



Multi-Criteria Decision Analysis

Methods and Software

Alessio Ishizaka Philippe Nemery

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Foreword

The growing recognition that decision makers will often try to achieve multiple, and usually conflicting, objectives has led during the last three decades to the development of multi-criteria decision analysis (MCDA). This is now a vast field of research, with its scientific community and its specialized journals, as well as a large and growing number of real-world applications, for supporting both public policy making and decisions by private corporations.

Students and practitioners coming to the field, however, will be surprised by the plethora of alternative methods, overloaded by the array of software available, and puzzled by the diversity of approaches that an analyst needs to choose from. For precisely these reasons, this book is a very welcome event for the field. Alessio Ishizaka and Philippe Nemery have managed to provide an accessible, but rigorous, introduction to the main existing MCDA methods available in the literature.

There are several features of the book that are particularly innovative. First, it provides a balanced assessment of each method, and positions them in terms of the type of evaluation that the decision requires (a single choice among alternatives, the ranking of all alternatives, the sorting of alternatives into categories, or the description of consequences) and the level of preference information that each method requires (from utility functions to no preference information). This taxonomy helps both researchers and practitioners in locating adequate methods for the problems they need to analyze.

Second, the methods are presented with the right level of formulation and axiomatization for an introductory course. This makes the book accessible to anyone with a basic quantitative background. Readers who wish to learn in greater depth about a particular method can enjoy the more advanced content covered ‘in the black box’ of each chapter.

Third, the book illustrates each method with widely available and free software. This has two major benefits. Readers can easily see how the method works in practice via an example, consolidating the knowledge and the theoretical content. They can also reflect on how the method could be used in practice, to facilitate real-world decision-making processes.

Fourth, instructors using the book, as well as readers, can benefit from the companion website (www.wiley.com/go/multi_criteria_decision_analysis) and the availability of software files and answers to exercises.

This book should therefore be useful reading for anyone who wants to learn more about MCDA, or for those MCDA researchers who want to learn more about other MCDA methods and how to use specialized software to support multi-criteria decision making.

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1

General introduction

1.1 Introduction

People face making decisions both in their professional and private lives. A manager in a company, for example, may need to evaluate suppliers and develop partnerships with the best ones. A household may need to choose an energy supplier for their family home. Students cannot ignore university rankings. Often candidates for a job vacancy are ‘ranked’ based on their experience, performance during the interview, etc.

As well as ranking and choice problems, there are also classification problems that have existed since classical times. In the fourth century BC, the ancient Greek philosopher Epicurus arranged human desires into two classes: vain desires (e.g. the desire for immortality) and natural desires (e.g. the desire for pleasure). These classifications were supposed to help in finding inner peace. Nowadays, classification problems occur naturally in daily life. A doctor, for instance, diagnoses a patient on the basis of their symptoms and assigns them to a pathology class to be able to prescribe the appropriate treatment. In enterprise, projects are often sorted into priority-based categories. Not long ago, a study showed that over 20 million Brazilians have moved from the lower social categories (D and E) to category C, the first tier of the middle class, and are now active consumers due to an increase in legal employment (Observador 2008). Hurricanes or cyclones are sorted into one of the five Saffir–Simpson categories based on their wind speed, superficial pressure and tide height.

All of these examples show that delicate decision problems arise frequently. Decision problems such as ranking, choice and sorting problems are often complex as they usually involve several criteria. People no longer consider only one criterion (e.g. price) when making a decision. To build long-term relationships, make sustainable and environmentally friendly decisions, companies consider multiple criteria in their decision process.

Table 1.1 Category of decision problems.

Decision	Time perspective	Novelty	Degree of structure	Automation
Strategic	long term	new	low	low
Tactical	medium term	adaptive	semi-structured	middle
Operational	short term	every day	well defined	high


Most of the time, there is no one, perfect option available to suit all the criteria: an ‘ideal’ option does not usually exist, and therefore a compromise must be found. To address this problem the decision maker can make use of naïve approaches such as a simple weighted sum. The weighted sum, described in Section 4.3.1, is a special case of a more complex method and can only be applied with the right precautions (correct normalization phase, independent criteria, etc.) to enable sensible outputs. In reality, this approach is unrefined as it assumes linearity of preferences which may not reflect the decision maker’s preferences. For example, it cannot be assumed that a wage of £4000 is twice as good as one of £2000. Some people would see their utility of preference improved by a factor of 5 with a wage of £4000. This cannot always be modelled with a weighted sum.

Multi-criteria decision analysis (MCDA) methods have been developed to support the decision maker in their unique and personal decision process. MCDA methods provide stepping-stones and techniques for finding a compromise solution. They have the distinction of placing the decision maker at the centre of the process. They are not automatable methods that lead to the same solution for every decision maker, but they incorporate subjective information. Subjective information, also known as preference information, is provided by the decision maker, which leads to the compromise solution.

MCDA is a discipline that encompasses mathematics, management, informatics, psychology, social science and economics. Its application is even wider as it can be used to solve any problem where a significant decision needs to be made. These decisions can be either tactical or strategic, depending on the time perspective of the consequences (Table 1.1).

A large number of methods have been developed to solve multi-criteria problems. This development is ongoing (Wallenius et al. 2008) and the number of academic MCDA-related publications is steadily increasing. This expansion is among others due to both the efficiency of researchers and the development of specific methods for the different types of problem encountered in MCDA. The software available, including spreadsheets containing method computations, *ad hoc* implementations, off-the-shelf, web or smartphone applications, has made MCDA methods more accessible and contributed to the growth in use of MCDA methods amongst researchers and the user community.

The aim of this book is to make MCDA methods even more intelligible to *novice* users such as students, or practitioners, but also to confirmed researchers. This book is ideal for people taking the first step into MCDA or specific MCDA methods. The cases studies and exercises effectively combine the mathematical and

practical approach. For each method described in this book, an intuitive explanation and interpretation of the method is set out, followed by a detailed description of the software best suited to the method. Free or free trial version software has been intentionally chosen, as it allows the reader to better understand the main ideas behind the methods by practising with the exercises in this book. Furthermore, the user has access to a Microsoft Excel spreadsheet containing an ‘implementation’ of each method. Software files and answers to the exercises can be downloaded from the companion website, indicated by the  icon in the book. The selected software and exercises allow the user to observe the impact of changes to the data on the results. The use of software enables the decision maker or analyst to communicate and justify decisions in a systematic way.

Each chapter contains a section (‘In the black box’) where scientific references and further reading are indicated for those interested in a more in-depth description or detailed understanding of the methods. Each chapter concludes with extensions of the methods to *other* decision problems, such as group decision or sorting problems.

This first chapter describes the different type of decision problems to be addressed in this book. This is followed by the introduction of the MCDA method best suited to solving these problems along with the corresponding software implementation. As several methods can solve similar problems, a section devoted to choosing an appropriate method has also been included. The chapter concludes with an outline of the book.

1.2 Decision problems

On any one day people face a plethora of different decisions. However, Roy (1981) has identified four main types of decision:

1. The choice problem. The goal is to select the single best option or reduce the group of options to a subset of equivalent or incomparable ‘good’ options. For example, a manager selecting the right person for a particular project.
2. The sorting problem. Options are sorted into ordered and predefined groups, called categories. The aim is to then regroup the options with similar behaviours or characteristics for descriptive, organizational or predictive reasons. For instance, employees can be evaluated for classification into different categories such as ‘outperforming employees’, ‘average-performing employees’ and ‘weak-performing employees’. Based on these classifications, necessary measures can be taken. Sorting methods are useful for repetitive or automatic use. They can also be used as an initial screening to reduce the number of options to be considered in a subsequent step.
3. The ranking problem. Options are ordered from best to worst by means of scores or pairwise comparisons, etc. The order can be partial if incomparable options are considered, or complete. A typical example is the ranking of universities according to several criteria, such as teaching quality, research expertise and career opportunities.

4. The description problem. The goal is to describe options and their consequences. This is usually done in the first step to understand the characteristics of the decision problem.

Additional problem types have also been proposed in the MCDA community:

5. Elimination problem. Bana e Costa (1996) proposed the elimination problem, a particular branch of the sorting problem.
6. Design problem. The goal is to identify or create a new action, which will meet the goals and aspirations of the decision maker (Keeney 1992)

To this list of problems the ‘elicitation problem’ can be added as it aims to elicit the preference parameters (or subjective information) for a specific MCDA method. Moreover, when the problem involves several decision makers, an appropriate group decision method needs to be used.

Many other decision problems exist, often combining several of the problems listed above. However, this book concentrates on the first four decision problems and presents extensions of some of the methods that allow, for example, group, elicitation and description problems also to be addressed.

1.3 MCDA methods

To solve the problems defined in the previous section, *ad hoc* methods have been developed. In this book, the most popular MCDA methods are described along with their variants. Table 1.2 presents these methods and the decision problems they solve. There are many more decision methods than those presented in Table 1.2, but this book confines itself to the most popular methods that have a supporting software package.

Table 1.2 MCDA problems and methods.

Chapter	Choice problems	Ranking problems	Sorting problems	Description problems
2	AHP	AHP	AHPSort	
3	ANP	ANP		
4	MAUT/UTA	MAUT/UTA	UTADIS	
5	MACBETH	MACBETH		
6	PROMETHEE	PROMETHEE	FlowSort	GAIA, FS-Gaia
7	ELECTRE I	ELECTRE III	ELECTRE-Tri	
8	TOPSIS	TOPSIS		
9	Goal Programming			
10	DEA	DEA		
11	Multi-methods platform that supports various MCDA methods			

Table 1.3 MCDA software programs.

Problems	MCDA Methods	Software
Ranking, description, choice	PROMETHEE – GAIA	Decision Lab, D-Sight, Smart Picker Pro , Visual Promethee
Ranking, choice	PROMETHEE ELECTRE UTA AHP ANP MACBETH TOPSIS DEA	DECERNS Electre IS, Electre III-IV Right Choice , UTA+, DECERNS MakeItRational , ExpertChoice, Decision Lens, HIPRE 3+, RightChoiceDSS, Criterium, EasyMind, Questfox, ChoiceResults, 123AHP, DECERNS Super Decisions , Decision Lens M-MACBETH DECERNS Win4DEAP , Efficiency Measurement System, DEA Solver Online, DEAFrontier, DEA-Solver PRO, Frontier Analyst
Choice	Goal Programming	-
Sorting, description	FlowSort - FS-GAIA	Smart Picker Pro
Sorting	ELECTRE-Tri UTADIS AHPSort	Electre Tri , IRIS - -

1.4 MCDA software

Researchers and commercial companies have developed various software programs over the last decade to help users structure and solve their decision problems. The aim of this book is not to describe all existing software, but to narrow the list down to the packages that apply to the methods described. A non-exhaustive list of the programs available is given in Table 1.3. The software packages represented in this book are in bold. Let us remark that the user has access to all the Microsoft Excel spreadsheets on the companion website.

1.5 Selection of MCDA methods

Considering the number of MCDA methods available, the decision maker is faced with the arduous task of selecting an appropriate decision support tool, and often

the choice can be difficult to justify. None of the methods are perfect nor can they be applied to all problems. Each method has its own limitations, particularities, hypotheses, premises and perspectives. Roy and Bouyssou (1993) say that ‘although the great diversity of MCDA procedures may be seen as a strong point, it can also be a weakness. Up to now, there has been no possibility of deciding whether one method makes more sense than another in a specific problem situation. A systematic axiomatic analysis of decision procedures and algorithms is yet to be carried out.’

Guitouni et al. (1999) propose an initial investigative framework for choosing an appropriate multi-criteria procedure; however, this approach is intended for experienced researchers. The next paragraphs give some guidance on selecting an appropriate method according to the decision problem, which will avoid an arbitrary adoption process.

There are different ways of choosing appropriate MCDA methods to solve specific problems. One way is to look at the required input information, that is, the data and parameters of the method and consequently the modelling effort, as well as looking at the outcomes and their granularity (Tables 1.4 and 1.5). This approach is supported by Guitouni et al. (1999).

If the ‘utility function’ for each criterion (a representation of the perceived utility given the performance of the option on a specific criterion) is known, then MAUT (Chapter 4) is recommended. However, the construction of the utility function requires a lot of effort, but if it is too difficult there are alternatives. Another way is by using pairwise comparisons between criteria and options. AHP (Chapter 2) and MACBETH (Chapter 5) support this approach. The difference is that comparisons are evaluated on a ratio scale for AHP and on an interval scale for MACBETH. The decision maker needs to know which scale is better suited to yield their preferences. The drawback is that a large quantity of information is needed.

Another alternative way is to define key parameters. For example, PROMETHEE (Chapter 6) only requires indifference and preference thresholds, whilst ELECTRE (Chapter 7) requires indifference, preference and veto thresholds. There exist so-called elicitation methods to help defining these parameters, but if the user wants to avoid those methods or parameters, TOPSIS (Chapter 8) can be used because only ideal and anti-ideal options are required. If criteria are dependent, ANP (Chapter 3) or the Choquet integral¹ can be used.

The modelling effort generally defines the richness of the output. One advantage to defining utility functions is that the options of the decision problem have a global score. Based on this score, it is possible to compare all options and rank them from best to worst, with equal rankings permitted. This is defined as a complete ranking. This approach is referred to as the full aggregation approach where a bad score on one criterion can be compensated by a good score on another criterion.

Outranking methods are based on pairwise comparisons. This means that the options are compared two-by-two by means of an outranking or preference degree. The preference or outranking degree reflects how much better one option is than

¹ This method has not been described in this book because it is not supported by a software package.

Table 1.4 Required inputs for MCDA ranking or choice method.


	Inputs	Effort input	MCDA method	Output
Ranking/choice problem	utility function	Very HIGH  Very LOW	MAUT	Complete ranking with scores
	pairwise comparisons on a ratio scale and interdependencies		ANP	Complete ranking with scores
	pairwise comparisons on an interval scale		MACBETH	Complete ranking with scores
	pairwise comparisons on a ratio scale		AHP	Complete ranking with scores
	indifference, preference and veto thresholds		ELECTRE	Partial and complete ranking (pairwise outranking degrees)
	indifference and preference thresholds		PROMETHEE	Partial and complete ranking (pairwise preference degrees and scores)
	ideal option and constraints		Goal programming	Feasible solution with deviation score
	ideal and anti-ideal option		TOPSIS	Complete ranking with closeness score
no subjective inputs required		DEA	Partial ranking with effectiveness score	

Table 1.5 Required inputs for MCDA sorting methods.

	Inputs	Effort Input	MCDA method	Output
Sorting method	utility function	HIGH	UTADIS	Classification with scoring
	pairwise comparisons on a ratio scale	↕	AHPSort	Classification with scoring
	indifference, preference and veto thresholds		ELECTRE-TRI	Classification with pairwise outranking degrees
	indifference and preference thresholds	LOW	FLWSORT	Classification with pairwise outranking degrees and scores

another. It is possible for some options to be incomparable. The comparison between two options is difficult as they have different profiles: one option may be better based on one set of criteria and the other better based on another set of criteria. These incomparabilities mean that a complete ranking is not always possible, which is referred to as a partial ranking. The incomparability is a consequence of the non-compensatory aspect of those methods. When facing a decision problem, it is important to define the type of output required from the beginning (presented in Tables 1.4 and 1.5).

Goal programming and data envelopment analysis (DEA) are also part of the MCDA family but are used in special cases. In goal programming, an ideal goal can be defined subject to feasibility constraints. DEA is mostly used for performance evaluation or benchmarking, where no subjective inputs are required.

1.6 Outline of the book

Following this introduction, in which general concepts of MCDA are explained, nine chapters describe the major MCDA methods. Each chapter can be read independently, and they are grouped into three sections, according to their approach:

- *Full aggregation approach* (or American school). A score is evaluated for each criterion and these are then synthesized into a global score. This approach assumes compensable scores, i.e. a bad score for one criterion is compensated for by a good score on another.
- *Outranking approach* (or French school). A bad score may not be compensated for by a better score. The order of the options may be partial because the notion of incomparability is allowed. Two options may have the same score, but their behaviour may be different and therefore incomparable.

- *Goal, aspiration or reference level approach.* This approach defines a goal on each criterion, and then identifies the closest options to the ideal goal or reference level.

Most chapters are divided into four sections, with the exception of specific MCDA methods, as extensions do not exist. Specific objectives are as follows:

- *Essential concepts.* The reader will be able to describe the essentials of the MCDA method.
- *Software.* The reader will be able to solve MCDA problems using the corresponding software.
- *In the black box.* The reader will understand the calculations behind the method. An exercise in Microsoft Excel facilitates this objective.
- *Extensions.* The reader will be able to describe the extensions of the MCDA methods to other decision problems, such as sorting or group decisions.

The book concludes with a description of the integrated software DECERNS, which incorporates six MCDA methods and a Geographical Information System. Linear programming, the underlying method for MACBETH and goal programming, is explained in the Appendix.

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Part I

FULL AGGREGATION APPROACH

2

Analytic hierarchy process

2.1 Introduction

This chapter explains the theory behind and practical uses of the analytic hierarchy process (AHP) method as well as its extensions. *MakeItRational*, a software package that helps to structure problems and calculate priorities using AHP, is described. Section 2.3 is designed for readers interested in the methodological background of AHP. Section 2.4 covers the extensions of AHP in group decision, sorting, scenarios with incomparability and large size problems.

The companion website provides illustrative examples with *Microsoft Excel*, and case studies and examples with *MakeItRational*.

2.2 Essential concepts of AHP

AHP was developed by Saaty (1977, 1980). It is a particularly useful method when the decision maker is unable to construct a utility function, otherwise MAUT is recommended (Chapter 4). To use AHP the user needs to complete four steps to obtain the ranking of the alternatives. As with any other MCDA method, the problem first has to be structured (Section 2.2.1). Following this, scores – or priorities, as they are known in AHP – are calculated based on the pairwise comparisons provided by the user (Section 2.2.2). The decision maker does not need to provide a numerical judgement; instead a relative verbal appreciation, more familiar to our daily live, is sufficient. There are two additional steps that can be carried out: a consistency check (Section 2.2.3) and a sensitivity analysis (Section 2.2.4). Both steps are optional but recommended as confirmation of the robustness of the results. The consistency check is common in all methods based on pairwise comparisons like AHP. The supporting software of *MakeItRational* facilitates the sensitivity analysis.

2.2.1 Problem structuring

AHP is based on the motto *divide and conquer*. Problems that require MCDA techniques are complex and, as a result, it is advantageous to break them down and solve one 'sub-problem' at a time. This breakdown is done in two phases of the decision process during:

- the problem structuring and
- the elicitation of priorities through pairwise comparisons.

The problem is structured according to a hierarchy (e.g. Figure 2.2) where the top element is the goal of the decision. The second level of the hierarchy represents the criteria, and the lowest level represents the alternatives. In more complex hierarchies, more levels can be added. These additional levels represent the sub-criteria. In any case, there are a minimum of three levels in the hierarchy.

Throughout this chapter, a shop location problem (Case Study 2.1) will be considered to illustrate the different steps of the AHP process.

Case Study 2.1

A businessman wants to open a new sports shop in one of three different locations:

- (a) **A shopping centre.** The shopping centre has a high concentration of a variety of shops and restaurants. It is a busy area, with a mix of customers and people walking around. Shops regularly use large displays and promotions to attract potential customers. As demand for these retail units is low, the rental costs are reasonable.
- (b) **The city centre.** The city centre is a busy area, and a meeting point for both young people and tourists. Attractions such as dance shows, clowns and market stalls are often organized, which attract a variety of visitors. The city centre has several small shops located at ground level in historical buildings, which suggests high rental costs. These shops have a high number of customers and are often in competition.
- (c) **A new industrial area.** The new industrial estate is in the suburbs of the city, where several businesses have recently been set up. Some buildings have been earmarked for small shops, but on the whole it has been difficult to attract tenants, which means that rental costs are currently low. Customers of the existing shops mainly work in the area and only a few customers come from the surrounding towns or cities to shop here.

Given the description of the problem, four criteria will be considered in making the final decision (Table 2.1).