hypothesis have been given by Wesnousky [1994], Hofmann [1996], Ishibe and Shimazaki [2009], and others. Arguments favoring the GR hypothesis have been made by Parsons and Geist [2009] and Naylor et al. [2009], among others. There are several problems associated with comparisons of the two hypotheses. First, sequences of CEs generally involve long intervals so that the data are relatively poor. Paleoseismic studies can be used, but again, there are concerns about data quality. In addition, it is necessary to specify the region to be considered in a comparison; in general, an arbitrary area must be specified.

A primary focus of this chapter is to consider the alternative GR and CE hypotheses in terms of seven $M \approx 6$ CEs that occurred on the Parkfield section of the San Andreas fault between 1857 and 2004. We will concentrate our attention on the CE cycle associated with the 1966 and 2004 events. We will consider the seismicity on and adjacent to the fault during the period $1971 - 2004$ (5 years after the 1966 earthquake) to $2009$ (5 years after the 2004 earthquake). We determine the GR statistics of aftershocks and background seismicity during the period. In the GR hypothesis, the 2004 main shock should lie on the GR scaling of the aftershocks and background seismicity; we will test this hypothesis.

Instrumental records of characteristics of CEs are rare. Historical and paleoseismic records are helpful, but their reliability can be questioned. An alternative approach is to use numerical simulations of CE occurrences to study their statistical behavior. We will use the Virtual California (VC) earthquake simulator to generate synthetic catalogs of earthquakes in Northern California. We will concentrate our attention on the Hayward fault and compare its simulated behavior with the behavior of the Parkfield segment of the San Andreas fault.

3. GLOBAL SEISMICITY

Earthquake frequency-magnitude statistics have been recognized to satisfy log-linear scaling both globally and regionally. Because of the many problems associated with the magnitudes of large earthquakes, the preferred approach to global seismicity is to use the Global Centroid Moment Tensor catalog. Using this catalog, the cumulative number of earthquakes per year $N(M) \cdot \text{yr}^{-1}$ with magnitudes greater than $M$ for the period 1977 to 2009 is given in Figure 1. There is a clear change in slope at $M = 7.55$. The best fit GR scaling from equation (2) in the range $5.5 < M < 7.55$ is $a = 8.09$ and $b = 1.0$. The best fit GR scaling from equation (2) for $M > 7.55$ is $a = 11.68$ and $b = 1.48$. This change in slope has been previously observed [Pacheco et al., 1992]; Rundle [1989] predicted the transition from $b = 1.0$ to $b = 1.5$ in the context of self-similarity. In both papers, the break at $M = 7.5$ is associated with the thickness of the brittle lithosphere and the transition from a 2-D to an approximately 1-D rupture. The largest earthquake during the last 100 years was the 1960 Chile earthquake, with $M \approx 9.5$. For this earthquake, the large-magnitude GR scaling, as shown in Figure 1, implies a recurrence time $\Delta t = 219$ years. Within the context of GR scaling, this earthquake was not unexpected.

From Figure 1, traditional GR scaling, where $b = 1$, appears to be valid within the magnitude domain $5.5 < M < 7.55$; GR scaling with $b = 1.5$ persists for $7.55 < b < 9.5$. Studies of seismicity in a deep gold mine in South Africa show GR scaling for magnitudes as small as $M = -4.4$ [Kwiatek et al., 2010], which implies that the lower-magnitude rollover in Figure 1 is due to the sensitivity limit of the global seismic network. For the parameter values, we have obtained within the domain where $b = 1.0$ and $a = 8.09$, that the earthquake magnitude $M$ is related to the rupture area $A_r$ by Aki [1981]

$$A_r = 64 \cdot 10^M \text{m}^2$$

with $A_r$ is in meters squared ($\text{m}^2$). Thus, the power law (fractal) scaling for global seismicity ($-4.4 < M < 7.5$) is valid for rupture areas from $2.55 \cdot 10^{-3} \text{m}^2 \leq A_r \leq 2024 \text{km}^2$.

4. CHARACTERISTIC EARTHQUAKES AT PARKFIELD

In the previous section, we considered the earthquakes that occur on many faults. We now turn to the frequency-magnitude statistics of earthquakes associated with a specific fault or fault segment. As previously noted, this is a controversial subject; there are two competing hypotheses. The first hypothesis is that the earthquakes associated with a fault or fault segment obey the GR scaling (equation (2)). In the competing CE hypothesis, large CEs on a fault, or fault segment, occur more frequently than power law scaling of smaller earthquakes would predict. There are several problems associated with comparisons of the two hypotheses.
First, sequences of CEs involve long temporal recurrence intervals, so the data are sparse. Additionally, it is necessary to specify the region adjacent to a fault or fault segment where earthquakes are considered. The best documented sequence of CEs occurred on the Parkfield segment of the San Andreas fault in California. Evidence suggests that earthquakes with $M \approx 6$ occurred in 1857, 1881, 1901, 1922, 1934, 1966, and 2004 [Bakun et al., 2005]. Based on seismograms, the 1922, 1934, 1966, and 2004 events were remarkably similar in intensity. However, the recurrence times, 12, 32, and 38 years, varied considerably.

The main focus of this chapter is to study the seismicity on the Parkfield segment of the San Andreas fault during a full earthquake cycle. Specifically, the cycle associated with the 28 June 1966 $M_s = 6.0$ and the 28 September 2004 $M_w = 5.96$ earthquakes. To take full advantage of the improvements of the seismic network, we will consider the cycle for the period 1972 to 2010. The quality of the data for the aftershocks of the 2004 earthquake is much better than the quality of the data for the aftershocks of the 1966 earthquake.

To isolate seismicity associated with the Parkfield section of the San Andreas fault, we confine our study to the region where the aftershocks of the 2004 earthquake were concentrated. This region is well defined, and we select the specific geometry from the work of Shcherbakov et al. [2006]. The region is elliptical, centered at 35.9°N and -120.5°W with semimajor and semiminor axes of 0.4° and 0.15°, respectively, oriented at 137°NW. Both the aftershocks and the elliptical region are shown in Figure 2.

We consider the full seismic cycle between 28 June 1971 (5 years after the 1966 Parkfield earthquake) and 28 September 2009 (5 years after the 2004 Parkfield earthquake). The cumulative frequency magnitude distribution for this time period and region is given in Figure 3; the $M = 5.96$ 2004 Parkfield event is included. The best fit GR scaling from equation (2) is also shown; assuming $b = 1.0$, we find $a = 5.65$. If the Parkfield earthquake was part of the GR scaling, we would require $a = 5.96$. Thus, the observed seismicity for the seismic cycle is a factor of 2 less than that required for full GR scaling. This would indicate that the 2004 Parkfield event was a CE relative to both aftershocks and background seismicity in the aftershock region.

Based on these scaling relations, invoking equation (2), there were $N_{CE} = 10^{3.65} = 4467$ $M > 2$ earthquakes during the CE cycle. If the GR hypothesis is valid, there would have been $N_{GR} = 10^{5.96} = 9120$ $M > 2$ earthquakes during the cycle. The earthquakes that occurred include the 2004 main
shock, aftershocks of this earthquake, and background seismicity. Based on the aftershock study of Shcherbakov et al. [2006], there were $N_{as} = 450 \ M > 2$ aftershocks. This leaves $N_{BG} = 4200 \ M > 2$ background earthquakes during the cycle. The background earthquakes constituted 44% of the number required by the GR hypothesis. The 2004 Parkfield main shock appears to be a well-defined CE, but there was considerable background seismicity.

Characteristic earthquakes are associated with quasiperiodicity, but can also have considerable variability. A measure of the variability of recurrence times on a fault or fault segment is the coefficient of variation $C_v$, the ratio of the standard deviation $\sigma$ to the mean $\mu$. For strictly periodic earthquakes on a fault or fault segment, we would have $\sigma = C_v = 0$. For the random (i.e., exponential with no memory) distribution of recurrence times, we would have $\sigma = \mu$ and $C_v = 1$. Ellsworth et al. [1999] analyzed 37 series of recurrent earthquakes and suggested a provisional generic value of the coefficient of variation, $C_v \approx 0.5$. A number of alternative statistical distributions have been proposed for this purpose. These include the exponential (Poisson), the lognormal, Brownian passage time (inverse Gaussian), and Weibull distributions [Davis et al., 1989; Sornette and Knopoff, 1997; Mathews et al., 2002]. We will primarily consider the Weibull distribution. The cumulative distribution function (CDF) for the Weibull distribution is

$$P(t) = 1 - \exp \left[ -\left( \frac{t}{\tau} \right)^\beta \right],$$

where $P(t)$ is the fraction of the recurrence times that are shorter than $t$, and $\beta$ and $\tau$ are fitting parameters. If $\beta = 1$, this is the Poisson (random) distribution. Reasons for preferring the Weibull distribution have been discussed by Abaimov et al. [2008].

The CDF of Parkfield recurrence times ($t = 12, 20, 21, 24, 32,$ and 38 years) is given in Figure 4. The mean,