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Ram N. Mohapatra
Debasis Giri
P. K. Saxena
P. D. Srivastava *Editors*

Mathematics and Computing 2013

International Conference in Haldia, India

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Ram N. Mohapatra · Debasis Giri
P. K. Saxena · P. D. Srivastava
Editors

Mathematics and Computing 2013

International Conference in Haldia, India

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Preface

With a view to have together at one place experts and professionals working in different aspects of research in Mathematics, the idea came to organize an event where people may deliberate upon theoretical as well as computational aspects of mathematics at a common forum. We are delighted to give shape to the idea and host this International Conference on Mathematics and Computing (ICMC 2013) at Haldia Institute of Technology, Haldia, in collaboration with Scientific Analysis Group (DRDO, Ministry of Defence) and Department of Mathematics, IIT Kharagpur.

Haldia is a city and a municipality in Purba Medinapur in the Indian state of West Bengal. Haldia Institute of Technology is one of the premier educational establishments in this part of the State and has had the privilege to have organized international conferences in the past as well.

With three tracks of presentations, contributory papers were called for at ICMC 2013 and 81 papers were submitted to the conference in response. The papers were reviewed on the basis of the significance, novelty, and technical quality. Of these, 22 papers were selected for presentation and publication in the Conference Proceedings.

The papers cover different topics including Cryptography, Algebra, Functional Analysis, Approximation Theory, Fluid Dynamics, etc. The event also included Tutorials on important topics of current thrust. It is expected that the conference will witness eminent personalities both from India and abroad (USA, Canada, Russia, Japan, Hong Kong, Turkey) delivering invited as well as tutorial talks. The prominent speakers from India were from Government R&D Organizations such as Defense Research and Development Organization, Industry and from Academic Institutions such as Indian Statistical Institute Kolkata, IIT Kharagpur, IMSc Chennai, IISc Bangalore, etc.

Three Tutorials were planned preceding the main conference to be given by Prof. Bimal Roy (ISI, Kolkata, India), Prof. M. L. Chaudhry (Royal Military College, Canada), Prof. Ram N. Mohapatra (University of Central Florida, USA), and Prof. A. Vasudevarao, IIT Kharagpur. There were seven invited talks delivered by experts like Prof. Ram N. Mohapatra (University of Central Florida, USA), Prof. R. Balasubramanian (IMSc, Chennai, India), Prof. V. A. Artamonov (Lomonosov Moscow State University, Russia), Prof. C. E. Venimadhavan (IISc, Bangalore, India), Prof. Duan Li (Chinese University of Hong Kong), Prof. Hiroshi Yanagihara (Yamaguchi University, Japan), Prof. Rifat Colak (Firat University, Turkey).

A conference of this kind would not have been possible without the support from different organizations and people across different committees. We are indebted to the Defense Research and Development Organization (DRDO), Ministry of Communication and Information Technology (MCIT), National Board for Higher Mathematics (NBHM), Cryptology Research Society of India (CRSI), Department of Science and Technology (DST), and the Council of Scientific and Industrial Research (CSIR) for sponsoring the event. Their support helped in significantly raising the profile of the conference.

All logistic and general organizational aspects were looked after locally by the Organizing Committee members from the Institute, who spent their time and energy in making the conference a success. The Technical Program Committee and External Reviewers helped in selecting the papers for presentations and working out the technical program. We acknowledge the support and help from all of them.

The organizers also express their hearty thanks to Springer for agreeing to publish the proceedings in its Mathematics and Statistics series.

Last but not the least; our sincere thanks go to all the authors who submitted papers to ICMC 2013 and to all speakers and participants.

We sincerely hope that the readers will find the proceedings stimulating and inspiring. Any constructive suggestions for improvement are welcome.

December 2013

Ram N. Mohapatra
Debasis Giri
P. K. Saxena
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Message from the General Chairs

As we all are aware, Mathematics has always been a discipline of interest not only to theoreticians but also to all practitioners irrespective of their specific profession. Be it Science, Technology, Economics, Commerce, or even Sociology, new Mathematical principles and models have been emerging and helping in new research and in drawing inferences from practical data as well as through logic. The past few decades have seen enormous growth in applications of Mathematics in different areas multidisciplinary in nature. Cryptography and signal processing are such areas, which have got more focus recently due to the need for securing communication while connecting with others. With emerging computing facilities and speeds, a phenomenal growth has happened in the problem solving area. Earlier, some observations were made and conjectures were drawn which remained conjectures till somebody could either prove it theoretically or found counter examples. But today, we can write algorithms and use computers for long calculations, verifications, or for generation of huge amounts of data. With available computing capabilities, we can find factors of very large integers of the size of hundreds of digits; we can find inverses of very large size matrices and solve a large set of linear equations, and so on. Thus, Mathematics and Computations have become more integrated areas of research these days, and it was thought to organize an event where thoughts may be shared by researchers and new challenging problems could be deliberated for solving these.

Apart from many other interdisciplinary areas of research, cryptography has emerged as one of the most important areas of research with discrete mathematics as a base. Several research groups are actively pursuing the research on different aspects of cryptology not only in terms of new cryptoprimitives and algorithms but a whole lot of concepts related to authentication, integrity, and security proofs/protocols are being developed, many times with open and competitive evaluation mechanism to evolve standards.

As conferences, seminars, and workshops are the mechanisms to share knowledge and new research results giving us a chance to get new innovative ideas for futuristic needs as threats and computational capabilities of adversaries are ever increasing, it was thought appropriate to organize the present conference focused on Mathematics and Computations covering theoretical as well as practical aspects of research, Cryptography being one of these.

Eminent personalities working in Mathematical Sciences and related areas were invited from abroad as well as from within the country to deliver invited talks and Tutorials for participants. The talks by these speakers covered a wide spectrum, viz., Number Theoretic Concepts, Cryptography, Algebraic Concepts like Quasi Groups and applications, etc. The conference was spread over 4 days (December 26–29, 2013) with the first day dedicated to Tutorials. The main conference was planned with special talks by experts and paper presentations in each session.

We hope that the conference met the aspirations of the participants and its objective of ideas and current research being shared and new targets/problems identified in the domain of Coding theory, Cryptography, Computational number theory, Algebra, Frame theory, Optimizations, Stochastic Processes, Compressive Sensing, Functional analysis, Complex variables etc., so that the researchers and students would get new directions to pursue their future research.

December 2013

P. K. Saxena
P. D. Srivastava

Message from the Program Chairs

It is a great pleasure for us to organize the International Conference on Mathematics and Computing-2013 to be held from December 26 to 29 at the Haldia Institute of Technology, Purba Medinipur, West Bengal, India. Our main goal is to provide an opportunity to the participants to learn about contemporary research in Mathematics and Computing and exchange ideas. With this aim in mind, we carefully chose the invited speakers and the tutorial speakers. It is our sincere hope that the conference will help the participants in their research and training and open new avenues for work in Mathematics and Computing.

On 26 December 2013, there will be tutorials. The conference will begin after a formal opening ceremony on 27 December 2013. There will be seven invited one-hour talks and 22, contributed half-hour talks. Our speakers/contributors come from Austria, Canada, Hong Kong, India, Japan, Philippines, Russia, Turkey, and USA.

After an initial call for papers, 81 papers were submitted in the conference. All submitted papers were sent to referees and after refereeing, 22 papers were recommended for publication. The proceedings of the conference will be published by Springer (Mathematics and Statistics Series).

We are grateful to the speakers, participants, referees, organizers, sponsors, and funding agencies for their support and help without which it would have been impossible to organize the conference. We owe our gratitude to the volunteers who work behind the scene in taking care of the details in making this conference a success.

December 2013

Ram N. Mohapatra
Debasis Giri

Contents

1 **Propagation of Water Waves in the Presence of Thin Vertical Barrier on the Bottom Undulation** 1
A. Choudhary and S. C. Martha

2 **Cryptanalysis of Multilanguage Encryption Techniques** 13
Prasanna Raghaw Mishra, Indivar Gupta and Navneet Gaba

3 **Signcryption with Delayed Identification** 23
Angsuman Das and Avishek Adhikari

4 **HDNM8: A Round-8 High Diffusion Block Cipher with Nonlinear Mixing Function** 41
Jaydeb Bhaumik and Dipanwita Roy Chowdhury

5 **Frames and Erasures** 57
Saliha Pehlivan

6 **Semi-inner Product: Application to Frame Theory and Numerical Range of Operators**. 77
N. K. Sahu and C. Nahak

7 **Multi-level Nonlinear Programming Problem with Some Multi-choice Parameter** 91
Avik Pradhan and M. P. Biswal

8 **A New Class of Rational Cubic Fractal Splines for Univariate Interpolation** 103
P. Viswanathan and A. K. B. Chand

9 **Applications of Compressive Sensing to Surveillance Problems** 121
Christopher Huff and Ram N. Mohapatra

10 Region of Variability for Some Subclasses of Univalent Functions 151
A. Vasudevarao

11 Ideal Cone: A New Method to Generate Complete Pareto Set of Multi-criteria Optimization Problems 171
Debdas Ghosh and Debjani Chakraborty

12 Fractional Programming Problem with Bounded Parameters 191
A. K. Bhurjee and G. Panda

13 Approximation Properties of Linear Positive Operators with the Help of Biorthogonal Polynomials 201
G. Icoz

14 Similarity-Based Reasoning Fuzzy Systems and Universal Approximation 215
Sayantan Mandal and Balasubramaniam Jayaram

15 Similarity Measure of Intuitionistic Fuzzy Numbers by the Centroid Point 231
Satyajit Das and Debashree Guha

16 Classification Rules for Exponential Populations Under Order Restrictions on Parameters 243
Nabakumar Jana, Somesh Kumar and Neeraj Misra

17 Solving the Exterior Bernoulli Problem Using the Shape Derivative Approach 251
Jerico B. Bacani and Gunther Peichl

18 Applications of the Hausdorff Measure of Noncompactness on the Space $l_p(r, s, t; B^{(m)}), 1 \leq p < \infty$ 271
Amit Maji and P. D. Srivastava

19 Some Geometric Properties of Generalized Cesàro–Musielak–Orlicz Sequence Spaces 283
Atanu Manna and P. D. Srivastava

20 Inverting the Transforms Arising in the *GI/M/*1 Risk Process Using Roots 297
Gopinath Panda, A. D. Banik and M. L. Chaudhry

21 On Quasi-ideals in Ternary Semirings 313
Manish Kant Dubey and Anuradha

22 Epidemiological Models: A Study of Two Retroviruses, HIV and HTLV-I 323
Dana Baxley, N. K. Sahu and Ram N. Mohapatra

About the Editors

Ram N. Mohapatra is Professor of Mathematics, University of Central Florida, Orlando, USA. He received his Ph.D. degree from the University of Jabalpur, India, in 1968. Earlier, he taught at Sambalpur University in India, American University in Beirut, Lebanon, University of Alberta, and York University, Canada, prior to coming to Orlando. His area of research is Mathematical Analysis and he is the author of two books, two edited monographs, and over 120 research papers. He referees articles for professional journals and serves as a member of the editorial board of a number of journals.

Debasis Giri is Professor in the Department of Computer Science and Engineering, Haldia Institute of Technology, India. His topics of interest include discrete mathematics, cryptography, information security, coding theory, advanced algorithms, design and analysis of algorithms, and formal languages and automata theory. His research interests include cryptography, network security, security in wireless sensor networks, and security in VANETs. Dr. Giri has delivered several talks and guest lectures at various universities and conferences. He is supervisor of three Ph.D. research scholars. Further, he guided many B.Tech. and M.Tech. students. He is associate editor of the *Journal of Security and Communication Networks* (Wiley), and the *Journal of Electrical and Computer Engineering Innovations*. Further, he is editorial board member and reviewer of many reputed international journals. He is also program committee member of many international conferences. He is life member of the Cryptology Research Society of India. He received his Ph.D. on "Cryptanalysis and Improvement of Protocols for digital signature, smart-card authentication and access control" from Indian Institute of Technology Kharagpur. He did both his M.Tech. and M.Sc. from Indian Institute of Technology Kharagpur. He secured tenth position in all India rank with percentile score of 98.42 in the Graduate Aptitude Test in Engineering (GATE) Examination in 1999. Dr. Giri has published more than 25 technical papers in several internal journals and proceedings.

P. K. Saxena is Director of one of the R&D labs of Defense Research Development Organization (DRDO) under the Ministry of Defense. Dr. Saxena, an outstanding scientist, has done his Ph.D. on "Radical theory of near rings" from Indian Institute of Technology Kanpur. Before joining DRDO, he taught

Mathematics at National Institute of Technology (NIT) Silchar and at National Defense Academy (NDA), Pune. Dr. Saxena has published about 62 research papers in several journals and conferences on interdisciplinary topics such as algebra, cryptology, fuzzy logic, artificial neural networks, and speech technology. He has led many important R&D projects and guided many students in important engineering projects. Three scholars have received the Ph.D. degree under his supervision and several others are registered. Dr. Saxena has delivered talks, guest lectures, keynote addresses at various forums apart from organizing international conferences as general chair. He has been in program committees of many international conferences. He has authored a book on *Cryptology* (in Hindi) which was awarded first prize by DRDO in 1997.

P. D. Srivastava is Professor of Mathematics at Indian Institute of Technology Kharagpur, India. During his 34 years of teaching career, he taught several courses such as functional analysis, topology, numerical analysis, measure theory, real analysis, complex analysis, and calculus to undergraduate and postgraduate students. Besides teaching, Professor Srivastava is equally devoted to research activities. He has approximately 51 papers to his credit published in several international journals. He has supervised 10 research scholars for the Ph.D. degree in Mathematics and one for PDF. Various universities have invited him for lectures and keynote addresses at their conferences. Various universities also invite him as an expert in the faculty selection as well as an expert to adjudicate Ph.D. theses. He is also reviewer for the *Mathematical Reviews* as well as paper referee for many journals. He did his Ph.D. from Indian Institute of Technology Kanpur and B.Sc., M.Sc. degrees from Kanpur University. Dr. Srivastava is not only an established researcher in his area but also a teacher par excellence. His style of lecture presentation and full command over the subject impresses students, which is reflected in the Students' Profile Forms (teaching assessment by students).

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Chapter 1
Propagation of Water Waves in the Presence of Thin Vertical Barrier on the Bottom Undulation

A. Choudhary and S. C. Martha

Abstract The problem involving diffraction of water waves by submerged thin vertical barrier over irregular bottom is examined using linearized theory of water waves. While formulating the problem mathematically, a mixed boundary value problem (BVP) occurs. The problem is solved using perturbation theory along with least-squares method and Green's integral theorem. The first order reflection and transmission coefficients are obtained in terms of integrals involving the shape function $c(x)$ representing the bottom undulation and the solution of the scattering problem by the submerged barrier. A special case of bottom undulation is considered to evaluate the first order reflection and transmission coefficients in detail. The numerical results of these coefficients are shown graphically.

Keywords Water wave scattering · Bottom undulation · Vertical barrier · Perturbation analysis · Least-squares method · Green's integral theorem · Reflection and transmission coefficients

1 Introduction

The interaction of water waves with vertical barriers has received considerable attention from many researchers. These problems are important due to their applications in ocean engineering such as breakwaters and wavemakers which protect a harbor or

A. Choudhary is grateful to the University Grants Commission (UGC), Government of India, for providing the Junior Research Fellowship for pursuing Ph.D. degree at the Indian Institute of Technology Ropar, India. S. C. Martha thanks the Indian Institute of Technology Ropar, India, for providing all necessary facilities.

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marinas from the rough sea. Dean [3] obtained the linearized solution of water wave scattering by the submerged plane barrier which extended infinitely downwards in deep water. Ursell [14] derived the solution of the problem of diffracted waves by thin vertical barrier partially immersed in deep water using the singular integral equation approach based on Havelock's expansion. Porter [12] solved the problem involving wave transmission through a gap in a vertical barrier in deep water using the complex variable method along with Green's integral theorem. Losada et al. [4] used Least-squares is bounded assolution of the problem involving scattering of water waves by different thin barriers. Mandal and Dolai [7] and Porter and Evans [13] obtained the solution of this problem using Galerkin approximation method.

The problems involving diffraction of water waves by undulating bottom topography of a seabed are also interesting to study because of their significance in finding the effect of naturally occurring bottom undulation such as sand ripples on wave propagation. Miles [11] derived the reflection and transmission coefficients using the finite cosine transform technique when oblique waves are incident to a cylindrical obstacle. Davies [1] discussed the problem of the reflection of incident waves by irregular bottom using Fourier transform technique. Davies and Heathershaw [2] compared the theoretical results of [1] by conducting experiments in wave tank. Martha et al. [10] solved the problem involving water wave scattering by small undulation on seabed using Fourier transform method and residue theorem.

However, looking at the present situation, the vertical barrier submerged on the undulating seabed will serve as an effective breakwater for coastal engineering. The literature in this direction is very limitted. Mandal and Gayen [8] solved one such problem using multi-term Galerkin approximation and Green's integral theorem.

In this paper, we discuss the problem involving diffraction of water waves by undulating bed topography and submerged vertical barrier. A mixed boundary value problem occurs while formulating the problem mathematically. On applying the perturbation analysis involving small parameter ε which characterizes the smallness of bottom undulation, two boundary value problems, namely BVP-I (by equating the coefficients of the powers of ε^0) and BVP-II (by equating the coefficients of the powers of ε), are obtained. The BVP-I corresponds to the problem of scattering of water waves by vertical barrier in water of uniform finite depth. The solution of the BVP-I is obtained by least-squares method for which the error is minimum. The zeroth order reflection and transmission coefficients involved in BVP-I are also determined. The BVP-II which involves the solution of BVP-I, represents the radiation problem in water of uniform finite depth. Green's integral theorem is used to obtain the solution of BVP-II and the first order reflection and transmission coefficients. The numerical results for these reflection and transmission coefficients are shown graphically.

2 Mathematical Formulation

A right-handed rectangular Cartesian coordinate system is employed in which x-axis is the position of undisturbed free surface and y-axis is taken positive vertically downwards from the origin. The bottom of the sea has small undulation and is

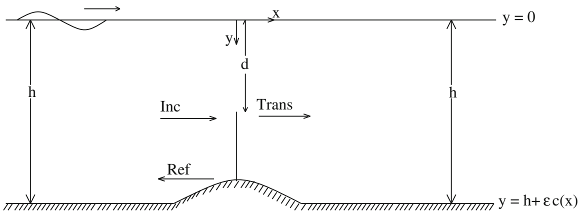

Fig. 1 Scattering of water waves by submerged vertical barrier with bottom undulation

described by $y = h + \varepsilon c(x)$, where $c(x)$ is a continuous bounded function describing the shape of the bottom undulation, h denotes the uniform finite depth of the ocean far to either side of the undulation of the bottom so that $c(x) \to 0$ as $|x| \to \infty$ and the nondimensional number $\varepsilon(\ll 1)$ gives the measure of smallness of the undulation. Consider a thin vertical barrier which is submerged on the bottom undulation, whose position is located at $x = 0$, $y \in L = [d, h]$ (Fig. 1).

It is assumed that the fluid is inviscid, incompressible and the motion is irrotational. If the motion is to be simple harmonic in time with angular frequency σ, then the velocity potential Φ which describes the fluid motion can be expressed as $\Phi(x, y, t) = Re\{\phi(x, y)e^{-i\sigma t}\}$. Then the complex valued potential ϕ satisfies the Laplace equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0, \text{ in the fluid region,} \tag{1}$$

along with conditions:

$$\text{Free surface condition: } \frac{\partial\phi}{\partial y} + K\phi = 0, \quad \text{on } y = 0, \quad \left(\text{with } K = \frac{\sigma^2}{g}\right) \tag{2}$$

$$\text{Bottom condition: } \frac{\partial\phi}{\partial n} = 0, \quad \text{on } y = h + \varepsilon c(x), \tag{3}$$

$$\text{Condition on barrier: } \frac{\partial\phi}{\partial x} = 0, \quad \text{on } x = 0, y \in L, \tag{4}$$

$$\text{Condition across gap: } \frac{\partial\phi}{\partial x}|_{x=0^-} = \frac{\partial\phi}{\partial x}|_{x=0^+}, \quad \text{on } x = 0, y \in \bar{L}, \tag{5}$$

$$\phi|_{x=0^-} = \phi|_{x=0^+}, \quad \text{on } x = 0, y \in \bar{L}, \tag{6}$$

$$\text{Edge condition: } r^{1/2}\nabla\phi \text{ is bounded as } r \to 0, \tag{7}$$

$$\text{Far-field behavior: } \phi(x, y) \sim \begin{cases} \phi_{inc}(x, y) + R\phi_{inc}(-x, y) & \text{as } x \to -\infty \\ T\phi_{inc}(x, y) & \text{as } x \to \infty. \end{cases} \tag{8}$$

where $\frac{\partial}{\partial n}$ is the normal derivative at a point (x, y) on the bottom, g is gravitational constant, r is the distance from a submerged end of the barrier, R is the reflection coefficient, and T is transmission coefficient, $\bar{L} = 0 \leq y \leq d$ and ϕ_{inc} denotes the incident wave.

The incident wave in a finite depth of water h can be written as

$$\phi_{inc}(x, y) = \psi_0(y)e^{i\hat{k_0}x}$$

where $\psi_0(y) = N_0^{-1} \cosh \hat{k_0}(h - y)$ with $N_0 = \left[\frac{4\hat{k_0}h}{2\hat{k_0}h + \sinh 2\hat{k_0}h}\right]^{-1/2}$,
$\hat{k_0}$ is the wave number of incident wave, the positive real root of the transcendental equation

$$K - k \tanh kh = 0. \tag{9}$$

3 Method of Solution

The bottom condition (3) can be approximated up to the first order of the small parameter ε as

$$\frac{\partial \phi}{\partial y} - \varepsilon \frac{\mathrm{d}}{\mathrm{d}x}\{c(x)\phi_x\} + O(\varepsilon^2) = 0 \quad \text{on } y = h. \tag{10}$$

The approximate boundary condition (10) suggests that ϕ, R, T can be expanded in terms of ε as given by

$$\begin{aligned} \phi(x, y) &= \phi_0 + \varepsilon\phi_1 + O(\varepsilon^2), \\ R &= R_0 + \varepsilon R_1 + O(\varepsilon^2), \\ T &= T_0 + \varepsilon T_1 + O(\varepsilon^2). \end{aligned} \tag{11}$$

Substituting the expressions of $\phi(x, y)$, R, and T from relation (11) into (1), (2), (4)–(8), and (10) and equating the coefficients of ε^0 and ε from both sides, the functions $\phi_0(x, y)$ and $\phi_1(x, y)$ satisfy the following BVPs:
BVP-I: The function $\phi_0(x, y)$ satisfies

$$\frac{\partial^2 \phi_0}{\partial x^2} + \frac{\partial^2 \phi_0}{\partial y^2} = 0 \quad \text{in the fluid region,} \tag{12}$$

$$\frac{\partial \phi_0}{\partial y} + K\phi_0 = 0, \quad \text{on } y = 0, \tag{13}$$

$$\frac{\partial\phi_0}{\partial y} = 0 \quad \text{on } y = h, \tag{14}$$

$$\frac{\partial\phi_0}{\partial x} = 0, \quad \text{on } x = 0, y \in L, \tag{15}$$

$$\frac{\partial\phi_0}{\partial x} \mid_{x=0^-} = \frac{\partial\phi_0}{\partial x} \mid_{x=0^+}, \quad \text{on } x = 0, y \in \bar{L}, \tag{16}$$

$$\phi_1 \mid_{x=0^-} = \phi_1 \mid_{x=0^+} \quad \text{on } x = 0, y \in \bar{L}, \tag{17}$$

$$r^{1/2}\nabla\phi_0 \quad \text{is bounded as } r \to 0, \tag{18}$$

$$\phi_0(x, y) \sim \begin{cases} (e^{i\hat{k_0}x} + R_0 e^{-i\hat{k_0}x})\psi_0(y) & \text{as } x \to -\infty \\ T_0 e^{i\hat{k_0}x}\psi_0(y) & \text{as } x \to \infty. \end{cases} \tag{19}$$

BVP-II: The function $\phi_1(x, y)$ satisfies

$$\frac{\partial^2\phi_1}{\partial x^2} + \frac{\partial^2\phi_1}{\partial y^2} = 0 \quad \text{in the fluid region}, \tag{20}$$

$$\frac{\partial\phi_1}{\partial y} + K\phi_1 = 0, \quad \text{on } y = 0, \tag{21}$$

$$\frac{\partial\phi_1}{\partial y} = \frac{d}{dx}\{c(x)\frac{\partial\phi_0}{\partial x}\} = p(x) \text{ (say) on } y = h, \tag{22}$$

$$\frac{\partial\phi_1}{\partial x} = 0, \quad \text{on } x = 0, y \in L, \tag{23}$$

$$\frac{\partial\phi_1}{\partial x} \mid_{x=0^-} = \frac{\partial\phi_1}{\partial x} \mid_{x=0^+}, \quad \text{on } x = 0, y \in \bar{L}, \tag{24}$$

$$\phi_1 \mid_{x=0^-} = \phi_1 \mid_{x=0^+} \quad \text{on } x = 0, \ y \in \bar{L}, \tag{25}$$

$$r^{1/2}\nabla\phi_1 \text{ is bounded as } r \to 0, \tag{26}$$

$$\phi_1(x, y) \sim \begin{cases} R_1 e^{-i\hat{k_0}x}\psi_0(y) & \text{as } x \to -\infty \\ T_1 e^{i\hat{k_0}x}\psi_0(y) & \text{as } x \to \infty. \end{cases} \tag{27}$$

Here, the BVP-I represents the scattering of water waves by thin vertical barriers in water of finite depth h. The solution for ϕ_0 can be expressed as

$$\phi_0(x, y) = \begin{cases} (e^{i\hat{k_0}x} + R_0 e^{-i\hat{k_0}x})\psi_0(y) + \sum_{n=1}^{\infty} A_n e^{k_n x}\psi_n(y) & \text{as } x \to -\infty, \\ T_0 e^{i\hat{k_0}x}\psi_0(y) + \sum_{n=1}^{\infty} B_n e^{-k_n x}\psi_n(y) & \text{as } x \to \infty, \end{cases} \tag{28}$$

where $\pm ik_n, (n = 1, 2, \ldots)$ are the purely imaginary roots of the transcendental Eq. (9), $A_n, B_n, (n = 1, 2, \ldots)$ are constants to be determined and $\psi_n(y) = N_n^{-1}$

$\cos k_n(h-y)$ with $N_n = \left[\dfrac{4k_n h}{2k_n h + \sin 2k_n h}\right]^{-1/2}$.

Now, using the boundary conditions (16) and (17), we obtain

$$R_0 + T_0 = 1 \text{ and } A_n = -B_n, \tag{29}$$

$$\text{and} \quad \frac{1}{h}\psi_0(y) + \frac{1}{h}\sum_{n=0}^{\infty} A_n \psi_n(y) = 0, \quad \text{on } x = 0, y \in \bar{L}, \tag{30}$$

where $A_0 = R_0 - 1$, $k_0 = -i\hat{k_0}$.
Again, using the boundary condition (15), we get

$$\sum_{n=0}^{\infty} A_n (k_n h) \frac{1}{h}\psi_n(y) = 0, \quad \text{on } x = 0, y \in L. \tag{31}$$

These two relations, (30) and (31), can be combined to make one fixed boundary condition which specifies the potential as given by

$$G(y) = 0, \ \ 0 < y < h, \tag{32}$$

where

$$G(y) = \frac{1}{h}\psi_0(y) + \frac{1}{h}\sum_{n=0}^{\infty} A_n \psi_n(y), \quad \text{on } y \in \bar{L},$$

and

$$G(y) = \sum_{n=0}^{\infty} A_n (k_n h) \frac{1}{h}\psi_n(y), \quad \text{on } y \in L.$$

The relation (32) represents an overdetermined system of equations which can be solved by applying least-squares method which requires

$$\text{Error} = \left(\int_o^h |G(y)|^2 \mathrm{d}y\right)^{1/2} = \left(\int_{y \in \bar{L}} |G(y)|^2 \mathrm{d}y + \int_{y \in L} |G(y)|^2 \mathrm{d}y\right)^{1/2} \tag{33}$$

to be minimum.
This error will be minimum when

$$\int_{y \in \bar{L}} G^*(y) \frac{\partial G(y)}{\partial A_m} \mathrm{d}y + \int_{y \in L} G^*(y) \frac{\partial G(y)}{\partial A_m} \mathrm{d}y = 0, \ \ \mathrm{m} = 0, 1, 2, \ldots \tag{34}$$

where $G^*(y)$ is the complex conjugate of $G(y)$.
Substituting the expressions of $G(y)$, $G^*(y)$ and their derivatives, we get

$$A_m - \sum_{n=0}^{\infty} A_n^* c_{nm} \left[1 - (k_m h)(k_n^* h)\right] = \delta_{0m} - c_{0m} \quad (\mathrm{m} = 0, 1, 2, \dots), \tag{35}$$

where

$$c_{nm} = \frac{1}{h} \int_d^h \psi_n(y)\psi_m(y)\mathrm{d}y = \delta_{nm} - \frac{1}{h} \int_0^d \psi_n(y)\psi_m(y)\mathrm{d}y.$$

Truncating the series for n and m, the system given by relation (35) can be solved numerically for $N + 1$ equations with $N + 1$ unknowns A_n.

The BVP-II which involves $\phi_0(x, y)$ the solution of BVP-I, represents the radiation problem. On applying Green's integral theorem to the functions $\phi_0(x, y)$ and $\phi_1(x, y)$ on the region bounded by

$$y = 0, 0 < x \le X; x = 0^+, 0 \le y \le d; x = 0^-, 0 \le y \le d; y = 0, -X \le x < 0;$$

$$x = -X, 0 \le y \le h; y = h, -X \le x \le X; x = X, 0 \le y \le h;$$

where X is positive, large, and tends to infinity, we obtain

$$R_1 = \frac{1}{2i\hat{k_0}} \int_{-\infty}^{\infty} c(x) \Big(\frac{\partial \phi_0(x, h)}{\partial x}\Big)^2 \mathrm{d}x. \tag{36}$$

Similarly, applying Green's integral theorem to the functions $\phi_0(-x, y)$ and $\phi_1(x, y)$ in the same region, we have

$$T_1 = -\frac{1}{2i\hat{k_0}} \int_{-\infty}^{\infty} c(x) \Big(\frac{\partial \phi_0(x, h)}{\partial x}\Big)\Big(\frac{\partial \phi_0(-x, h)}{\partial x}\Big) \mathrm{d}x. \tag{37}$$

These R_1 and T_1 can be evaluated when the shape function $c(x)$ is known.

3.1 Particular Cases

Case (i): In the absence of vertical barrier, the problem assumed here will be the problem involving scattering of water waves by bottom undulation only, and in this