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Marco Ceccarelli *Editors*

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Editors

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Preface

MEDER 2015, IFToMM International Symposium on Mechanism Design for Robotics, is the third event of a series that started in 2010 as a specific conference activity on mechanisms for robots. The first event was held at Universidad Panamericana de Ciudad de Mexico, Mexico in September 2010 and the second was held at Beihang University, Beijing, China in October 2012.

The aim of the MEDER Symposium is to bring together researchers, industry professionals and students from broad ranges of disciplines dealing with mechanism for robots, in an intimate, collegial and stimulating environment. Again, in the 2015 MEDER the event received increased attention, since the proceedings contain contributions by authors from all over the world.

The proceedings of MEDER 2015 Symposium contains 42 papers, which were selected after review for oral presentation. These papers cover several aspects of the wide field of Mechanism Design for Robotics, from theoretical studies up to practical applications through new robot designs and prototypes. They are authored mainly from the IFToMM community coming from China, Denmark, France, Germany, Italy, Japan, Mexico, The Netherlands, Norway, Russia, Singapore, Spain, UK and USA.

We express grateful thanks to the MEDER Symposium International Scientific Committee members including Marco Ceccarelli, Chair (University of Cassino, Italy), Juan Carretero (University of New Brunswick, Canada), Lu Zhen (Beihang University, China), Pierre Larochelle (Florida Institute of Technology, USA), Ding Xilun (Beihang University, China), Grigore Gogu (French Institute for Advanced Mechanics, France), I-Ming Chen (Nanyang Technological University, Singapore), Mario Acevedo (Panamerican University, Mexico), Teresa Zielinska (Warsaw University of Technology, Poland), Joseph Rooney (Open University, UK), Atsuo Takanishi (Waseda University, Japan), Alfonso Hernandez (Bilbao University, Spain) for cooperating enthusiastically for the success of the MEDER 2015 event.

We thank the authors who contributed with very interesting papers on several subjects, covering many fields of Mechanism Design for Robotics and additionally for their cooperation in revising papers in due time in agreement with the reviewers' comments. We are grateful to the reviewers for the time and efforts they spent in

evaluating the papers within a given schedule that has permitted the publication of this proceedings volume. These reviewers are Zheng-Hua Tan (Aalborg University, Denmark), Xuping Zhang (Aarhus University, Denmark), Marco Ceccarelli and Giuseppe Carbone (University of Cassino and South Latium, Italy), Juan Antonio Carretero (University of New Brunswick, Canada), Grigore GOGU (French Institute of Advanced Mechanics in Clermont-Ferrand, France), John Hayes (Carleton University, Canada), Ronghua Li (Dalian Jiaotong University, China), Carl Nelson (University of Nebraska–Lincoln, USA), Latifah Nurahmi (IRCCyN, France), Alba Perez Gracia (Idaho State University, USA), Teresa Zielinsks (Warsaw University of Technology, Poland), Alfonso Hernandez (University of the Basque Country, Spain), Volkert van der Wijk (University of Twente, The Netherlands) and Tao Li (Institute of Advanced Manufacturing Technology, China).

We thank Aalborg University (AAU), in particular, the Department of Mechanical and Manufacturing Engineering and Aalborg U Robotics, for having hosted the MEDER 2015 event. We would like to thank our colleagues at AAU for their efforts and support in the symposium organizing. We thank the local organizing committee members including Ole Madsen, Jørgen Kepler, Guanglei Wu, Ewa Kolakowska and Christoffer Eg Sloth, who put their prestigious time into the event to make it successful. We also extend our thanks for help by the LARM Laboratory of Robotics and Mechatronics of University of Cassino.

We thank The IFToMM (International Federation for the Promotion of Mechanism and Machine Science) for sponsorship of two Young Delegation Travel grants. The symposium received generous support from local sponsors, namely the Thomas B. Thriges Fund and the Danish RoboCluster, which were critical to make this symposium of low registration cost possible.

We thank the publisher Springer and its Editorial staff for accepting and helping in the publication of this Proceedings volume, since its early steps in 2014.

We are grateful to our families, as without their patience and comprehension it would not have been possible for us to organize MEDER 2015, IFToMM International Symposium on Mechanism Design for Robotics and this Proceedings volume.

Aalborg | Shaoping Bai
Cassino | Marco Ceccarelli
March 2015

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Finger Mechanisms for Robotic Hands

M. Ceccarelli

Abstract The problem of grasping with robots is solved by using suitable finger mechanisms that are inspired from structures in nature. A variety of solutions have been experienced and are used in a variety of designs all around the world. This paper discusses a survey of possibilities by addressing attention to characteristics and problems in the design and operation of those finger mechanisms. The author's experience with LARM hand is reported to show practical results in attaching the problem of improving efficient solutions with better finger mechanisms.

Keywords Artificial hands · Finger anatomy · Finger mechanisms · Kinematic design

1 Introduction

Manipulation of objects with fingered robotic hands is an aspect which can be used in many applications, [6], also in industry and service contexts and it attracts still great interest as indicated in [1, 4, 16].

Since a recent past, in order to develop anthropomorphic finger mechanisms researchers have used two different approaches: complex mechanisms in order to perform manipulation tasks with high dexterity, or design of mechanisms with a reduced number of degrees of freedoms (DOFs) and actuators with less performances but a fairly simplified device operation.

Using underactuated mechanisms it is possible to achieve an adaptive grasp that mimics the human grasping action for which it is possible to consider two kinds of structures, namely using flexibility of links or designing underactuated mechanisms as pointed out in [8]. A mechanism is defined underactuated when its number of

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actuators is smaller than the number of degrees of freedom of the mechanism. It is possible to identify two types of underactuated finger mechanisms, depending on whatever a tendon or a link transmission is used. Example of tendon finger mechanisms is presented in [2], and a pulley-cable solution is described in [3].

When large grasping forces are required, underactuated linkage mechanisms are usually preferred, like in [2] where a 1-DOF mechanism with suitable four-bar linkages and flexible elements is used to move all phalanxes of fingers, in [17] where an underactuated linkage with force control is studied or in [19] where a 5 fingered underactuated prosthetic hand is reported.

Since the end of 1990s at LARM in Cassino design and research activities have been carried out in order to design a low-cost easy-operation robotic hand with anthropomorphic fingers, denominated LARM Hand [5].

In this paper the design of a new underactuated finger mechanism has been proposed for LARM Hand, as focused on requirements referring to 1-DOF, anthropomorphic grasp, and mechanism's compact size.

2 Fingers in Nature

Fingers in nature are related to hands and feet when considering a grasp in a broad sense as a task by which interaction is with an object to be manipulated or with the ground to be in contact with, respectively.

Such a variety of task configurations has generated a variety of finger anatomy but with solutions that still show a common structure. The anatomy of fingers in hands both in humans and primates is illustrated in the example in Fig. 1, [11]. The anatomy is characterized by a bone structure with a serial kinematic configuration and a muscle complex system by which each bone is moved through two or more counteracting muscular tendons. The main terminology of human anatomy is reported in Fig. 1 and it is usually used also for artificial hands.

In Fig. 2, examples of finger configurations are reported as referring to horses and cats with a structure that still shows sequential bones and parallel architecture muscle system. The similarity with human anatomy is recalled by the similar terminology of the parts in those other fingers.

Summarizing, the anatomy of fingers in nature shows the following design characteristics:

- A serial bone-link structure whose main purpose is rigidity and load capability
- An actuation muscle system aiming to rotate each bone-link independently but with coordinated movements with other finger links

The operation of fingers in nature is devoted to grasp and interaction with objects by exerting proper force during a proper motion for pure mechanical purposes.

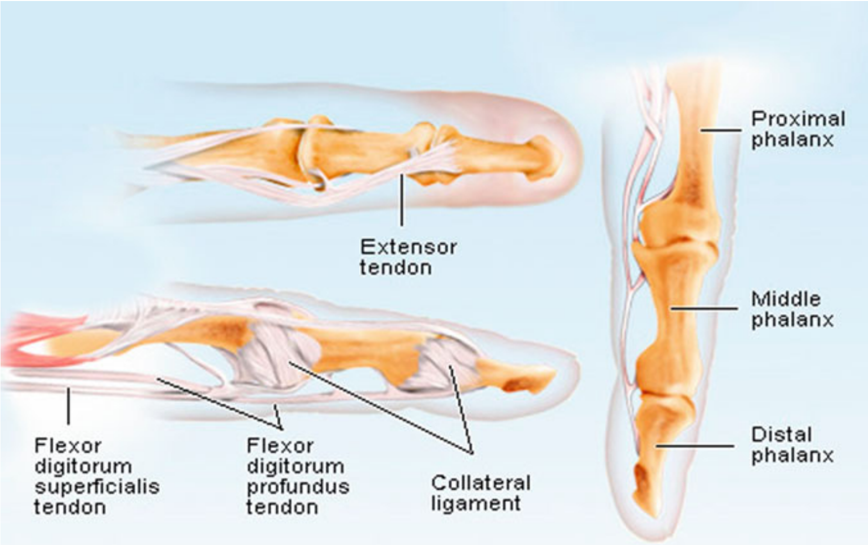

Fig. 1 Anatomy of human fingers, [11]

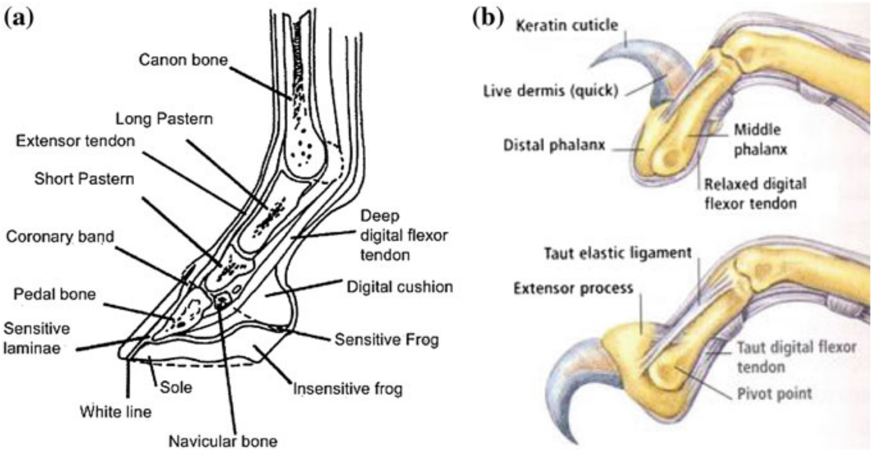

Fig. 2 Anatomy structures in: **a** horse fingers, [9]; **b** cat fingers, [12]

3 Requirements for Artificial Fingers

Artificial fingers are conceived to replicate the design and operation of fingers in nature for performing grasping actions at the most. Artificial fingers are developed for automatic machinery, and recently mainly for robotic systems, [6]. Some applications are also directed to prosthesis implementations. In this paper attention is focused on artificial fingers for robots.

Requirements can be outlined by looking both to the mimicking design purposes and operation peculiarities. However, in general common requirements can be identified mainly in the aspects for:

1. Motion properties in:
   - Grasping configurations
   - Smooth approaching motion
   - Adaptable motion configuration to object shapes
   - Reconfigurable grasping configurations
   - Workspace ranges
   - Limited motion impacts again objects to be grasped
2. Force capability in:
   - Stable grasping configurations
   - Efficient transmission of input power to grasping forces
   - Distribution of grasping forces among several contacts with grasped object
   - Positions of application points of the grasping forces
   - Adjustable grasping forces
3. Mechanical design in:
   - Stiff or compliant structure at grasp
   - Phalanx shape for adaptability to object shape
   - Room for sensors
   - Compact design versus human-like solutions
   - Lightweight solution with smart or traditional materials
   - Low-friction joints
   - Location of power source

In general, the size of grasping devices for robots is designed as function of prescribed grasps with a given set of objects. In particular, for humanoid robots artificial hands are usually of hum-like size but different smaller or larger solutions can be required in other robotic systems and manipulators.

The above-mentioned aspects can be considered with proper numerical ranges of requirements after a careful analysis of peculiarities and aims in design procedures for the adopted finger solution structure. A general procedure can be outlined as in the block diagram of Fig. 3 where the core procedure application of mechanism design procedures with specific algorithms taking into account the grasping purposes and features.

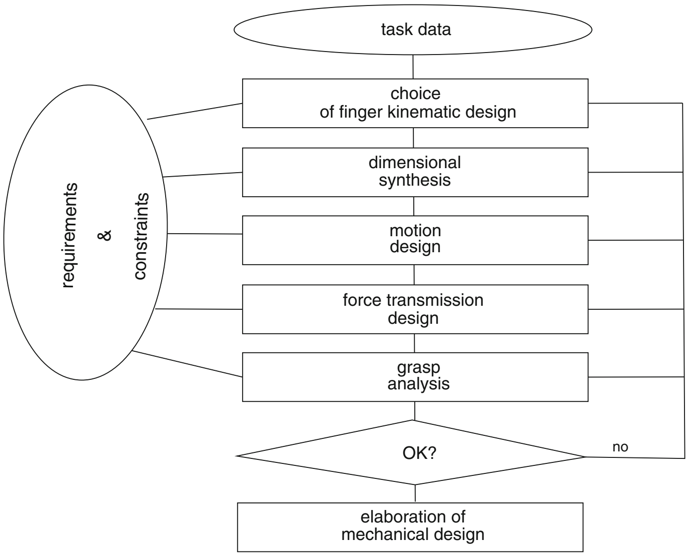

Fig. 3 A flowchart for design procedure of finger mechanisms

4 Mechanisms for Fingers

Artificial fingers are conceived as inspired by natural solutions so that in general they show structures with phalanx bodies with serial chain configuration. Figure 4 summarizes such an inspiration by looking also at the phalanx actuation with motion joint angles θi (i = 1, …, 3) and joint torque τi (i = 1, …, 3) with serial design, Fig. 4a or parallel architectures, Fig. 4b. Indeed a variety of actuating mechanisms are used to obtain actuation solutions with phalanx bodies that are even part of them.

Examples of the most used mechanisms for finger design are reported in Figs. 5, 6, 7 as referring to design finger structures which are based on linkages, tendons, or direct drives, respectively.

In Fig. 5 an artificial human hand is shown with the fingers that are designed with linkages as a product that is available in the market even at very low cost and fairly easy robot implementation, [15]. Phalanxes are links of the linkage and the finger mechanism shows a kinematic chain that can have 1 or more DOFs. In general linkages are used in finger structures to obtain 1 DOF finger mechanism whose actuator can be active on the first joint like in the example in Fig. 5.

Figure 6 shows a general scheme of tendon driven finger design where each phalanx body is actuated by two antagonist cables whose pulling gives motion and force to the phalanxes, [14]. This solution is clearly inspired from the human anatomy and it is the most common used structure in robotic hand for humanoid

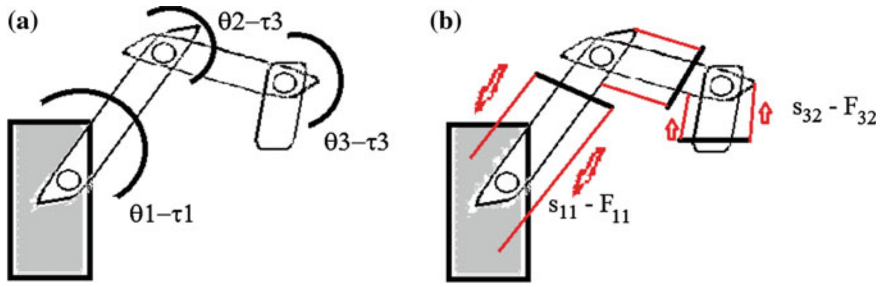

Fig. 4 Kinematic main structures for finger mechanisms: **a** serial chain as inspired by bone structure; **b** parallel architecture as inspired by muscle actuators

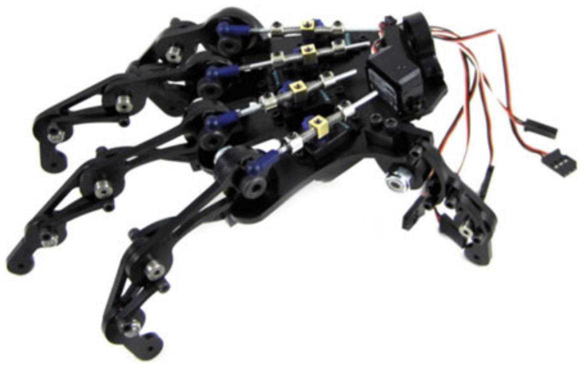

Fig. 5 Finger linkage mechanisms with rigid links in an artificial hand, [15]

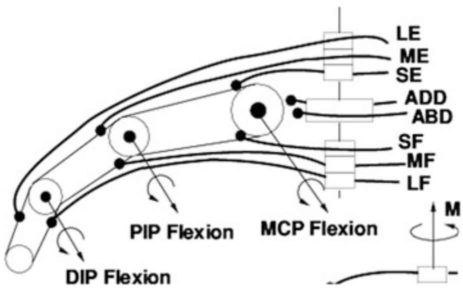

Fig. 6 A scheme of finger mechanism with tendon/cable actuation of the phalanxes, [14]

robots. Complexity refers to the multiple actuators that are needed to act in coordination for the antagonism operations.

Figure 7 shows examples in which the phalanx body is the main issue to obtain grasping adaptability to object characteristics. In Fig. 7a, [13], deformable material

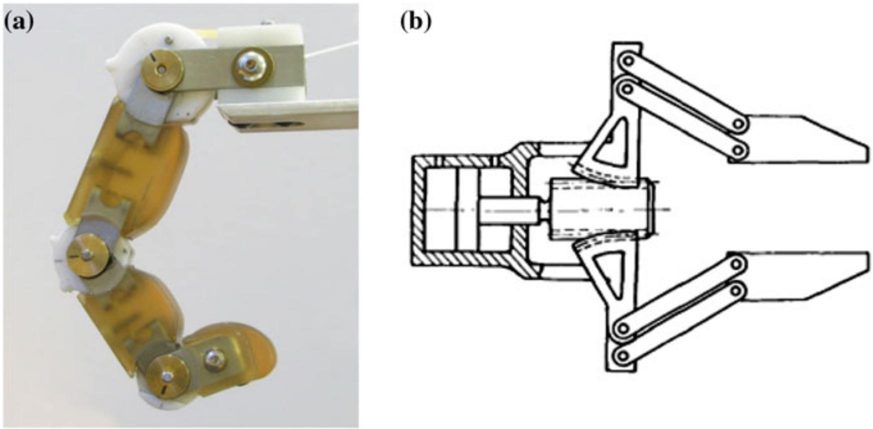

Fig. 7 Examples of finger mechanisms with phalanx bodies: **a** as deformable body, [13]; **b** as rigid link, [10]

is used as active cover of phalanx bodies where as in Fig. 7b, [10], rigid link fingertip is used in a gripper for grasping rigid bodies with possibility to shape its geometry to facilitate multiple contacts with grasped objects.

5 LARM Solutions and New Designs

A last version of LARM Hand is reported in Fig. 8a, with three 1-DOF fingers, a palm, and a standard flange for connection with robots [5]. The size of this prototype is 1.2 times larger than an average human hand as summarized in Table 1 [7]. The actuation system consists of three DC motors with a reduction gear train on each axis. The 1-DOF human-like finger mechanism for LARM Hand is designed according to the scheme in Fig. 8b. Each finger is composed of two cross two

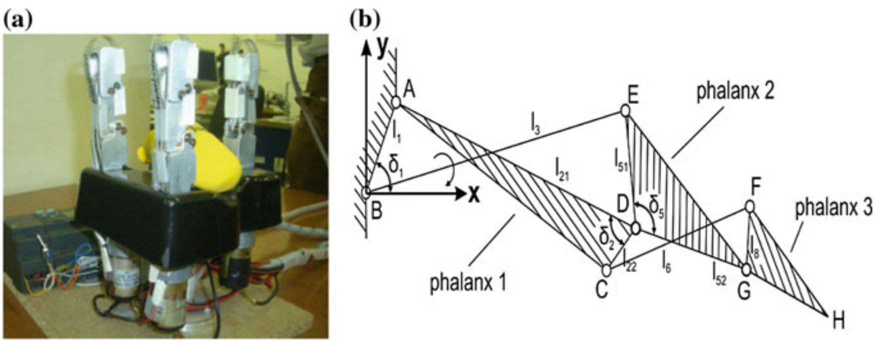

Fig. 8 LARM Hand IV: **a** prototype built in 2007; **b** finger mechanism's scheme [5]

Table 1 Structural parameters of the LARM Hand IV in Fig. 8

(mm)								(deg)		
l_1	l_{21}	l_{22}	l_3	l_{51}	l_{52}	l_6	l_8	δ_1	δ_2	δ_5
8.8	24.1	3.9	28.5	6	19.9	25	6.9	83.5	51	129

four-bar linkages. Phalanx 1 is the input bar of the first four-bar linkage and is also the base frame of the second four-bar linkage. Phalanx 2 is the input bar of the second four-bar linkage and it is also the coupler of the first four-bar linkage. Phalanx 3 is the coupler of the second four-linkage. The linkage design is characterized by a limited grasping adaptability that is determined by the linkage proportions for the finger configurations. In order to improve the capability of grasping objects with different sizes and shapes, solutions with underactuated mechanisms have been considered in previous works, [17, 18, 20].

Underactuation with spring elements as passive joints can be considered as convenient solution for artificial finger designs, [2]. Thus, at LARM attempts have been worked out to define suitable solutions with slight modifications of the original LARM finger design with the aim to improve the grasping adaptability to object shapes but by preserving the original features of 1 DOF actuation and linkage configurations within the finger body.

In particular, Fig. 9 shows a solution with torsional springs in order to obtain underactuated behavior of the four-bar linkages, [18]. Figure 10 refers to a first design for underactuating the LARM finger mechanism by using a linear spring within the body of phalanx 2, [17].

In order to design a new 1-DOF underactuated finger for LARM Hand, new kinematic solutions have been developed by using considerations in [8]. In [20] in order to obtain a new underactuated finger mechanism for LARM Hand a mechanism search has been worked out to identify several possible solutions.

The selected solution for a new finger mechanism is shown in Fig. 11. It is composed by 8 links and 9 revolute joints. Phalanxes are respectively links 2, 6 and 8. This mechanism has a limited manufacturing complexity because of the reduced number of bodies and linkage structure. Because of underactuation this mechanism is able to grasp objects with different shapes remaining within the finger body during its movement.

The mechanism operation can be described according to characteristic configurations for specific contacts by using suitable virtual equivalent mechanisms that have been used in [17, 18] to characterize the underactuated behavior of a finger

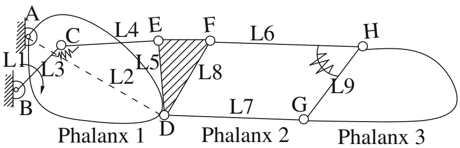

Fig. 9 Underactuated LARM finger mechanism with torsional springs, [18]

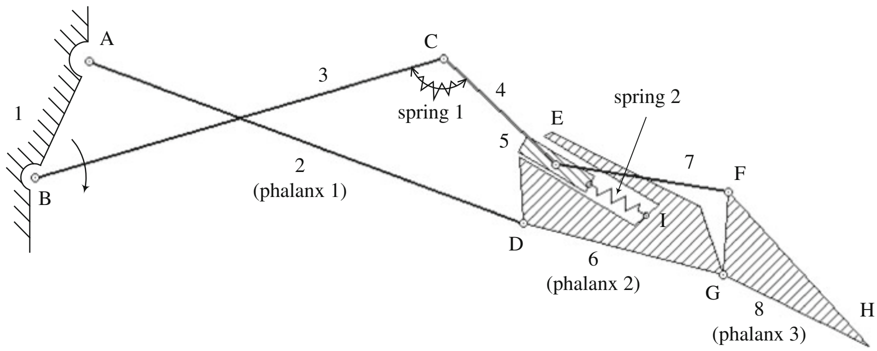

Fig. 10 Underactuated LARM finger mechanism with a linear spring, [17]

mechanism. Namely, equivalent mechanisms can be identified for the cases: no phalanxes in contact; only phalanx 1 in contact with an object; phalanxes 1 and 2 in contact. Referring to the first situation, a phalanx is free when there is no contact force and a torque acts on it. Generally a phalanx is free before it will touch an object. In this case, links that are connected by spring can be considered as a single virtual link. Here links 3 and 4 can be considered as acting as one virtual link 9 as shown by dashed line segment BD in Fig. 11b. Link 6 and 7 can be considered as another virtual link 10 through segment FI. Therefore, the proposed finger mechanism can be simplified as the equivalent mechanism of Fig. 11b, which recall the original linkage in LARM Hand, shown in Fig. 8b. In the second situation, phalanx 1 is stopped while phalanxes 2 and 3 are free. In this case, link 2 and joints E and F are fixed and they act as a virtual base as shown in Fig. 11b. Spring 1 will start to be deformed because of motor push. But spring 2 will be not activated because phalanxes 2 and 3 are free. Links 6 and 7 can still be considered as one single virtual link 10. Thus, the finger mechanism can be simplified as the equivalent mechanism in Fig. 11b. When phalanx 2 is stopped because in contact with object, link 6 and joints E, F and G are fixed and phalanxes 1 and 2 act as a virtual base. Thus, also spring 2 will start to be deformed.

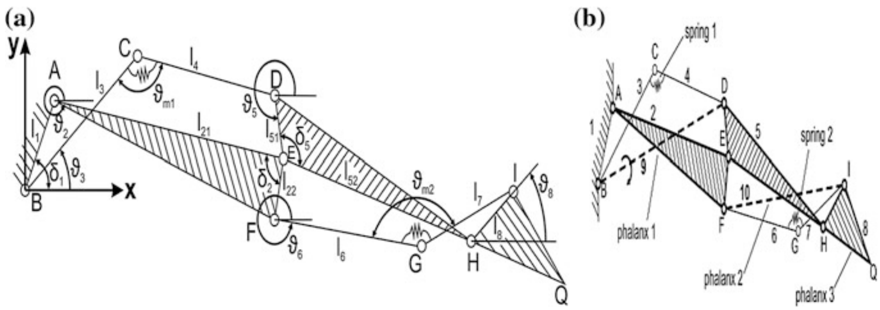

Fig. 11 A new underactuated solution for LARM Hand fingers: **a** a scheme with structural parameters; **b** an equivalent mechanisms during functioning

The experience of LARM finger designs with underactuated linkages has been developed satisfactorily by considering requirements in a procedure like in Fig. 3 and by preserving the peculiarities of the original LARM design. Nevertheless, although underactuated linkages with springs shows feasible operations, practical mechanical design with convenient handsome features is still a problem and further design research is under development.

6 Conclusions

In this paper finger design has been approached by looking at the mechanism design for finger structures that can be also the driving systems as inspired from anatomy of fingers in nature. The role of the finger mechanism is outlined by discussing requirements and peculiarities for the grasping tasks and by referring to the common general topologies that are used in artificial robot hands. In particular the experiences with LARM Hand are reported to outline future developments with solutions that can be based on underactuated linkages.

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Dimensional Synthesis of One-Jointed Multi-fingered Hands

Alba Perez-Gracia

Abstract Wristed, multi-fingered hands can be designed for specific tasks, leading to an optimized performance and simplicity. In this work we present the design of the simplest family of multi-fingered hands, with one revolute joint at the wrist and a set of fingers attached to the palm with a single revolute joint each. It is shown that hands with two to five fingers can be designed for meaningful tasks, and that two arbitrary positions can be defined at most for these hands, yielding a good tool for pick-and-place, or grasp-and-release, tasks. For each of those possible designs, dimensional synthesis is performed and an algebraic solution is derived. It is proved that two solutions exist for the general case of this family of hands. Coupled actuation for the grasp-and-release task can be easily implemented for these hands, to create an underactuated design able to be driven with a single actuator. Some examples are presented.

Keywords Robotic hands · Kinematic synthesis

1 Introduction

Kinematic chains with a tree topology consist of several common joints that branch to a number of serial chains, each of them corresponding to a different end-effector. A typical example of a kinematic chain with a tree topology is a wristed, multi-fingered hand.

Compared to other topologies, the tree topologies have not been so widely studied. Kinematic analysis for applications in modular robots and robotic hands can be found in [1, 10, 11], and dynamic analysis is found in [2, 3]. Structural

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synthesis for multiple fingers with no wrist, considering grasping and manipulation requirements, are found in [4]. The first reference to kinematic design of tree topologies is found in [7].

The kinematic synthesis of these topologies presents particular challenges that are different of those that appear in single serial chains or in closed-loop systems. In particular, the kinematic synthesis of multi-fingered hands has been explored also in [9] and more extensively in [8].

When dealing with exact kinematic synthesis, one of the first steps is to define the task in order to size the workspace of the system. In particular, we consider revolute joints without joint limits. In the case of tree topologies, consider a task having the same number of positions for each of the multiple end-effectors; this means that we are dealing with a coordinated action of all those fingertips, denoted as a *simultaneous task*. It has been proved [5] that not all tasks are possible for all fingers, as the relative motion between fingers needs to be considered.

In [5], a chart of solvable multi-fingered hands with identical fingers was presented. Here we focus on the simplest case of that chart, a family of hands with one revolute joint at the wrist and a series of fingers, joined to a single palm by one revolute joint each. For this family of hands, it is possible to obtain a closed, algebraic solution for the design equations. This provides a fast calculation method, as well as information on the number of solutions and the effect of pre-defined constraints. The method is demonstrated with an example.

2 Multi-fingered Hands

A tree topology for a kinematic chain has a set of common joints splitting on several chains, possibly in several stages, and ending in multiple end-effectors [6]. The tree topology is represented as a rooted tree graph; for this we follow the approach of Tsai [13], the root vertex being fixed with respect to a reference system. In tree topologies, a vertex can be connected to several edges defining several branches.

Wristed, multi-fingered hands are kinematic chains with a tree or hybrid topology. For our synthesis formulation, the internal loops in the hand structure are substituted using a reduction process [8], so that the hand has a tree topology with links that are ternary or above.

Tree topologies are denoted as $W-(B_1, B_2, \ldots, B_b)$, where W are the common joints and the dash indicates a branching or splitting stage, with the branches contained in the parenthesis, each branch B_i characterized by its type and number of joints. In the case of using just revolute joints, the joint type is dropped and only the number of joints is indicated. Figure 1 shows the compacted graph for the $R-(R, R, R, R, R)$, or $1-(1,1,1,1,1)$ chain. This is one of the member of the one-jointed hand family, in particular the five-fingered hand. The root vertex is indicated with a double circle.

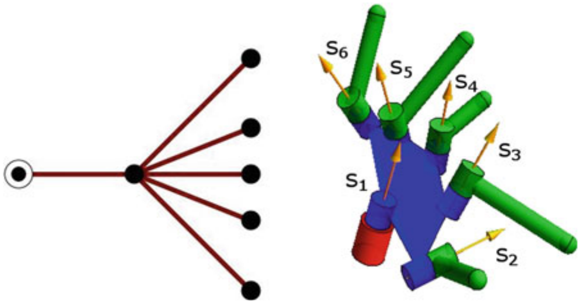


Fig. 1 The tree graph of the $1-(1,1,1,1,1)$ hand, *left*; the kinematic sketch, *right*

3 Dimensional Synthesis of Multi-fingered Hands

Given a desired hand topology with b branches and joint axes S_i, and a simultaneous task for each fingertip, characterized by a set of m_p finite positions $\hat{P}^b_{1k}$ and m_v velocities $\dot{\mathsf{P}}^b_k$, kinematic synthesis is applied by equating the forward kinematics equations of each branch to the relative displacement of the corresponding fingertip. Similarly, velocities can be defined for some of those task positions,

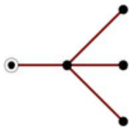
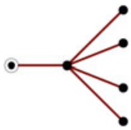
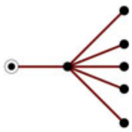
$$\mathbf{F}\left(\mathsf{S},\Delta\theta,\dot{\theta}\right)=\begin{cases}\hat{P}^c_{1k}-\prod_{i=1}^{n_c} e^{\frac{\Delta\theta^k_{i,c}}{2}\mathsf{S}_{i,c}}, & k=2,\dots,m_p \\ & c=1,\dots,b \\ \dot{\mathsf{P}}^c_k-\sum_{i=1}^{n_c}\dot{\theta}^k_{i,c}\,\mathsf{S}^k_{i,c}, & k=1,\dots,m_v \\ & c=1,\dots,b\end{cases} \tag{1}$$

In most of the cases, this set of equations is solved using numerical methods to obtain a kinematic design.

4 Single-Jointed, Single-Branching Hands

The solvability of this family of hands was studied in [5]. In Table 1, all possible one-jointed trees consisting of revolute joints and able to perform a meaningful task are presented. The first row in the table contains the serial $R-R$ robot, which is known [12] to be solvable for $m=3$ positions, yielding two real solutions. The last row presents the $1-(1,1,1,1,1)$ hand, with five fingers. Notice that any general hand of these characteristics with more than five fingers will not be solvable for a useful task, as it will not be able to reach an initial and final positions. For all the cases in the middle, from two to four fingers, the hand will be able to reach an initial and a final position, as their solvability calculations show two plus one incompletely specified position.

Table 1 Topologies with 1 common joint and 1-jointed branches, single branching

Topology	Graph	Graph solvability
$1-(1)$		Solvable, $m=3$
$1-(1,1)$		Solvable, $m=2.33$
$1-(1,1,1)$		Solvable, $m=2.14$
$1-(1,1,1,1)$		Solvable, $m=2.05$
$1-(1,1,1,1,1)$		Solvable, $m=2$

5 Dimensional Synthesis for the 1 − (1, 1, 1, 1, 1) Hand

The $1-(1,1,1,1,1)$ hand consists of a single revolute joint at the wrist and a palm spanning five fingers, each of them being a fingertip link connected to the palm by a single revolute joint. Figure 1 shows the reduced tree graph for the hand and the kinematic sketch with the notation used for the axes.

The six joints have Plucker coordinates $\mathsf{S}_i = \mathbf{s}_i + \varepsilon \mathbf{s}_{i0}$, for $i = 0, \ldots, 5$. The rotation angle about each joint is θ_i.

As stated in Table 1, this kinematic system can be solved for exact dimensional synthesis for $m = 2$ positions. In what follows, the algebraic solution is derived.

5.1 Design Equations

Let $\hat{P}_{ij}$ be the ith displacement assigned to finger j. For this robot, $i = 1, 2$ and $j = 1, \ldots, 5$. Construct the relative displacements from first to second position for each finger j, $\hat{P}_j = \hat{P}_{2j}\hat{P}_{1j}^{-1}$, $j = 1, \ldots, 5$, where the composition of displacements is used.

The forward kinematics equations for relative displacements are constructed, for each finger, as the composition of screw displacements about each joint axes along the serial chain. The screw displacements about the joint axes are easily computed as the exponential of the unit twist, and so the product of exponentials is

$$e^{\hat{S}_0 \frac{\Delta\theta_0}{2}} e^{\hat{S}_j \frac{\Delta\theta_j}{2}}, \quad j = 1, \ldots, 5, \tag{2}$$

where $\Delta\theta_j = \theta_j^2 - \theta_j^1$ is the relative angle to transform from first to second position, for each finger j.

Impose now that each of the fingers has to be able to perform the desired relative displacement $\hat{P}_j$ from first to second position, to create a set of equations to solve for the design parameters of the robotic hand,

$$e^{\hat{S}_0 \frac{\Delta\theta_0}{2}} e^{\hat{S}_j \frac{\Delta\theta_j}{2}} = \hat{P}_j, \quad j = 1, \ldots, 5. \tag{3}$$

This is a set of six independent equations per finger times five fingers, for a total of 30 independent equations. When using unit dual quaternions to express the displacements, the total set has 40 equations. The parameters to solve for are the Plucker coordinates of the axes and the relative joint angles to reach the relative transformation.

5.2 Algebraic Solution

A revolute joint has five independent parameters, and is unable to perform a general displacement. Each finger of the $1 - (1, 1, 1, 1, 1)$ hand is a serial $R - R$ chain in which the first joint cannot be fully arbitrary. This hand can be seen as five $R - R$ chains in which each chain needs to fully define the second joint axis and a single parameter of the common first joint axes, in order to perform a general displacement.

Using this rationale, solve for the second joint axes of each finger serial chain,

$$e^{\hat{S}_j \frac{\Delta\theta_j}{2}} = \left(e^{\hat{S}_0 \frac{\Delta\theta_0}{2}}\right) * \hat{P}_j, \quad j = 1, \ldots, 5, \tag{4}$$

where $*$ denotes the conjugate unit dual quaternion. The expansion of this equation in dual quaternion form yields:

$$\begin{aligned}&\left(\cos\frac{\Delta\theta_j}{2}+\varepsilon\sin\frac{\Delta\theta_j}{2}\right)\hat{S}_j\\&=\left(\cos\frac{\Delta\theta_0}{2}-\sin\frac{\Delta\theta_0}{2}S_0\right)\left(p_{j0}\ +_{j7}\ +\ \mathsf{P}_j\right),\quad j=1,\ldots,5,\end{aligned}\tag{5}$$

where,

$$\hat{S}_j=0+\varepsilon 0+\mathsf{S}_j=0+s_{jx}i+s_{jy}j+s_{jz}k+\varepsilon\left(s^0_{jx}i+s^0_{jy}j+s^0_{jz}k+0\right)\tag{6}$$

is the pure dual quaternion that represents a geometric line in the Clifford algebra. The solution for all S_j, $j=1,\ldots,5$, is obtained as a function of the first, common joint axis, S_0. These equations are also used to fully define this first axes.

Notice that the last component of the dual quaternion must be equal to zero for each joint solution, according to Eq. (5). This forces the product $\left(e^{\hat{S}_0\frac{\Delta\theta_0}{2}}\right)*\hat{P}_j$ to have also last component equal to zero, and creates one equation on the parameters of the first axis S_0 from each finger equations,

$$\cos\frac{\Delta\theta_0}{2}p_{j7}+\sin\frac{\Delta\theta_0}{2}\left(\mathbf{s}_0\cdot\mathbf{p}^0_j+\mathbf{s}^0_0\cdot\mathbf{p}_j\right)=0,\quad j=1,\ldots,5.\tag{7}$$

These are five equations in the five independent unknowns of the rotation about S_0 by $\Delta\theta_0$. Imposing that the joint angle has to be the same for the simultaneous task,

$$\cot\frac{\Delta\theta_0}{2}=\frac{-(\mathbf{s}_0\cdot\mathbf{p}^0_j+\mathbf{s}^0_0\cdot\mathbf{p}_j)}{p_{j7}},\quad j=1,\ldots,5,\tag{8}$$

we end up with four linear equations in the Plucker coordinates of the axes. Those are six parameters subject to two Plucker constraints plus the four linear equations from (8),

$$\begin{aligned}&\frac{\mathbf{s}_0\cdot\mathbf{p}^0_j+\mathbf{s}^0_0\cdot\mathbf{p}_j}{p_{j7}}=\frac{\mathbf{s}_0\cdot\mathbf{p}^0_{j+1}+\mathbf{s}^0_0\cdot\mathbf{p}_{j+1}}{p_{(j+1),7}},\quad j=1,\ldots,4,\\&\mathbf{s}_0\cdot\mathbf{s}_0=1,\quad \mathbf{s}_0\cdot\mathbf{s}^0_0=0,\end{aligned}\tag{9}$$

with at most four solutions. However two of the solutions correspond to the double covering of $SO(3)$, and the final number of different solutions is two.

The two solutions for the Plucker coordinates of the first joint S_0 are used to compute a single rotation angle about this first joint for each solution, using Eq. (8). These values allow us to create the screw transformation about the first joint, and the rest of the joint axes and angles can be calculated using Eq. (4).