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Mechanisms,
Transmissions and
Applications
Proceedings of the Third MeTrApp Conference 2015

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Burkhard Corves • Erwin-Christian Lovasz Mathias Hüsing Editors

# Mechanisms, Transmissions and Applications 

Proceedings of the Third MeTrApp Conference 2015

Springer

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## Preface

MeTrApp 2015 is already the third edition of a conference that started in 2011 as a workshop organized by the University of Timisoara in Romania. The second edition was organized as a conference in Bilbao, Spain, by the University of the Basque Country. Now this book is already the third of its kind presenting the collection of scientific papers that were presented on the occasion of the Third Conference on Mechanisms, Transmissions and Applications organized by RWTH Aachen University in Aachen, Germany.

The driving force behind this now well-established conference series is the International Federation for the Promotion of Mechanism and Machine Science namely its two Technical Committees "Linkages and Mechanical Controls" and "Gearing and Transmissions".

The aim of this Third Conference on Mechanisms, Transmissions and Applications is to offer a stage for original research presentations for researchers, scientists, industry experts, and students in the fields of mechanisms and transmissions with special emphasis on industrial applications in order to stimulate the exchange of new and innovative ideas. By collecting the peer-reviewed papers that were funnelled through a rigorous two-stage review process, within the Springer Mechanism and Machine Science Series, we take the chance to present and share the outcome of this conference with interested professionals and scientists who are at the front line of mechanism and machine theory. Thus, the content of this book is subdivided into different sections that cover the topics Mechanism and Machine Design, Mechanical Transmissions, Industrial Applications, VDI-Guidelines, Biomechanics and Medical Engineering, Robotics, Mechatronics, and Dynamics of Mechanisms and Machines.

In total, we received 41 papers, which were carefully reviewed by three reviewers per paper in a double-review process. Finally, 35 papers were accepted for presentation during the conference and for publication in this book. Thus, we want to express our thanks to the reviewers who contributed to this process with their experience and scientific background. Only through their effort was it possible to organize a thorough yet speedy review process.

Many thanks also go to the authors for their enthusiasm about this conference and to all who helped in organizing this publication as well as the conference itself. We thank the German Research Foundation for awarding a grant in order to host this conference, as well as RWTH Aachen University for supporting this conference. We also thank the staff at Springer for their support through all stages of preparing this book.

We very much hope that this book also inspires those who could not attend the conference to contribute or attend one of the next issues of MeTrApp.

February 2015
Burkhard Corves
Erwin-Christian Lovasz
Mathias Hüsing

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## Part I <br> Mechanism and Machine Design

# The Infinitesimal Burmester Lines in Spatial Movement 

Delun Wang, Wei Wang, Huimin Dong and Son Lin


#### Abstract

The paper studies the infinitesimal Burmester lines of the moving body at any instant, according to the invariance of the constraint ruled surface of the binary link C-C. A line-trajectory is expressed by the invariants of axodes of a rigid body in spatial motion. The Euler-Savary analogue of a line-trajectory in spatial movement is described in the Frenet frame of axodes. Both the stationary line congruence of constant axis curvature and the Ball line are revealed. The degenerated cases of the infinitesimal Burmester lines, the characteristic lines $L_{\mathrm{HC}}$ and $L_{\mathrm{RC}}$, are discussed according to the H-C curvature and R-C curvature. An numerical example of a spatial linkage RCCC is given to show some of above results.


Keywords Curvature theory • Ruled surface • Invariant • Axode • Kinematics

## 1 Introduction

The kinematic synthesis of linkages is essentially to locate the special points or lines in the moving body, which trace constraint curves or ruled surfaces of the binary links with specific curvature properties, such as the Burmester points in planar motion. The curvature theory provides the clues to locate the special lines in the moving body by estimating the approximation of a general ruled surface and a

[^0]constraint ruled surface. The infinitesimal Burmester lines in spatial motion are not completely defined and readily located yet in the literature. The constraint curves and ruled surfaces in spatial linkages have been introduced in Ref. [1]. Distelli [2] previously presented the Euler-Savary equation of the straight line in spatial motion. The dual numbers, dual vectors and dual matrices are adopted to study the trajectories of line in space. Yang [3] defined characteristic scalars to reflect the infinitesimal properties of ruled surface. Kose [4] adopted dual vector calculus to study the invariants of a line trajectory in spatial motion. The canonical coordinate system was adopted by Veldkamp [5], Kirson [6], Roth [7] to study instantaneous spatial motion. Ting [8] presented a unified algebraic approach for the modeling of the instantaneous motion of all linear elements. McCarthy [9] derived three curvature parameters, invariants of a ruled surface, to study their local shape by the moving frame.

For the complexity of line-trajectories, the kinematic invariants of a ruled surface, three construction parameters [10], are naturally preferred to describe the local geometrical properties of a ruled surface. The construction parameters of both the moving and fixed axodes are used to reveal the intrinsic property of a rigid body in spatial motion, which occupy a similar important fundamental roles in spatial kinematics $[11,12]$ both in the expressional forms and the contents as the centrodes do in planar motion. Hence, this paper studies the infinitesimal Burmester line of the moving body in spatial motion at an instant, corresponding to binary link C-C, in terms of the invariance of the constraint ruled surfaces [1].

## 2 Constraint Ruled Surfaces of Spatial Linkages

In linkages, a binary link with two kinematic pairs kinematically connects a moving body to a base link, or fixed frame. The line of the moving body, or the axis/ guidance line of a moving joint, is constrained and defined as a characteristic line, and its trajectory in the base link is called a constraint ruled surface.

For a binary link R-C, the moving joint C is a cylindrical pair and the fixed joint R is a revolute pair. Its constraint ruled surface, a hyperboloid of one sheet, is $\Sigma_{\mathrm{RC}}$ with $L_{\mathrm{RC}}$ as the characteristic line. The binary links with fixed joint R, such as R-C, R-R, R-H and R-P, have the same constraint ruled surface, or correspond to the same characteristic line on the moving body. Therefore, a cylindrical pair C can be taken as a typical moving joint for the binary links with moving joints $\mathrm{C}, \mathrm{R}, \mathrm{H}$ and P. Hereinafter, all constraint ruled surfaces are represented by a symbol $\Sigma$ with subscripts. With C-pair as the moving joint of the binary links, three current kinematic joints, C, R, H, of spatial linkages are listed in Table 1.

Table 1 Binary (generalized) links and their constraint surfaces

| Binary links | Constraint ruled surfaces | Symbols of lines | Symbols of surfaces |
| :--- | :--- | :--- | :--- |
| C-C | Constant axis ruled surface | $L_{\mathrm{CC}}$ | $\Sigma_{\mathrm{CC}}$ |
| H-C | Helicoid | $L_{\mathrm{HC}}$ | $\Sigma_{\mathrm{HC}}$ |
| R-C | Hyperboloid | $L_{\mathrm{RC}}$ | $\Sigma_{\mathrm{RC}}$ |

## 3 Infinitesimal Burmester Lines

In order to describe the spatial movement of a rigid body relative to the fixed body, the moving Cartesian coordinate system $\left\{\mathbf{O}_{m} ; \mathbf{i}_{m}, \mathbf{j}_{m}, \mathbf{k}_{m}\right\}$ is established on the moving body and the fixed system $\left\{\mathbf{O}_{f}, \mathbf{i}_{f}, \mathbf{j}_{f}, \mathbf{k}_{f}\right\}$ is on the fixed body. As we known, there is an instantaneous screw axis (ISA) for the spatial motion of the moving body relative to the fixed body. For all instants, the ISA traces a moving axode $\Sigma_{m}$ in $\left\{\mathbf{O}_{m} ; \mathbf{i}_{m}, \mathbf{j}_{m}, \mathbf{k}_{m}\right\}$ and fixed axode $\Sigma_{f}$ in $\left\{\mathbf{O}_{f} ; \mathbf{i}_{f}, \mathbf{j}_{f}, \mathbf{k}_{f}\right\}$.

### 3.1 The Axodes of a Rigid Body in Spatial Movement

The fixed axode $\Sigma_{f}$ is a ruled surface, whose vector expression can be written by an adjoint approach [11] in $\left\{\mathbf{O}_{f} ; \mathbf{i}_{f}, \mathbf{j}_{f}, \mathbf{k}_{f}\right\}$ as $\mathbf{R}_{f}=\boldsymbol{\rho}_{f}+\mu \mathbf{s}_{f}$, where $\boldsymbol{\rho}_{f}$ is the vector of the striction curve of $\Sigma_{f}, \mathbf{s}_{f}$ is the unit vector of the generator of $\Sigma_{f}$. The construction parameters $\alpha_{f}, \beta_{f}, \gamma_{f}$ of $\Sigma_{f}$ can be derived by Frenet formulas [10].

On the other hand, the moving axode $\Sigma_{m}$ is also a ruled surface, whose vector expression can be written in $\left\{\mathbf{O}_{m} ; \mathbf{i}_{m}, \mathbf{j}_{m}, \mathbf{k}_{m}\right\}$ as $\mathbf{R}_{m}=\boldsymbol{\rho}_{m}+\mu \mathbf{s}_{m}$, where $\boldsymbol{\rho}_{m}$ is the vector of the striction curve of $\Sigma_{m}, \mathbf{s}_{m}$ is the unit vector of the generator of $\Sigma_{m}$. The construction parameters $\alpha_{m}, \beta_{m}, \gamma_{m}$ of $\Sigma_{m}$ can be derived by Frenet formulas. The induced construction parameters $\alpha^{*}, \beta^{*}, \gamma^{*}$, as key kinematic invariants for the spatial movement, can be obtained as $\alpha^{*}=\alpha_{f}-\alpha_{m}, \beta^{*}=\beta_{f}-\beta_{m}, \gamma^{*}=\gamma_{f}-\gamma_{m}=0$. The arc lengths of the spherical image curves of $\mathbf{s}_{f}$ and $\mathbf{s}_{m}$ are respectively $\sigma_{f}$ and $\sigma_{m}$.

We designate $\sigma$ to represent $\sigma_{f}$ and $\sigma_{m}$ for short since they are equal to each other. The properties of axodes are presented in Ref. [11], which lay the groundwork for the following spatial kinematics in this paper.

### 3.2 A Line-Trajectory and Its Frenet Frame

For a moving body in spatial motion, a point $P$ with Cartesian coordinates $\left(x_{P m}, y_{P m}, z_{P m}\right)$ in $\left\{\mathbf{O}_{m} ; \mathbf{i}_{m}, \mathbf{j}_{m}, \mathbf{k}_{m}\right\}$ traces a spatial trajectory $\Gamma_{P}$ in $\left\{\mathbf{O}_{f} ; \mathbf{i}_{f}, \mathbf{j}_{f}, \mathbf{k}_{f}\right\}$. A line $L$ with unit directional vector $\boldsymbol{l}\left(\delta_{l}, \theta_{l}\right)$ in Frenet frame $\left\{\boldsymbol{\rho}_{m} ; \mathbf{E}_{1 m}, \mathbf{E}_{2 m}, \mathbf{E}_{3 m}\right\}$ of $\Sigma_{m}$ passes through $P . \delta_{l}$ is the inclined angle between $\mathbf{l}$ and $\mathbf{E}_{1 m}$, and $\theta_{l}$ is the

Fig. 1 A line-trajectory adjoint to ISA

directional angle of the projection vector of $\mathbf{I}$ on the plane $\boldsymbol{\rho}_{m}-\mathbf{E}_{2 m} \mathbf{E}_{3 m}$. The position of $L$ is located by parameters $(p, h)$ in $\left\{\boldsymbol{\rho}_{m} ; \mathbf{E}_{1 m}, \mathbf{E}_{2 m}, \mathbf{E}_{3 m}\right\}$, as shown in Fig. 1.

At an instant, $L$ corresponds to ISA and traces a trajectory-ruled surface $\Sigma_{l}$ in $\left\{\mathbf{O}_{f}, \mathbf{i}_{f}, \mathbf{j}_{f}, \mathbf{k}_{f}\right\}$, which can be expressed by

$$
\left\{\begin{array}{l}
\Sigma_{l}: \mathbf{R}_{l}=\mathbf{\rho}_{l}+\mu \mathbf{l}=\boldsymbol{\rho}_{f}+p \mathbf{E}_{1 f}+H \mathbf{E}_{1 f} \times \mathbf{E}_{1}+\mu \mathbf{l},  \tag{1}\\
H=h / \sqrt{1-l_{1}^{2}}, \quad \mathbf{l}=\left(l_{1}, l_{2}, l_{3}\right)=\left(\cos \delta_{l}, \sin \delta_{l} \cos \theta_{l}, \sin \delta_{l} \sin \theta_{l}\right)
\end{array}\right.
$$

The line $L$ is described by the four parameters $\left(l_{1}, l_{2}, p, h\right)$ in Frenet frame $\left\{\boldsymbol{\rho}_{f}\right.$; $\left.\mathbf{E}_{1 f}, \mathbf{E}_{2 f}, \mathbf{E}_{3 f}\right\}$ of $\Sigma_{f}$. The differential of the arc length $\sigma_{l}$ of the spherical image curve of $\mathbf{I}$ is $\mathrm{d} \sigma_{l}=|\mathrm{d} \mathbf{l} / \mathrm{d} \sigma| \mathrm{d} \sigma=\beta^{*} \sqrt{1-l_{1}^{2}} \mathrm{~d} \sigma$. The vector equation of striction curve of $\Sigma_{l}$ is $\boldsymbol{\rho}_{l}=\boldsymbol{\rho}_{f}+p \mathbf{E}_{1 f}+h \mathbf{E}_{1 f} \times \mathbf{E}_{1} /\left|\mathbf{E}_{1 f} \times \mathbf{E}_{1}\right|$ and the derivative of the striction directrix distance $b_{l}$ is $\mathrm{d} b_{l} / \mathrm{d} \sigma=\left(l_{2} p-l_{1} l_{3} H+l_{3} \gamma\right) /\left(1-l_{1}^{2}\right)$. The Frenet frame $\left\{\boldsymbol{\rho}_{l} ; \mathbf{E}_{1}, \mathbf{E}_{2}, \mathbf{E}_{3}\right\}$ of $\Sigma_{l}$ can be established, and the three construction parameters can be derived by the Frenet formulas of a ruled surface as

$$
\begin{equation*}
\alpha_{l}=\frac{l_{1} \alpha^{*}-\beta^{*}\left(1-l_{1}^{2}\right) H+d b_{l} / d \sigma}{\beta^{*}\left(1-l_{1}^{2}\right)^{1 / 2}}, \quad \beta_{l}=\frac{l_{3}+\beta^{*} l_{1}\left(1-l_{1}^{2}\right)}{\beta^{*}\left(1-l_{1}^{2}\right)^{3 / 2}}, \quad \gamma_{l}=\frac{\alpha^{*}}{\beta^{*}}+l_{1} H \tag{2}
\end{equation*}
$$

### 3.3 The Infinitesimal Burmester Lines

### 3.3.1 Constant Axis Curvature

Based on the properties of the constraint ruled surface, we can discuss the infinitesimal Burmester lines of the rigid body in spatial movement. As presented in Ref. [1], the constant axis curvature or the $\mathrm{C}-\mathrm{C}$ curvature includes $\beta_{l}=$ constant and

Fig. 2 Euler-Savary analogue of a line-trajectory

$\alpha_{l}-\beta_{l} \cdot \gamma_{l}=$ constant. This implies the spherical image curve of $\Sigma_{l}$ is a circle, and the striction curve of $\Sigma_{l}$ is a cylindrical curve.

For the constant axis curvature surface $\Sigma_{\mathrm{CC}}$ of $\Sigma_{l}$, the position can be located by the normal of $\Sigma_{l}$ at the striction point, or the fixed point $A$ on the fixed axis $L_{A}$ of $\Sigma_{\mathrm{CC}}$. The position vector of the fixed point $A$ is

$$
\begin{equation*}
\mathbf{R}_{A}=\boldsymbol{\rho}_{l}+\eta \mathbf{E}_{2}, \quad \eta=\left(\alpha_{l}-\beta_{l} \gamma_{l}\right) /\left(1+\beta_{l}^{2}\right) \tag{3}
\end{equation*}
$$

where $\eta$ is the distance between the fixed point $A$ and the striction point $B$ along the common normal, as shown in Fig. 2. Based on the Eq. (3), both the direction and the position of $\Sigma_{\mathrm{CC}}$ can be expressed as

$$
\left\{\begin{array}{l}
\cot \left(\delta-\delta_{l}\right)+\cot \delta_{l}=-\beta^{*} / \sin \theta_{l}  \tag{4}\\
\frac{\eta+h}{\sin ^{2}\left(\delta-\delta_{l}\right)}-\frac{h}{\sin ^{2} \delta_{l}}=-\frac{\alpha^{*}-\beta^{*} \gamma}{\sin \theta_{l}}+\frac{\cos \theta_{l}}{\sin ^{2} \theta_{l}} \beta^{*} p
\end{array}\right.
$$

At an instant, the Eq. (4) is called the Euler-Savary analogue of a line-trajectory in spatial movement.

### 3.3.2 Stationary Line Congruence

If a line-trajectory $\Sigma_{l}$ contacts $\Sigma_{\mathrm{CC}}$ in the third order, the condition equations $\mathrm{d} \beta_{l} /$ $\mathrm{d} \sigma=0$ and $\mathrm{d} \eta / \mathrm{d} \sigma=0$ should be satisfied, which are

$$
\left\{\begin{array}{l}
\cot \delta_{l}=1 /\left(M \sin \theta_{l}\right)+1 /\left(N \cos \theta_{l}\right)  \tag{5}\\
a_{11} p+a_{12} H+a_{13}=0
\end{array}\right.
$$

where $1 / M=\left(\beta_{m}-\beta^{*}\right) / 3,1 / N=\left(\mathrm{d} \beta^{*} / \mathrm{d} \sigma\right) / 3 \beta^{*}$. The coefficients $a_{11}, a_{12}, a_{13}$ are all functions of $\left(l_{1}, l_{2}, l_{3}\right)$ and the induced construction parameters of axodes. All such lines of the moving body constitute stationary line congruence of constant axis
curvature. Any line of the line congruence has a constant inclination angle and the equivalent distance with a fixed line at four infinitesimal positions.

In particular, if the line-trajectory $\Sigma_{l}$ has the properties that $\beta_{l}=0, \eta=0, \mathrm{~d} \beta_{l} /$ $\mathrm{d} \sigma=0$ and $\mathrm{d} \eta / \mathrm{d} \sigma=0$, the line $L$ is regarded as Ball line. It intersects with a fixed line orthogonally at four infinitesimal positions. The directions of the Ball lines are determined by an eight-degree algebraic equation of $\cos \delta_{l}$, which can be solved to get the eight sets of directional angles. Actually, there may exist four directions of Ball lines due to the symmetrical spherical image of unit direction vector of lines.

### 3.3.3 Infinitesimal Burmester Lines

A line $L$ of a rigid body in spatial motion, whose trajectory $\Sigma_{l}$ has the properties as $\mathrm{d} \beta_{l} / \mathrm{d} \sigma=0, \mathrm{~d} \eta / \mathrm{d} \sigma=0, \mathrm{~d}^{2} \beta_{l} / \mathrm{d} \sigma^{2}=0$ and $\mathrm{d}^{2} \eta / \mathrm{d} \sigma^{2}=0$, is called constant axis line, or infinitesimal Burmester line. The condition equations are

$$
\left\{\begin{array}{l}
\cot \delta_{l}=1 /\left(M \sin \theta_{l}\right)+1 /\left(N \cos \theta_{l}\right)  \tag{6}\\
\left(1+\tan ^{2} \theta_{l}\right)\left[\frac{2-\beta_{m} M}{M^{2}}+\frac{d M / d \sigma+3 M / N}{M^{2}} \tan \theta_{l}+\frac{1+d N / d \sigma}{N^{2}} \tan ^{2} \theta_{l}\right. \\
\left.+\frac{\beta_{m}-1 / M}{N} \tan ^{3} \theta_{l}-\frac{1}{N^{2}} \tan ^{4} \theta_{l}\right]+\tan ^{2} \theta_{l}=0 \\
a_{11} p+a_{12} H+a_{13}=0 \\
a_{21} p+a_{22} H+a_{23}=0
\end{array}\right.
$$

where $a_{21}, a_{22}, a_{23}$ are also functions of $\left(l_{1}, l_{2}, l_{3}, \alpha^{*}, \beta^{*}, \gamma^{*}\right)$. At any instant, there exist at most six directions of the Burmester lines in the moving body, whose five infinitesimal successive positions locate on a constant axis ruled surface $\Sigma_{\mathrm{CC}}$.

If the fixed joint C is replaced by H -pair or R-pair, the constant axis ruled surface $\Sigma_{\text {CC }}$ degenerates to be a constant parameter surface in geometrical shape, such as a helicoid and a hyperboloid of one sheet, denoted by $\Sigma_{\mathrm{HC}}$ and $\Sigma_{\mathrm{RC}}$ respectively, and the characteristic line $L_{\mathrm{CC}}$ becomes $L_{\mathrm{HC}}$ and $L_{\mathrm{RC}}$.

Based on the geometrical properties [1] of $\Sigma_{\mathrm{HC}}, \alpha_{l}=$ const, $\beta_{l}=\mathrm{const}$ and $\gamma_{l}=$ const are referred to as the constant curvature or $\mathbf{H}-\mathbf{C}$ curvature. If a linetrajectory $\Sigma_{l}$ contacts $\Sigma_{\mathrm{HC}}$ in second order, the condition equation $\mathrm{d} \gamma_{l} / \mathrm{d} \sigma=0$ should be satisfied. There exist infinite $\left(\infty^{3}\right)$ such lines in the moving body at an instant. If a line-trajectory $\Sigma_{l}$ contacts $\Sigma_{\mathrm{HC}}$ in third order, the contact conditions are $\mathrm{d} \alpha_{l} /$ $\mathrm{d} \sigma=\mathrm{d} \beta_{l} / \mathrm{d} \sigma=\mathrm{d} \gamma_{l} / \mathrm{d} \sigma=0$ and $\mathrm{d}^{2} \gamma_{l} / \mathrm{d} \sigma^{2}=0$. There exist finite such lines in the moving body at an instant.

Based on the geometrical properties [1] of $\Sigma_{\mathrm{RC}}, \alpha_{l}=$ const, $\beta_{l}=$ const, $\gamma_{l}=$ const and $\alpha_{l} \beta_{l}+\gamma_{l}=0$ are defined as the hyperbolic curvature or $\mathbf{R}$-C curvature. If a linetrajectory $\Sigma_{l}$ contacts $\Sigma_{\mathrm{RC}}$ in second order, the condition equations $\alpha_{l} \beta_{l}+\gamma_{l}=0$ and $\mathrm{d} \gamma_{l} / \mathrm{d} \sigma=0$ should be satisfied. There exist infinite such lines in the moving body at an instant. If a line-trajectory $\Sigma_{l}$ contacts $\Sigma_{\mathrm{RC}}$ in third order, the contact conditions are $\alpha_{l} \beta_{l}+\gamma_{l}=0, \mathrm{~d} \alpha_{l} / \mathrm{d} \sigma=\mathrm{d} \beta_{l} / \mathrm{d} \sigma=\mathrm{d} \gamma_{l} / \mathrm{d} \sigma=0$ and $\mathrm{d}^{2} \gamma_{l} / \mathrm{d} \sigma^{2}=0$. The line may not exist because four parameters of the line should meet five constraint equations.

## 4 Numerical Example

A spatial RCCC linkage with parameters $\alpha_{01}=30^{\circ}, \alpha_{12}=55^{\circ}, \alpha_{23}=45^{\circ}, \alpha_{30}=60^{\circ}$, $a_{0}=5, a_{1}=2, a_{2}=4, a_{3}=3, h_{0}=0$ is given in Ref. [13]. The Cartesian coordinate systems are respectively built up on different links by the Denavit-Hartenberg appointment. With the motion of the linkage determined by the input angle $\theta_{1}$, the ISA traces the moving axode $\Sigma_{m}$ and fixed axode $\Sigma_{f}$ in the coupler link and frame link. At the instant $\theta_{1}=1.00$, the construction parameters of $\Sigma_{f}$ and $\Sigma_{m}$ are $\alpha_{f}=-3.4818, \beta_{f}=0.4993, \gamma_{f}=0.8023$ and $\alpha_{m}=-6.0744, \beta_{m}=-0.0498, \gamma_{m}=0.8023$, so the induced construction parameters are $\alpha^{*}=2.5926$ and $\beta^{*}=0.5490$.

For the coupler link 2 of the RCCC linkage, there are at most six infinitesimal Burmester lines at any instant, whose positions and orientations can be calculated through Eq. (6). At the instant $\theta_{1}=1.00$, we can locate four infinitesimal Burmester lines in the coupler link, whose parameters are listed in Table 2.

In order to show the positions of the Burmester lines, we chose the coordinate plane $\mathbf{R}_{\mathrm{C}}-\mathbf{x}_{2} \mathbf{y}_{2}$ of the coupler link as a reference plane, as shown in Fig. 3a. The four Burmester lines intersect the reference plane at four reference points, as shown in Fig. 3b. Obviously, two of the four Burmester lines are just the two axes $\mathbf{z}_{1}$ and $\mathbf{z}_{2}$ of the C-pairs of the coupler link with points $B^{\prime}$ and $C$ as the reference points.

Table 2 Parameters of the Burmester lines at instant $\theta_{1}=1.00$

| Burmester <br> lines | Parameters $\left(\delta_{l}, \theta_{l}, p, H\right)$ in Frenet <br> frame of $\Sigma_{m}$ |  |  |  |
| :--- | :--- | :--- | :--- | ---: |
|  | $\delta_{l}$ | $\theta_{l}$, | $p$ | $H$ |
| $L_{B 1}$ | 1.1305 | 4.5703 | 5.2179 | 2.9518 |
| $L_{B 2}$ | 0.7009 | 6.1193 | 4.5109 | 11.7035 |
| $L_{B 3}$ | 1.3182 | 5.5787 | 0.0982 | 2.2858 |
| $L_{B 4}$ | 1.3262 | 5.5571 | 13.8430 | 7.2734 |



Fig. 3 The orientation and position of the infinitesimal Burmester lines. a RCCC linkage, b Instantaneous Burmester lines

## 5 Conclusions

Based on the constraint ruled surfaces of the binary link C-C in spatial linkages and their generalized curvatures, the Burmester lines of a rigid body at five infinitesimal successive spatial positions are defined for the binary link C-C and concisely located in the Frenet frame of axodes. There are at most six infinitesimal Burmester lines at an instant for a spatial motion. The curvatures of a line trajectory in spatial movement are developed to the generalized curvatures for the constraint ruled surfaces of the binary links in spatial linkages, which provide a solid ground for the curvature theory of the line trajectory in spatial movement.

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# Educational and Research Kinematic Capabilities of GIM Software 

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#### Abstract

In this paper an educational and research software named GIM is presented. This software has been developed with the aim of approaching the difficulties students usually encounter when facing up to kinematic analysis of mechanisms. A deep understanding of the kinematic analysis is necessary to go a step further into design and synthesis of mechanisms. In order to support and complement the theoretical lectures, GIM software is used during the practical exercises, serving as an educational complementary tool reinforcing the knowledge acquired by the students.


Keywords Motion simulation - Computational kinematics • Mechanism synthesis - General purpose software

## 1 Introduction

In the teaching of subjects related to Machine Theory, supporting and complementing theoretical lectures with a simulation and analysis software, helps the students to understand deeply and visually the theoretical bases of the Mechanisms

[^1]Science. In the Department of Mechanical Engineering of the University of the Basque Country (UPV/EHU) two main Bachelor subjects can be highlighted in this field: Applied Mechanics [1, 2] and Kinematics of Mechanisms [3]. In these subjects, GIM software is used.

GIM is a registered software created by the COMPMECH Research Group (www.ehu.es/compmech). The software has been developed focusing, not only on educational purposes but also on research in the field of computational kinematics and mechanism design applications. The software presented in this article also has potential to be used by students of Master Degrees in Mechanical Engineering and other subjects related to Robotics, Mechanism Design, etc.

GIM has been developed in a modular structure. After defining the kinematic structure of a linkage in the Geometry module, the user can perform the motion simulation in the Motion module. GIM is mainly oriented to the field of kinematic analysis, motion simulation and dimensional synthesis of planar mechanisms. In any case, it also includes other modules for workspace and singularity evaluation [4] and static analysis of mechanical structures.

Currently other Universities are using different kinematic softwares during their lessons. In RWTH Aachen, IGM students use the interactive geometry software Cinderella (freeware tool provided by Springer) during mechanism lectures [5]. Other Spanish Universities use well known commercial softwares as GeoGebra [6] or ADAMS [7]. In this paper, we will present the main GIM capabilities used mainly by Bachelor and Master student but also by some of our PhD students.

## 2 Kinematic Analysis

In this section are presented and briefly described the main capabilities of the socalled Motion module. This module is able to simulate the motion of any n-dof planar mechanism with any kind of revolute and prismatic joints. Also the disk-disk and disk-line rolling and cam contacts can be modelled.

### 2.1 Position, Velocity and Acceleration Problems

Figure 1 shows an example of the most elemental results that can be depicted when the motion of a mechanism is obtained and simulated. Obviously, once the geometry has been defined, the first step to compute the motion is to define as many actuators in the mechanism as degrees of freedom. There are different types of actuators, i.e. rotary ones, linear ones, as well as several input function types, i.e. polynomial or sinusoidal, available to control the position, velocity and acceleration of each actuator.

The trajectories, velocities (v) and accelerations (a) of any point, and the angular velocities ( $\omega$ ) and accelerations ( $\alpha$ ) of all elements can be drawn. Also the center of


Fig. 1 General motion simulation results


Fig. 2 Relative motion composition in velocities and accelerations
curvature ( C ) of the trajectory and the intrinsic components of the acceleration $\left(\mathrm{a}_{\mathrm{T}}, \mathrm{a}_{\mathrm{N}}\right)$ can be represented. From an academic point of view, the compliance of all the well-known properties of these magnitudes can be directly observed and understood along the whole motion, e.g. the velocity is always tangent to the trajectory, or the normal acceleration points always towards center of curvature.

All motion results can be obtained not only in the fixed frame, but also with respect to any relative reference. Such a capability, as shown in Fig. 2, provides the best support for explaining (or understanding) the frame ( $\mathrm{v}_{\mathrm{F}}, \mathrm{a}_{\mathrm{F}}$ ) and relative ( $\mathrm{v}_{\mathrm{R}}, \mathrm{a}_{\mathrm{R}}$ ) motion compositions ( $\mathrm{a}_{\mathrm{C}}$ is Coriolis term).

In the same way, the user can check how the specific properties of the rolling motion, with or without sliding, are verified, e.g., as shown in Fig. 3, when a rolling motion with no sliding component is performed, contact points in both elements have the same velocity and the same projection of the acceleration over the contact tangent line. Those properties are not satisfied when exist sliding in the rolling.

### 2.2 Kinematic Geometry

GIM software is able to compute and represent the main kinematic geometrical entities of any element of the mechanism, such as the instantaneous center of rotation $(\mathrm{P})$, the pole of accelerations $(\mathrm{Q})$, the fixed and moving centrodes (c.f. and


Fig. 3 Velocities and accelerations of contact points in a pure rolling and in a cam joint


Fig. 4 Rolling motion sequence between the fixed and moving centrodes
c.m.) and the inflection and Bresse circles (c.i. and c.b.). When studying these entities, students have an interactive tool at hand which facilitates a deeper understanding.

For example, as shown in Fig. 4, during the motion simulation, it is checked that the fixed and moving centrodes are always tangent at the instantaneous center of rotation, and the moving centrode moves welded the element, rolling over the fixed one. In the same way, as this tool enables to drag the coupler point of an element to any position in the moving plane, it can be observed that the pole of velocity has acceleration and the pole of acceleration has velocity (Fig. 5, up). Also, any point on the inflection circle is passing through an inflection point on its trajectory and lacks of normal acceleration, as well as any point on the Bresse circle lacks of tangent acceleration (Fig. 5, down).

### 2.3 Advanced Computations

Apart from the basic kinematic geometry, some advances features can be issued. Here are found some examples. Figure 6 shows the envelope of a line that moves fixed to the coupler element of a mechanism. As it is known, in positions where


Fig. 5 Instantaneous centers of rotation and acceleration and inflection and Bresse circles


Fig. 6 Moving line envelope and return circle
such a line passes through the return pole ( R ), the envelope curve has a cusp point (c.r. is the return circle).

For some simple mechanisms, as the 4-bar linkage, the cubic of stationary curvature (c.s.c.) and the pivot point curve (p.p.c.) can be traced. When the coupler point is located at the intersection between the cubic of stationary curvature and the inflection circle (Ball point, B) it is achieved a quasi-straight-line trajectory in the proximity of such a point (Fig. 7).

Fig. 7 Cubic of stationary curvature, pivot point curve and ball point


## 3 Dimensional Synthesis

The Motion module presented in the previous section has a general purpose and can solve a large amount of planar mechanisms. Apart from this module, the program provides another one that deals with the dimensional synthesis of one specific mechanism, the four-bar linkage. Three traditional synthesis problem types have been addressed, i.e. path generation, rigid body guiding and function generation.

### 3.1 Path Generation Synthesis

This synthesis is based on the use of precision points and consists in determining a mechanism whose coupler point trajectory passes exactly through some specific positions. With this software tool, the user can drag the precision points to change their position and the computation of the mechanism that fulfills all conditions is done in real time. This non-linear problem admits many different solutions for the same input values. The designer has the full control to visualize each of them, as depicted in Fig. 8.


Fig. 8 Synthesis with three precision points. Different solutions for the same data


Fig. 9 Synthesis with four precision points. Order error and loop error


Fig. 10 Synthesis with five precision points. Different solutions for the same data

The program offers the option for making the mechanism synthesis being defined three, four, or five precision points of the trajectory. This ability is shown in Figs. 8, 9 and 10. From the academic point of view, one of main advantages is the possibility of checking that some solutions do not accomplish the right order in the sequence of precision points, order error, and also, in the case of Grashof's four bar linkages, could appear the so-called branch error, that means that some points belong to the disconnected path corresponding to the crossed quadrilateral.

Also, the software allows the visualization of the classical geometrical constructions for making the synthesis. User can choose between displaying or not such auxiliary constructions. This feature is valuable for an academic purpose.

### 3.2 Rigid Body Guiding

The solid element guiding synthesis computes the mechanism that is able to fully locate an element in a set of desired postures (position and orientation). Figure 11 shows the three alternatives given in the program. The first one allows the user to specify three postures and the relative positions of the floating joints (the shape of the coupler). Using the second option, apart from three desired postures, positions of fixed joints can be specified. Finally, solid element guiding for four target postures can be done.


Fig. 11 Synthesis for solid element guiding


Fig. 12 Four-bar mechanism cognate and translational mechanisms

### 3.3 Additional Functionalities

In the Synthesis module the user can find some extra features related with the design alternatives based in the four-bar linkage. Some of them are presented in Fig. 12. Apart from the crossed mechanism, cognates, which trace the same coupler trajectory, are obtained. Based on them, the so-called 1-dof translational mechanisms are obtained. The two known versions can be represented: the redundant one and the non-redundant one.

## 4 Conclusions

The software presented in this paper allows students and researchers to model and analyse in a quick and simple way n-dof planar linkages. Using the software capabilities, the user is able to carry out a deep kinematic performance analysis of the whole mechanism. GIM software has proven to be a very effective tool to complement and reinforce the theoretical concepts explained during the lectures of subject related to Mechanism and Machine Science.

GIM software can be freely downloaded from the COMPMECH web site in the following link: www.ehu.es/compmech/software.


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