INDUSTRIAL MOTION CONTROL
MOTOR SELECTION, DRIVES,
CONTROLLER TUNING,
APPLICATIONS

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WILEY
## Contents

Preface ix  

1 Introduction 1  
1.1 Components of a Motion Control System 3  
\hspace{0.5cm} 1.1.1 Human–Machine Interface 3  
\hspace{0.5cm} 1.1.2 Motion Controller 4  
\hspace{0.5cm} 1.1.3 Drives 6  
\hspace{0.5cm} 1.1.4 Actuators 7  
\hspace{0.5cm} 1.1.5 Transmission Mechanisms 7  
\hspace{0.5cm} 1.1.6 Feedback 7  
References 9  

2 Motion Profile 11  
2.1 Kinematics: Basic Concepts 11  
2.2 Common Motion Profiles 15  
\hspace{0.5cm} 2.2.1 Trapezoidal Velocity Profile 15  
\hspace{0.5cm} 2.2.2 S-curve Velocity Profile 21  
2.3 Multiaxis Motion 28  
\hspace{0.5cm} 2.3.1 Slew Motion 28  
\hspace{0.5cm} 2.3.2 Interpolated Motion 29  
Problems 30  
References 34  

3 Drive-Train Design 35  
3.1 Inertia and Torque Reflection 36  
\hspace{0.5cm} 3.1.1 Gearbox Ratio 36  
\hspace{0.5cm} 3.1.2 Reflected Inertia 38  
\hspace{0.5cm} 3.1.3 Reflected Torque 39  
\hspace{0.5cm} 3.1.4 Efficiency 39  
\hspace{0.5cm} 3.1.5 Total Inertia 40  
3.2 Inertia Ratio 41  
\hspace{0.5cm} 3.2.1 Targeted Practical Inertia Ratio 43  
3.3 Transmission Mechanisms 43
3.3.1 Load and Inertia Reflection through Transmission Mechanisms 44
3.3.2 Pulley-and-Belt 45
3.3.3 Lead Screw 47
3.3.4 Rack-and-Pinion Drive 52
3.3.5 Belt-Drive for Linear Motion 53
3.3.6 Conveyor 54
3.4 Torque Required for the Motion 56
3.4.1 Acceleration (Peak) Torque 57
3.4.2 Running Torque 57
3.4.3 Deceleration Torque 58
3.4.4 Continuous (RMS) Torque 58
3.5 Motor Torque–Speed Curves 62
3.5.1 Torque–Speed Curves for AC Servomotors 63
3.5.2 Torque–Speed Curves for AC Induction Motors 64
3.6 Motor Sizing Process 67
3.7 Motor Selection for Direct Drive 68
3.8 Motor and Transmission Selection 69
3.9 Gearboxes
3.9.1 Planetary Servo Gearheads 72
3.9.2 Worm Gear SpeedReducers 73
3.10 Servo Motor and Gearhead Selection 75
3.11 AC Induction Motor and Gearbox Selection 89
3.12 Motor, Gearbox, and Transmission Mechanism Selection 96
Problems 100
References 105

4 Electric Motors 107
4.1 Underlying Concepts 107
4.1.1 Electrical and Mechanical Cycles 109
4.1.2 Three-Phase Windings 110
4.2 Rotating Magnetic Field 110
4.2.1 Hall Sensors 110
4.2.2 Six-Step Commutation 111
4.3 AC Servo Motors 114
4.3.1 Rotor 114
4.3.2 Stator 115
4.3.3 Sinusoidal Commutation 119
4.3.4 Torque Generation with Sinusoidal Commutation 123
4.3.5 Six-Step Commutation of AC Servo Motors 124
4.3.6 Motor Phasing with Encoders and Hall Sensors 125
4.4 AC Induction Motors 126
4.4.1 Stator 126
4.4.2 Rotor 127
4.4.3 Motor Operation 128
4.4.4 Constant Speed Operation Directly Across-the-Line 129
4.4.5 Variable Speed Operation with a VFD 131
4.5 Mathematical Models 132
  4.5.1 AC Servo Motor Model 134
  4.5.2 AC Induction Motor Model 140
Problems 145
References 146

5 Sensors and Control Devices 148
  5.1 Optical Encoders 148
    5.1.1 Incremental Encoder 149
    5.1.2 SinCos Encoders 152
    5.1.3 Absolute Encoder 154
    5.1.4 Serial Encoder Communications 157
    5.1.5 Velocity Estimation 160
  5.2 Detection Sensors 162
    5.2.1 Limit Switches 162
    5.2.2 Proximity Sensors 162
    5.2.3 Photoelectric Sensors 163
    5.2.4 Ultrasonic Sensors 164
    5.2.5 The Concept of Sinking and Sourcing 165
    5.2.6 Three-Wire Sensors 167
  5.3 Pilot Control Devices 168
    5.3.1 Push Buttons 169
    5.3.2 Selector Switches 170
    5.3.3 Indicator Lights 170
  5.4 Control Devices for AC Induction Motors 171
    5.4.1 Motor Control Circuit 172
Problems 174
References 175

6 AC Drives 177
  6.1 Drive Electronics 177
    6.1.1 Converter and DC Link 178
    6.1.2 Inverter 180
  6.2 Basic Control Structures 188
    6.2.1 Cascaded Velocity and Position Loops 188
    6.2.2 Single-Loop PID Position Control 192
    6.2.3 Cascaded Loops with Feedforward Control 201
  6.3 Inner Loop 207
    6.3.1 Inner Loop for AC Induction Motors 209
    6.3.2 Inner Loop for AC Servo Motors 210
  6.4 Simulation Models of Controllers 210
    6.4.1 Simulation Model for Vector Control of an AC Induction Motor 211
    6.4.2 Simulation Model for Vector Control of an AC Servo Motor 215
  6.5 Tuning 215
    6.5.1 Tuning a PI Controller 219
    6.5.2 Tuning a PID Position Controller 223
6.5.3 Tuning a Cascaded Velocity/Position Controller with Feedforward Gains 232

Problems 242
References 245

7 Motion Controller Programming and Applications 247
7.1 Move Modes 247
7.1.1 Linear Moves 248
7.1.2 Circular Moves 248
7.1.3 Contour Moves 248

7.2 Programming 249
7.2.1 Motion Programs 250
7.2.2 PLC Functionality 250

7.3 Single-Axis Motion 253
7.3.1 Jogging 254
7.3.2 Homing 254

7.4 Multiaxis Motion 256
7.4.1 Multiple Motors Driving One Axis 256
7.4.2 Coordinated Motion of Two or More Axes 257
7.4.3 Following Using Master/Slave Synchronization 258
7.4.4 Tension Control 273
7.4.5 Kinematics 278

Problems 284
References 288

Appendix A Overview of Control Theory 289
A.1 System Configurations 289
A.2 Analysis Tools 289
A.2.1 Transfer Functions 291
A.2.2 Block Diagrams 292

A.3 Transient Response 293
A.3.1 First-Order System Response 293
A.3.2 Second-Order System Response 293

A.4 Steady-State Errors 297
References 298

Index 299
Preface

Over the past couple of decades, the academic community has made significant advances in developing educational materials and laboratory exercises for fundamental mechatronics and controls education. Students learn mathematical control theory, board-level electronics, interfacing, and microprocessors supplemented with educational laboratory equipment. As new mechanical and electrical engineering graduates become practicing engineers, many are engaged in projects where knowledge of industrial motion control technology is an absolute must since industrial automation is designed primarily around specialized motion control hardware and software.

This book is an introduction to industrial motion control, which is a widely used technology found in every conceivable industry. It is the heart of just about any automated machinery and process. Industrial motion control applications use specialized equipment and require system design and integration where control is just one aspect. To design such systems, engineers need to be familiar with industrial motion control products; be able to bring together control theory, kinematics, dynamics, electronics, simulation, programming and machine design; apply interdisciplinary knowledge; and deal with practical application issues. Most of these topics are already covered in engineering courses in typical undergraduate curricula but in a compartmentalized nature, which makes it difficult to grasp the connections between them.

As I wrote this book, my goal was to bring together theory, industrial machine design examples, industrial motion control products and practical guidelines. The context of studying industrial motion control systems naturally brought separately taught topics together and often crossed disciplinary lines. The content came from my personal experience in developing and teaching mechatronics and automation courses, working with undergraduate students and from many discussions with engineers in the motion control industry. For example, even though many types of motors are available, I chose to concentrate on three-phase AC servo and induction motors based on input from the motion control industry. By no means this is a comprehensive book on any of the topics covered. It is not an in-depth examination of control theory, motor design, or power electronics. Rather, it is a balanced coverage of theory and practical concepts. Much of this material is available in manufacturer data sheets, manuals, product catalogs, fragments in various college courses, websites, trade magazines, and as know-how among practicing engineers. The book presents these pieces in a cohesive way to provide the fundamentals while supplementing them with solved examples based on practical applications.
The book starts with an introduction to the building blocks of a typical motion control system in Chapter 1. A block diagram is provided and the basic function of each building block is explained.

Chapter 2 examines how the motion profile is generated when an axis of a machine makes a move. After an overview of basic kinematics, two common motion profiles are explained. The chapter concludes with two approaches for multiaxis coordination.

As the mechanical design of each axis and the overall machine are significant factors in achieving the desired motion, Chapter 3 focuses on drive-train design. Concepts of inertia reflection, torque reflection, and inertia ratio are introduced. Five types of transmission mechanisms are explored in depth. Torque–speed curves of motors, gearboxes, and motor selection procedures for different types of motors and axes with transmission mechanisms are provided.

Electric motors are by far the most commonly used actuators in industrial motion control. Chapter 4 begins with fundamental concepts such as electrical cycle, mechanical cycle, poles, and three-phase windings. Construction and operational details of AC servo and induction motors are provided. Torque generation performance of AC servo motors with sinusoidal and six-step commutation is compared. The chapter concludes with mathematical and simulation models for both types of motors.

Motion control systems employ an assortment of sensors and control components along with the motion controller. Chapter 5 starts with the presentation of various types of optical encoders for position measurement, limit switches, proximity sensors, photoelectric sensors, and ultrasonic sensors. Sinking or sourcing designations for sensor compatibility to I/O cards are explained. Next, control devices including push buttons, selector switches, and indicator lights are presented. The chapter concludes with an overview of motor starters, contactors, overload relays, soft-starters, and a three-wire motor control circuit.

A drive is the link between the motor and the controller. It amplifies small command signals generated by the controller to high-power voltage and current levels necessary to operate a motor. Chapter 6 begins by presenting the building blocks of drive electronics. The popular pulse width modulation (PWM) control technique is explained. Then, basic closed-loop control structures implemented in the drive are introduced. Single-loop PID position control and cascaded velocity and position loops with feedforward control are explored in depth. Mathematical and simulation models of the controllers are provided. Control algorithms use gains that must be tuned so that the servo system for each axis can follow its commanded trajectory as closely as possible. The chapter concludes by providing tuning procedures for the control algorithms presented earlier and includes practical ways to address integrator saturation.

The book concludes with Chapter 7, which is about programming and motion control applications. Linear, circular, and contour move modes of a motion controller are explored. The chapter continues by introducing algorithms for basic programmable logic controller (PLC) functionality that are commonly used in motion controller programs. The chapter concludes by reviewing how a motion controller can control a non-Cartesian machine, such as a robot, by computing its forward and inverse kinematics in real-time.

One of the challenges in writing a book like this is the variety of motion controller hardware and software in the market and their proprietary nature. Each controller manufacturer has its own programming language and programming environment for their products. Since the programming details are very specific to each hardware, I attempted to provide algorithmic outlines rather than complete programs in a specific programming language or structure. The product manuals and manufacturer suggestions must be closely followed in adapting these
algorithms to a specific choice of motion controller hardware. Another challenge was the digital control systems implemented in the controllers. These sampled data systems would be modeled using the $z$-transform ($z$-domain). However, almost all undergraduate engineering programs include only coverage of the continuous-time systems modeled using the Laplace transform ($s$-domain). Therefore, I chose to use the $s$-domain in presenting the control system models. This approach provides a good approximation since today’s controllers have very fast sampling frequencies and the mechanical systems they control have relatively slow dynamics.

As I presented concepts, especially for the solved drive-train examples, I used data for industrial products from datasheets and catalogs. Today, most of these resources are available on the Internet as provided in the references of the chapters. Over time, manufacturers will change their products and these catalogs may not be available anymore. However, the theoretical coverage and the practical selection procedures should equally apply to the similar, newer future products.

This book is intended to be an introduction to the topic for undergraduate mechanical and electrical engineering students. Since many practicing engineers are involved in motion control systems, it should also be a resource for system design engineers, project managers, industrial engineers, manufacturing engineers, product managers, field engineers, mechanical engineers, electrical engineers, and programmers in the industry. For example, the tuning procedures in Chapter 4 were demonstrated using mathematical simulations. But if a real system with a motion controller is available, then these procedures can simply be used to tune the controller of the real system without the need for any of the simulations. Similarly, Chapter 7 provides algorithms for common types of motion control applications such as winding. These algorithms can be a starting point to develop the control programs in the programming language of the real system. As stated earlier, the product manuals and manufacturer suggestions must be closely followed in adapting these algorithms to a specific choice of real motion controller hardware.

I am indebted to many people who helped me through the journey of writing this book. Many thanks to Mr. Ken Brown, President of Applied Motion Systems, Inc., Mr. Ed Diehl, President of Concept Systems, Inc., for our discussions about motion control systems, for the valuable suggestions and materials they provided. Special thanks goes to Mr. Dimitri Dimitri, President of Delta Tau Data Systems, Inc., and Mr. Curtis Wilson, Vice President of Engineering and Research at Delta Tau Data Systems, Inc. for their contributions to our laboratory, to this book and for their in-depth technical guidance. I would also like to acknowledge the contributions by Mr. Dean Ehnes, Mechanical Engineer, Mr. John Tollefson, Electrical Engineer, and Mr. Brian Hutton, General Manager of Columbia/Okura, LLC through insightful discussions about material handling systems, motors, and for ideas for example problems. My colleague Dr. Xiaodong Liang from electrical engineering provided a detailed review of the chapters on motors and electric drives, which was much appreciated. I received tremendous help from Mr. Ben Spence, my former student from a long time ago and now Systems Engineer at Applied Motion Systems, Inc. I learned a lot from him as we spent countless hours working together on laboratory machines and discussing many technical aspects of motion control. Also, I am grateful to Mr. Matthew Bailie, Mechanical Engineer at Applied Motion Systems, Inc. for his guidance about the motor sizing procedures and for ideas for some of the example problems. Many thanks to the companies that provided the product photographs used in the book. I would like to thank Mr. Paul Petralia, Senior Editor at John Wiley & Sons and the Wiley staffs Sandra Grayson, Clive Lawson and Siva Raman Krishnamoorty, for their guidance
throughout the process of developing this book. I had the privilege of working with many outstanding students who learned this material as I tried it out in courses I taught in the last few years. I really appreciated their valuable feedback, suggestions, patience, and enthusiasm.

As I discovered through this process, writing a textbook is a major undertaking. I am very grateful to my wife for her endless patience, support, and encouragement while I was writing this book for the last 3 years.

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1

Introduction

Motion control is widely used in all types of industries including packaging, assembly, textile, paper, printing, food processing, wood products, machinery, electronics, and semiconductor manufacturing. It is the heart of just about any automated machinery and process. Motion control involves controlling mechanical movements of a load. For example, in case of an inkjet printer, the load is the ink cartridge that has to be moved back-and-forth across the paper with high speed and precision. On the other hand, in a paper-converting machine, the load can be the large parent roll of paper that is loaded into the machine for processing. In this case, the load rotates as the paper is unwound from the roll and rewound into smaller processed rolls such as embossed paper towels.

Each motor moves a segment of the mechanical components of the machine. The segment of the machine along with the motor that moves it is called an axis. Considering an inkjet printer as an example, the mechanical components involved in the sliding motion of the print cartridge and the motor driving them collectively make up an axis of the machine. Another axis of the printer consists of all mechanical components and the motor that feed paper into the printer. In case of the paper-converting machine, the mandrel that holds the roll of paper, the pulleys, and belts that connect it to the motor and the motor make up an axis.

A typical motion control system manages position, velocity, torque, and acceleration of an axis. Often the machine consists of multiple axes whose position and/or velocity must be controlled in a synchronized fashion. For example, the X-axis and Y-axis of the table of a CNC machine need to be controlled in a coordinated way so that the machine can cut a round corner in the work piece. The ability to precisely control and coordinate complex motions of multiple axes enables design of industrial machines such as those in Figure 1.1.

Prior to the programmable motion controllers, coordination was achieved through mechanical means [14]. A central line shaft was connected to a large electric motor or an engine that ran at constant speed. This motion source was then used to drive all the axes of the machine by coupling them to the line shaft through pulleys, belts, gears, cams, and linkages (Figure 1.2). Clutches and brakes were used to start or stop the individual axes. The gear ratios between the line shaft and the individual axes determined the speed of each axis. Drive-trains, which were often long shafts, transferred the coordinated motion to the appropriate part of the
Complex machines with multiple axes are made possible with the ability of the controller to precisely coordinate motion of all axes. (a) Foil and wire winding machine (Reproduced by permission of Broomfield, Inc. [3]). (b) Pressure sensitive labeling machine (Reproduced by permission of Tronics America, Inc. [19]).

Machine. Complex machinery required sophisticated mechanical designs. Backlash, wear, and deflections in the long shafts were problematic. The biggest challenge was when a change in the product had to be introduced into the production system. It required physically changing the gear reducers, which was costly and very time consuming. Also, realigning the machine for accurate timing was difficult after drive-train changes.

As computers became main-stream equipment through the inexpensive availability of electronics, microprocessors, and digital signal processors, coordination of motion in multiaxes
machines began to shift into a computer-controlled paradigm. In a modern multiaxis machine, each axis has its own motor and electric drive. Coordination between the axes is now achieved through electronic gearing in software. The drive-trains with long shafts are replaced by short and much more rigid shafts and couplings between the motor and the mechanism it drives. The motion controller interprets a program and generates position commands to the drives of the axes. These motion profiles are updated in real-time as the drives commutate the motors and close the control loops. In today’s technology it is typical for an ordinary motion controller to coordinate up to eight axes at a time. Controllers with 60+ axis capabilities are available.

### 1.1 Components of a Motion Control System

The complex, high-speed, high-precision control required for the multiaxis coordinated motion is implemented using a specialized computer called **motion controller**. As shown in Figure 1.3, a complete motion control system consists of:

1. Human–machine interface (HMI),
2. Motion controller
3. Drives
4. Actuators
5. Transmission mechanisms

#### 1.1.1 Human–Machine Interface

The HMI is used to communicate with the motion controller. The HMI may serve two main functions: (i) Operating the machine controlled by the motion controller, and (ii) Programming the motion controller.

Control panels as shown in Figure 1.4a with pilot lights, push buttons, indicators, digital readouts, and analog gauges are common hardware-based HMIs to serve the purpose of operating a machine. Chapter 5 discusses operator interface devices such as pilot lights, push buttons,
1.1.2 Motion Controller

The motion controller is the “brains” of the system. It generates motion profiles for all axes, monitors I/O, and closes feedback loops. As presented in Chapter 2, the controller generates the motion profile for an axis based on the desired motion parameters defined by the user or the programmer. While the machine is running, it receives feedback from each axis motor. If there is a difference (following error) between the generated profile and the actual position or
velocity of an axis, the controller generates correction commands, which are sent to the drive for that axis. Chapter 6 discusses various control algorithms used to act on the following error to generate command signals to eliminate the error. As discussed in Chapter 7, the controller can also generate and manage complex motion profiles including electronic camming, linear interpolation, circular interpolation, contouring, and master–slave coordination.

Motion controllers are available in different form factors (Figure 1.5). The integrated form factor incorporates the computer, the drive electronics for the axes, and the machine I/O into a single unit. This unit is called motion controller or drive. In a modular system, the computer, the drives, and the machine I/O are separate units connected to each other via some type of communication link. In this case, just the computer is called the motion controller.

A complete motion controller consists of the following:

1. Computer
   - Interpretation of user programs
   - Trajectory generation
   - Closing the servo loops
   - Command generation for the drives (amplifiers)
   - Monitoring axis limits, safety interlocks
   - Handling interrupts and errors such as excessive following (position) error.

**Figure 1.5** Motion controllers can have the various form factors. (a) Integrated form factor (Reproduced by permission of Delta Tau Data Systems, Inc. [5]). (b) Modular form factor. (Left) motors and drives [16], (Right) controller [17] (Courtesy of Rockwell Automation, Inc.)
2. I/O for each axis
   - Motor power output
   - Servo I/O for command output to amplifiers
   - Input terminals for feedback signals from motor or other external sensors
   - Axis limits, homing signals, and registration.

3. Machine I/O
   - Digital input terminals for various sensors such as operator buttons and proximity sensors
   - Digital output terminals to drive external devices (usually through relays)
   - Analog inputs (often optional) for analog sensors such as pressure, force
   - Analog outputs (often optional) to drive analog devices.

4. Communication
   - Network communications with other peripheral devices, the host computer, and/or supervisory system of the plant using protocols such as DeviceNet®, Profibus®, ControlNet®, EtherNet/IP®, or EtherCAT®
   - USB or serial port communications, and
   - HMI communications.

1.1.3 Drives

The command signals generated by the controller are small signals. The drive (Figure 1.6) amplifies these signals to high-power voltage and current levels necessary to operate a motor. Therefore, the drive is also called an amplifier. The drive closes the current loop of the servo system as discussed in Chapter 6. Therefore, it must be selected to match the type of motor

![Figure 1.6](image)

**Figure 1.6** Drives are used to provide high voltage and current levels necessary to operate motors. (a) Digital servo drive (Reproduced by permission of ADVANCED Motion Controls®) [1]. (b) AC drive (Courtesy of Rockwell Automation, Inc.) [18]
to be driven. In recent trends, the line between a drive and a controller continues to blur as the drives perform many of the complex functions of a controller. They are expected to handle motor feedback and not only close the current but also the velocity and position loops.

### 1.1.4 Actuators

An actuator is a device that provides the energy to move a load. Motion control systems can be built using hydraulic, pneumatic, or electromechanical (motor) technologies. This book presents three-phase AC servo and induction motors (Figure 1.7). Underlying concepts of electromechanical operation of these motors along with mathematical models are presented in Chapter 6. Specific control algorithms implemented in the drive to control each type of motor are explored in Chapter 6. When a machine for a motion control application is designed, motors must be carefully selected for proper operation of the machine. Chapter 3 presents torque–speed curves for each of these motors and design procedures for proper motor sizing.

### 1.1.5 Transmission Mechanisms

A transmission mechanism is used to connect the load to the motor of an axis. It helps meet the motion profile requirements. Chapter 3 presents gearboxes (Figure 1.8), lead/ball-screw drives, linear belt drives, pulley-and-belt drives, and conveyors. When a load is coupled to a motor through a transmission mechanism, the load inertia and torque are reflected through the mechanism to the motor. Chapter 3 provides extensive discussion on the mathematical models for this. Motor, gearbox, and transmission selection procedures are also provided.

### 1.1.6 Feedback

Feedback devices are used to measure the position or speed of the load. Also, the drive and the controller use feedback to determine how much current needs to be applied to each phase of the motor as explained in detail in Chapters 4 and 6. Most common feedback devices are resolvers, tachometers, and encoders. Chapter 5 explains encoders, which can be rotary or

![Figure 1.7](image-url) **Figure 1.7** AC servo and induction motors are used in motion control applications as actuators. (a) AC servo motors (Reproduced by permission of Emerson Industrial Automation [8]). (b) AC induction motor (Reproduced by permission of Marathon™ Motors, A Regal Brand) [12]
Gearboxes are used in motion control applications to help achieve speed and torque requirements. (a) In line gearhead for servo motors (Reproduced by permission of DieQua Corp. [6]). (b) Right-angle worm gear reducer for AC induction motors (Reproduced by permission of Cone Drive Operations, Inc. [4])

Encoders are used in motion control applications as feedback devices. (a) Rotary encoder (Reproduced by permission of US Digital Corp. [20]). (b) Linear encoder (Reproduced by permission of Heidenhain Corp. [9])

Linear as shown in Figure 1.9. In addition, encoders can be incremental or absolute. Selection of the feedback device depends on the desired accuracy, cost, and environmental conditions of the machine.

A different type of feedback is provided to the controller from detection sensors such as proximity switches, limit switches, or photoelectric sensors. These devices detect presence or absence of an object. For example, a photoelectric sensor such as the one in Figure 1.10 may detect arrival of a product on a conveyor and signal the motion controller to start running the conveyor.
Introduction

Figure 1.10  Photoelectric sensors are used for detection of presence of an object [11]

References


Motion Profile

A moving object follows a trajectory. In an automatic machine, motion may involve a single axis moving along a straight line. In more complex cases, such as moving the cutting tool of a CNC milling machine along a circular path, coordinated motion of multiple axes is required.

When an axis of the machine needs to move from point “A” to “B”, a trajectory connecting these points needs to be generated. In motion control, the trajectory is also called motion profile. The motion profile should lead to smooth acceleration of the axis from point “A” to a constant operational speed. After moving at this speed for a while, the axis needs to smoothly decelerate to come to a stop at point “B”.

Motion controller generates the motion profile at regular intervals to create velocity and position commands for the servo control system of each motor. Each servo control system then regulates its motor signals to move its axis along the desired profile.

This chapter begins with an overview of basic kinematics. Then, the two most common motion profiles, namely trapezoidal and S-curve velocity profiles, are presented. The chapter concludes with a discussion of the slew and interpolated motion approaches for multiaxis coordination.

2.1 Kinematics: Basic Concepts

Kinematics is the study of motion without considering the forces causing that motion. It governs the relationships between time, position, velocity, and acceleration. Studying the kinematics of a machine is needed for motion profile calculations but also in choosing the right motors for the axes during the design of the machine.

As an axis is moving from point “A” to “B”, its position, \( s(t) \), along the trajectory is a function of time. Velocity, \( v(t) \), is the change of position \( s(t) \) in a given time interval. It is defined as

\[
v(t) = \frac{ds}{dt}
\]

Similarly, acceleration, \( a(t) \), is the change of velocity \( v(t) \) in a given time interval:

\[
a(t) = \frac{dv}{dt}
\]
These equations can also be written in the integration form as

\[ s = \int v(t) \, dt \quad (2.2) \]
\[ v = \int a(t) \, dt \quad (2.3) \]

Recall that integration of a function is the summation of infinitesimal elements under the curve of the function. Therefore, it is equal to the area under the curve. Then, from Equation (2.2), position at time \( t \) is equal to the area under the velocity curve up to time \( t \) as shown in Figure 2.1.

Slope of a curve at a point can be found using differentiation. Then, from Equation (2.1) acceleration is the slope of the velocity curve as shown in Figure 2.1.

![Figure 2.1 Basic relationships between position, velocity, and acceleration](image_url)
Geometric Rules for Motion Profile

1. Position at time $t$ is equal to the area under the velocity curve up to time $t$
2. Acceleration is the slope of the velocity curve

For a more general case, the following can be derived from Equations (2.2) and (2.3):

$$v = v_0 + a(t - t_0)$$
$$s = s_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$  \hspace{1cm} (2.4)

where $t_0$ is the initial time, $v_0$ is the initial velocity, and $s_0$ is the initial position. The acceleration “$a$” is constant.

**EXAMPLE 2.1.1**

Given the velocity profile in Figure 2.2, find the position and acceleration at $t = 5$ s.

*Figure 2.2 Velocity profile for acceleration*
Slope of the velocity profile is the acceleration. Therefore,

\[
a = \frac{10}{5} = 2 \text{ in/s}^2
\]

The triangular area under the velocity curve up to \( t = 5 \) s is the position reached at \( t = 5 \) s. Hence,

\[
s = \frac{1}{2} (10 \cdot 5)
\]

\[
= 25 \text{ in/s}
\]

**EXAMPLE 2.1.2**

An axis is traveling at a speed of 10 in/s. At \( t = 5 \) s it starts to slow down as given by the velocity profile in Figure 2.3. What is the axis position when it stops? Assume that the axis starts decelerating at 25 in.

![Figure 2.3 Velocity profile for deceleration](image-url)
Solution
Slope of the velocity profile is the acceleration. In this case, the slope is negative since the axis is decelerating. Therefore,

\[ a = \frac{-10}{10} = -1 \text{ in/s}^2 \]

The triangular area under the velocity curve is the position reached at \( t = 15 \text{ s} \). Hence,

\[ \Delta s = \frac{1}{2} \cdot 10(15 - 5) = 50 \text{ in} \]

The axis travels 50 in while it is slowing down. Since it started its deceleration at 25 in, the axis will be at \( s = 75 \text{ in} \) when it stops.

2.2 Common Motion Profiles
There are two commonly used motion profiles:

- Trapezoidal velocity profile
- S-curve velocity profile.

The trapezoidal velocity profile is popular due to its simplicity. The S-curve velocity profile leads to smoother motion.

2.2.1 Trapezoidal Velocity Profile
Figure 2.4 shows the trapezoidal velocity profile, the resulting acceleration, and position profiles. It also shows the jerk (also called jolt), which is the derivative of acceleration. As it can be seen, the jerk is infinite at four points in this motion profile due to the discontinuities in the acceleration at corners of the velocity profiles. There are three distinct phases in the motion: (1) acceleration, (2) constant velocity (zero acceleration), and (3) deceleration.

To move an axis of the machine, usually the following desired motion parameters are known:

- Move velocity \( v_m \)
- Acceleration \( a \)
- Distance \( s \) to be traveled by the axis.

The desired motion profile can be programmed into the motion controller by first specifying the move velocity and move time for the motion. Then, the program commands the axis to move through distance \( s \).
2.2.1.1 Geometric Approach

To calculate the move time $t_m$, we can apply the geometric rules to Figure 2.4 starting with the slope of the velocity curve $a = \frac{v_m}{t_a}$:

$$t_a = t_d = \frac{v_m}{a} \quad (2.5)$$

Figure 2.4  Trapezoidal velocity profile and associated position, acceleration, and jerk profiles to move an axis from 0 to position L